### Course 7: Reduction for reanalysis

- Reanalysis: section 10.4
- Start with motivation
- Then on blackboard
  - Parameter, nominal model selection
  - $b_p$  residual vector columns of  $\Delta K$  or  $\Delta KT_0$
  - Complex modulus example
  - Residue iteration
  - Issues with reduction
- Back with a few illustrations

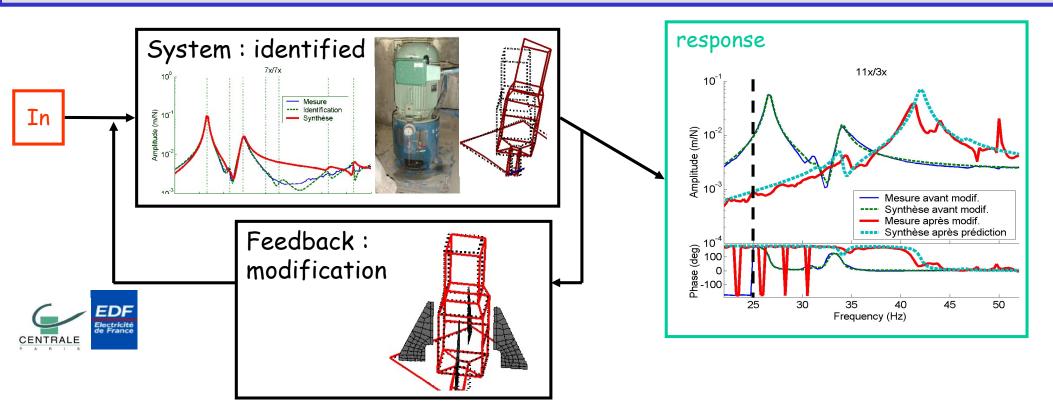






http://savoir.ensam.eu/moodle/course/view.php?id=1874 http://savoir.ensam.eu/moodle/course/view.php?id=490

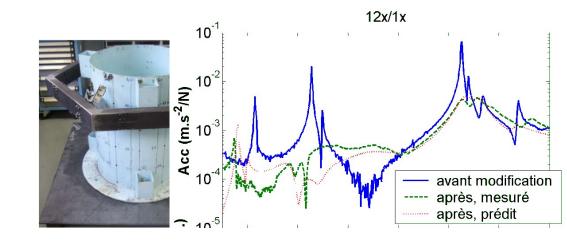
### Reanalysis: model 1 config -> prediction other



Test: example

SDM: structural dynamics modification

- distributed mass, stiffness or damping modifications
- Restricted to low frequency global modes



### Reanalysis: computational example

Environment

Design point

System States

#### Squeal applications

- 8-15 components
- Multiple interfaces/parameters
- 300-600 modes

Design exploration 1000 points

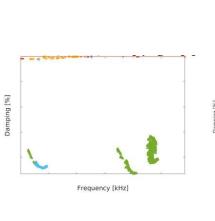
Full 80 days CPU, 22 TB

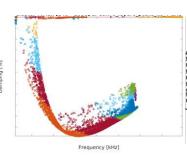
CMT a few hours off-line learning, <1h exploration</li>

