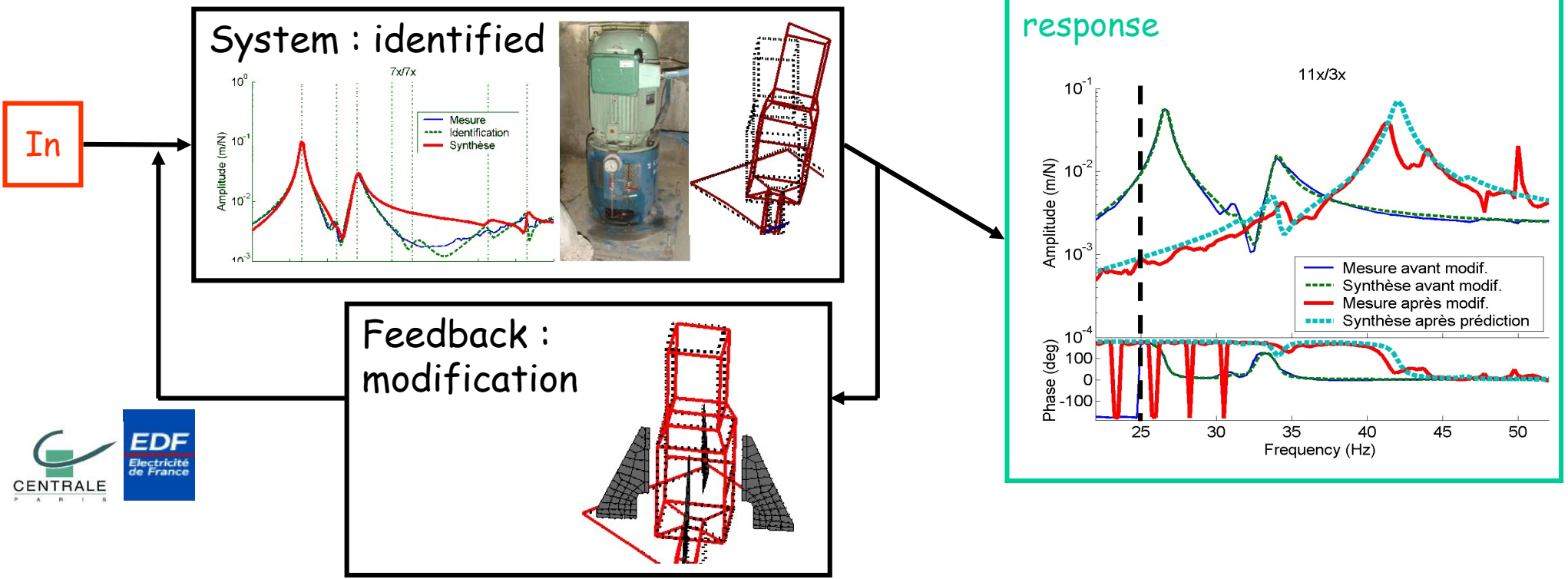


Course 7 : Reduction for reanalysis

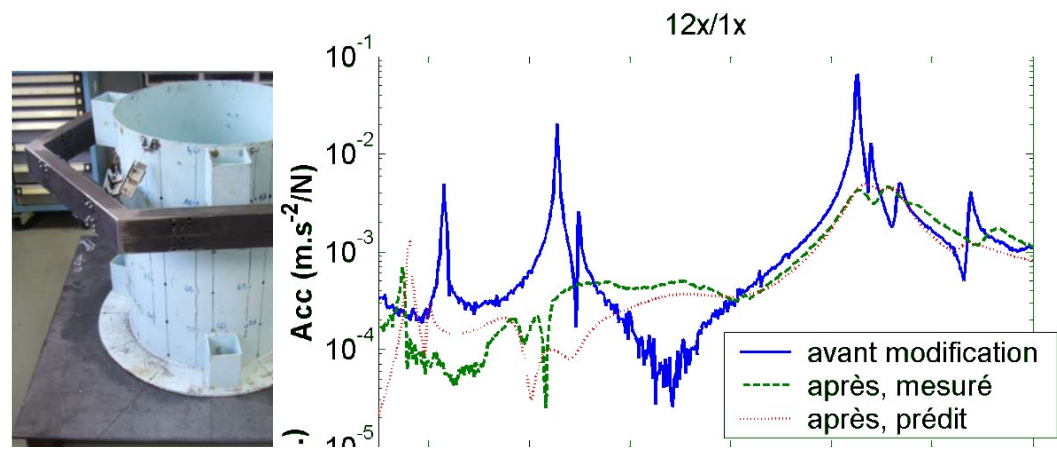
- Reanalysis : section 10.4
- Start with motivation
- Then on blackboard
 - Parameter, nominal model selection
 - b_p residual vector columns of ΔK or $\Delta K T_0$
 - Complex modulus example
 - Residue iteration
 - Issues with reduction
- Back with a few illustrations

Reanalysis : model 1 config -> prediction other



Test : example

- SDM : structural dynamics modification