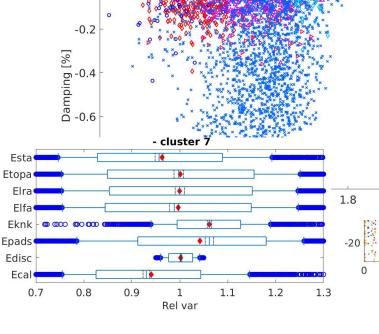
Exploitation

DOE: basic edges, LHS

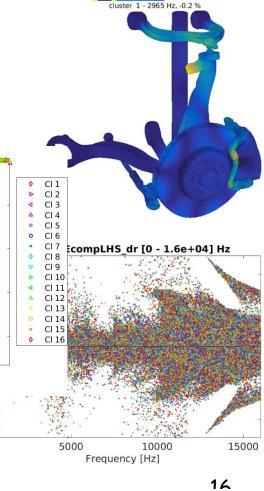
Clustering & parameters





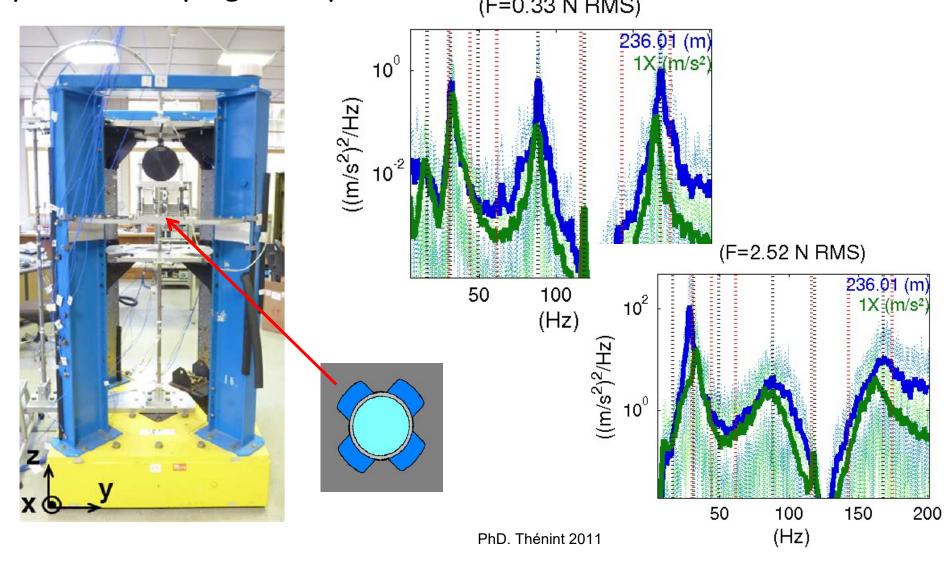


Cluster MMek Iwtol1e-2

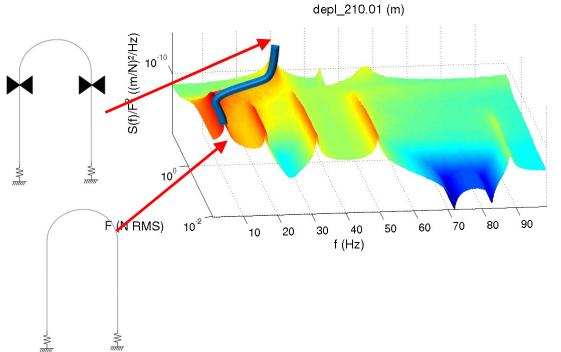


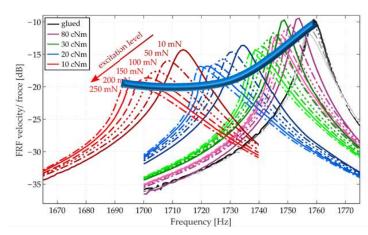
## Energy coupling & NL system

Bending beam with gap & contact
Apparent damping with pure contact
(F=0.33 N RMS)

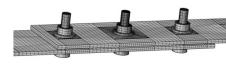


## Damping occurs in the transition



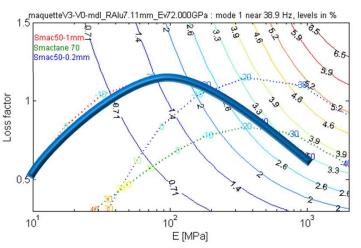


FEMTo-ST Orion Testbed





- Contact with broadband excitation level
- Friction: amplitude/normal force dependence
- Viscoelasticity: temperature/frequency dependence