- distributed mass, stiffness or damping modifications
 - Restricted to low frequency global modes



Reanalysis : computational example

Squeal applications

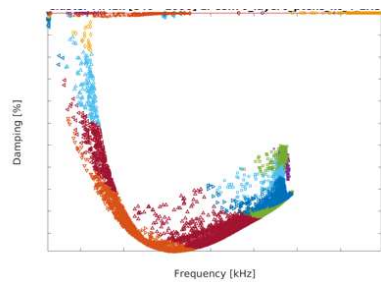
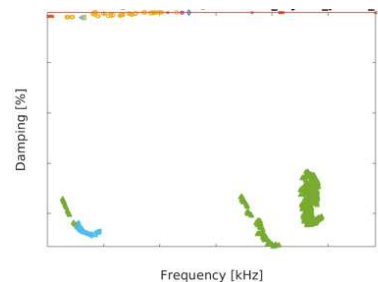
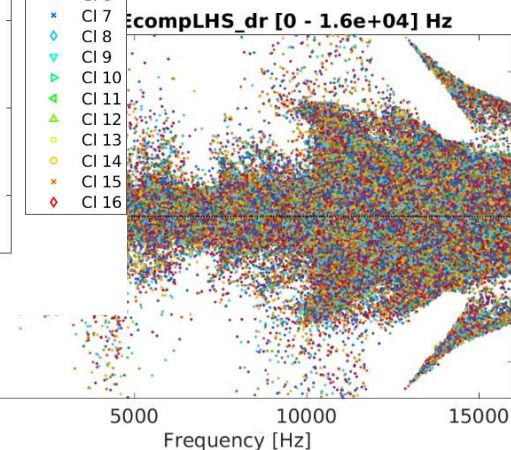
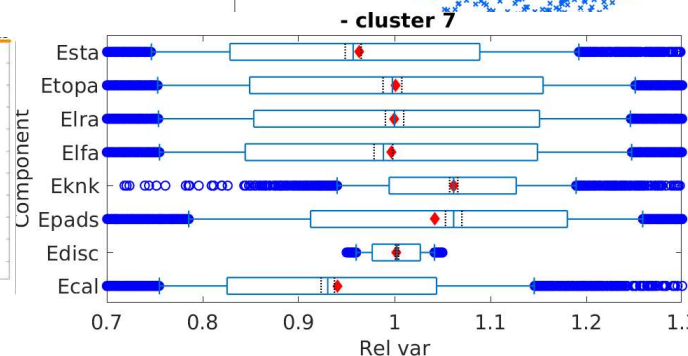
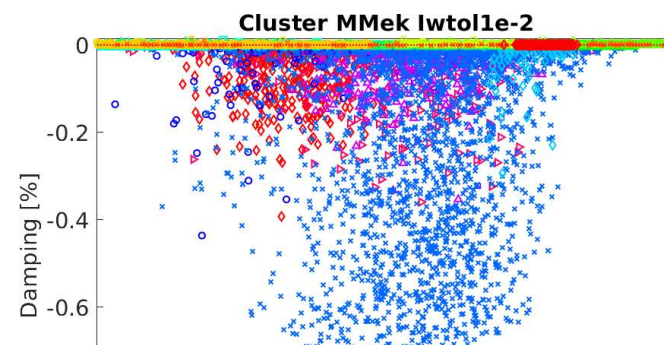
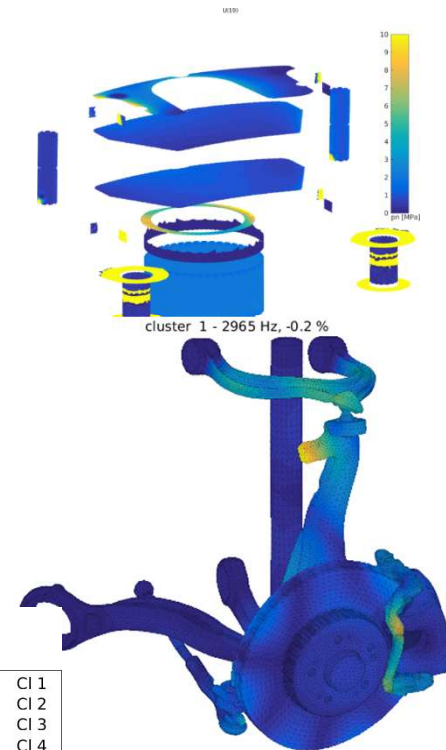
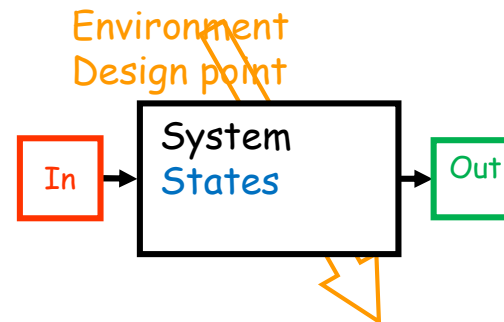
- 8-15 components
- Multiple interfaces/parameters
- 300-600 modes