ENSAM/PhD. Hammami 2014 Hammami/Balmes/Guskov, MSSP 2015



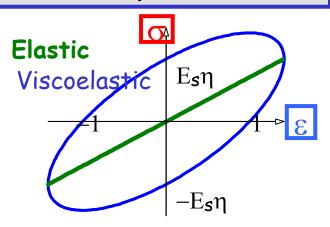


### Dissipation models: viscoelasticity

- Viscoelasticity (Bolzmannian material)
- Stress function of strain history independent of amplitude
- Time convolution  $\sigma(t) = \int_{-\infty}^{t} C(\tau) \epsilon(t \tau)$
- Frequency complex modulus  $\sigma = E(\omega, T, \epsilon_S)\epsilon$

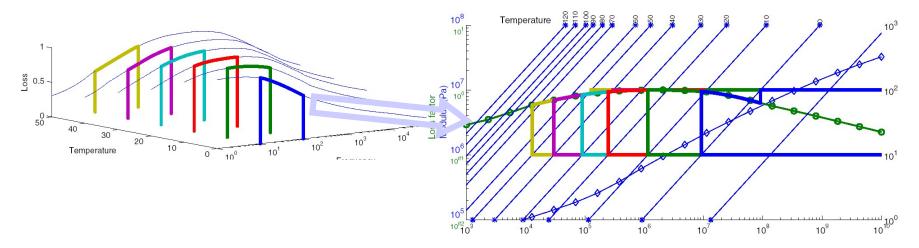
$$E = E'(1 + i\eta)$$

E' storage modulus,  $\eta$  loss factor

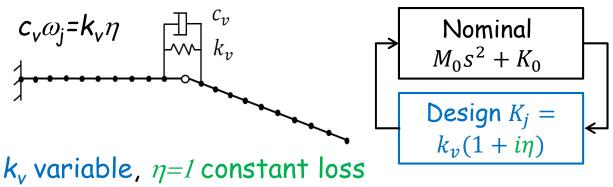


Temperature/frequency equivalence

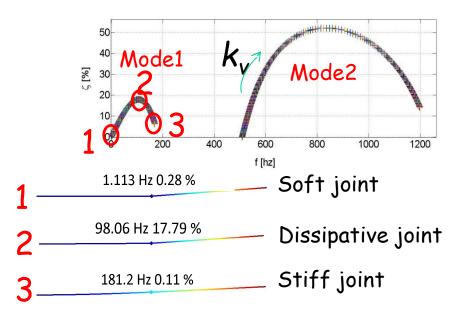
$$E(\omega, T, \epsilon_S) = E(\omega \alpha(T, \epsilon_S))$$



## A mechanical root locus (coupling)



Output :  $f(k_v, \eta) \zeta(k_v, \eta)$ 



Damping can only exist if joint is « working »



Energy fraction in joint must be  $\ll$  sufficient  $\gg$  (depends on  $k_v$ )

Related ideas: electro-mechanical coupling coefficient, pole/zero distance, modal strain energy

#### On blackboard

- Parameter, nominal model selection
- $b_p$  residual vector columns of  $\Delta K$  or  $\Delta K T_0$
- Complex modulus example
- Residue iteration
- Issues with reduction

Related but not perfectly matching: section 5.4

#### Families of reduced models

 $\begin{array}{ll} \textbf{p} & \text{physical parameters (geometry, constit.)} \\ Z(\textbf{p.s}) = [M(\textbf{p}) \textbf{s}^2 + \textbf{K}(\textbf{p})] & \text{finite element model} \\ Z_R(\textbf{p.s}) = \textbf{J}^T Z T & \text{reduced dynamic model} \\ T(\textbf{p}) = [\phi_1 \dots \phi_{NR \ K} \ K^{-1} \textbf{b}] & \text{modal model} \\ \{y\} = [H] \{u\} & \text{transfer functions} \end{array}$ 

Kinematic reduction  $\Leftrightarrow$  choice fixed Ritz basis  $\{q\} = [T]\{q_R\}$ 

### Response surface / reduction

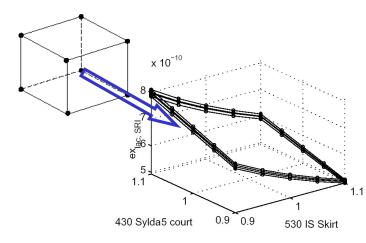
All predict I/O relation but

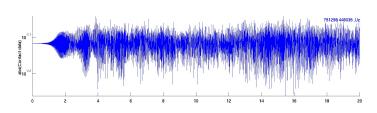
Response surface methodologies

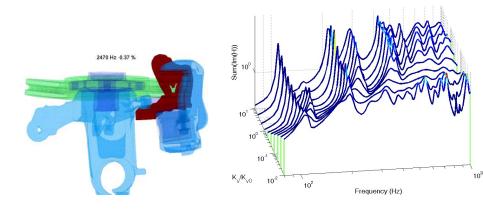
- no knowledge of internal state
   PGD (higher dimension variable separation)
- time and space are coupled
   Fixed basis reanalysis
- Response surface for system matrices
   T<sup>T</sup>Z(p)T≈f(p,T<sup>T</sup>M<sub>i</sub>T)



- still dynamic model
- restitution  $\{q\}=[T]\{q_R\}$ knowledge of all internal states







#### First order enhancement to MSE

 $[Z(E(s),s)] \{q\} = \{F\}$  Damped viscoelastic resp. rewritten as

$$[Z(E_o,s)]{q} = {F}-[\sum_{(E(s)-E_o)/E_o}[Im(Z-Z_o)]]{q}$$

Tangent system, internal loads

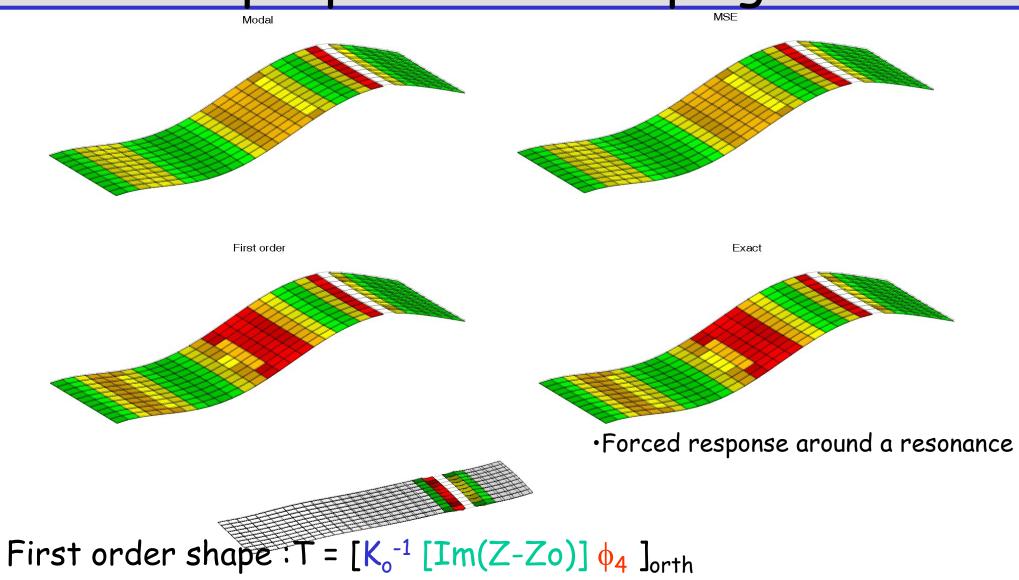
#### Basis contains

- Modes to represent nominal resonances
- ·Flexibility to viscoelastic loads associated with nominal modes

$$T = \begin{bmatrix} \phi_{1:NM} & K_o^{-1} & Im(Z-Z_o) \end{bmatrix} \phi_{1:NM}$$
Modes static response to unit load
MSE

Principle of reduction (assumptions on excitation space & freq) unchanged

## Non proportional damping model



### Reanalysis

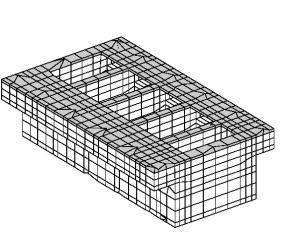
Reanalysis Multi-model bases

$$\left[\left[T^TK(p)T\right] - \omega_{jR}^2(p)\left[T^TM(p)T\right]\right]\left\{\phi_{jR}(p)\right\} = \left\{0\right\}$$

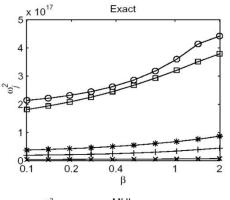
•Modes  $T=[\Phi(p_0)]$ 

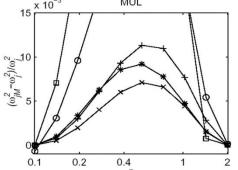
•Multi-model  $T=[\Phi(p_1) \Phi(p_2)]$ 

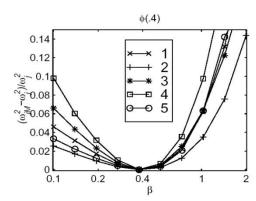
•Modes + sensitivities  $T=[\Phi(p_0) \partial \Phi(p_0)/\partial p]$ 

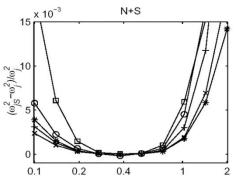


Frequencies



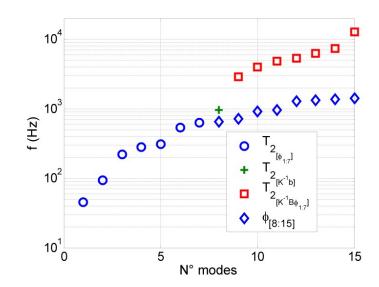




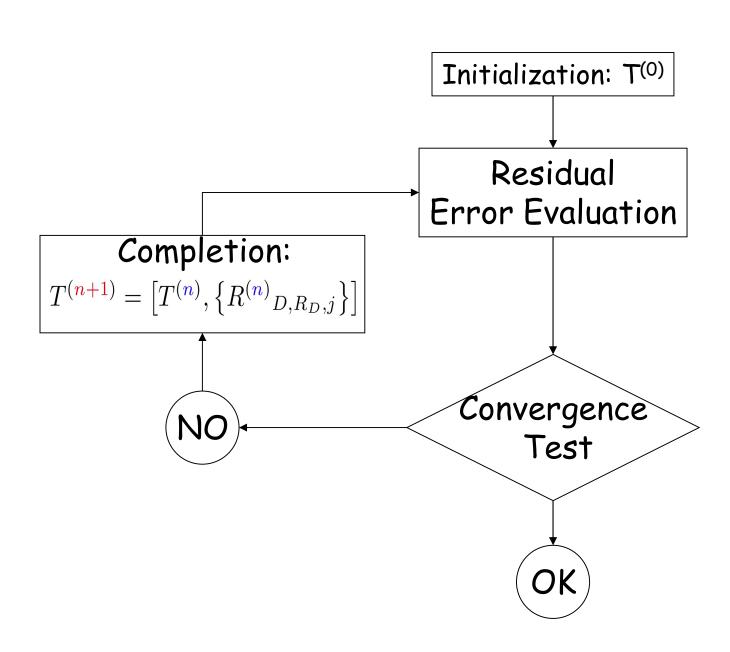


# Reduction Bases/Starting Point

- Modal  $T=[\Phi(p_0)]$
- Modal + static Responses to input  $T=[\Phi(p_0) K^{-1}b]$
- Modal + static responses to representative loads  $T=[\Phi(p_0) \ K^{-1}[b \ R_j] \ \Phi(p_k)]$
- Frequencies of corrections □
   differ from modes ◆: the added
   information is VERY different



#### Iterative Procedure

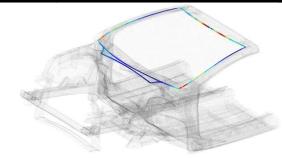


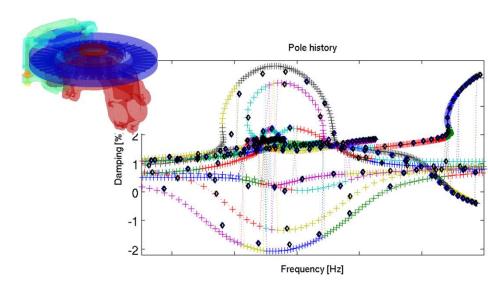
## Fixed basis: enormous cost reduction

- Windshield joint complex modes at 500 design points for ½ cost of direct solver
- Campbell diagram: 200 rotations speeds for the cost of 4.
- Squeal instabilities as function of pressure: few pressures sufficient for interpolation

14	1.000			
12				
10 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0 0				
Freduency 8	00000000000000000000000000000000000000	3-0-0-0-1 3-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0-0	7	
2	******	Full assembly Multi-model		
0 2	4 6 Rotation speed	8	10	

Ψ,λ	SOL107	2200s
Φ,ω	SOL103	300s
Ψ,λ Reduced	First order Error <4%	490s
Ψ,λ(500*Τ)	SOL107	~ 12 days
Ψ,λ(500*T) reduced	First order Error small	~1000s



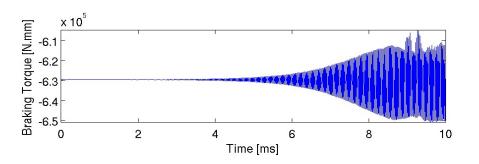


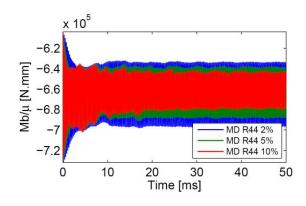
### Reanalysis: squeal example

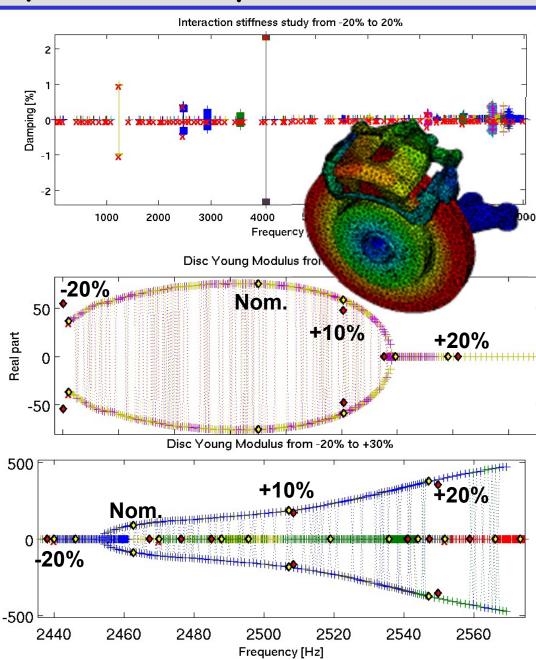
Real part

### Objectives:

- complex mode stability
- · Transient simulation







## Other applications

- Multi-stage cyclic symmetry (SNECMA).
  - Which stage, which diameter, ...
  - Mistuning (which blade)
- Damping design (PSA)
  - Fixed system modes, component redesign