Design exploration 1000 points

- Full 80 days CPU, 22 TB
- CMT a few hours off-line learning, <1h exploration

Exploitation

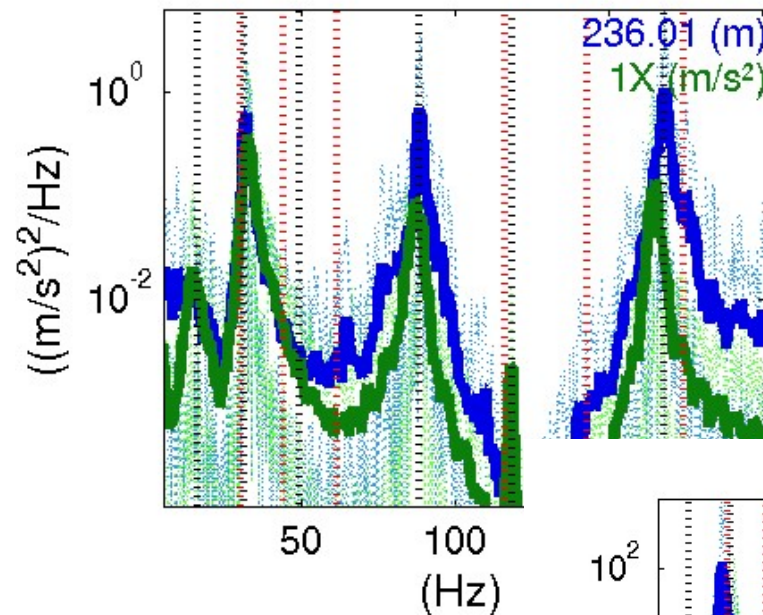
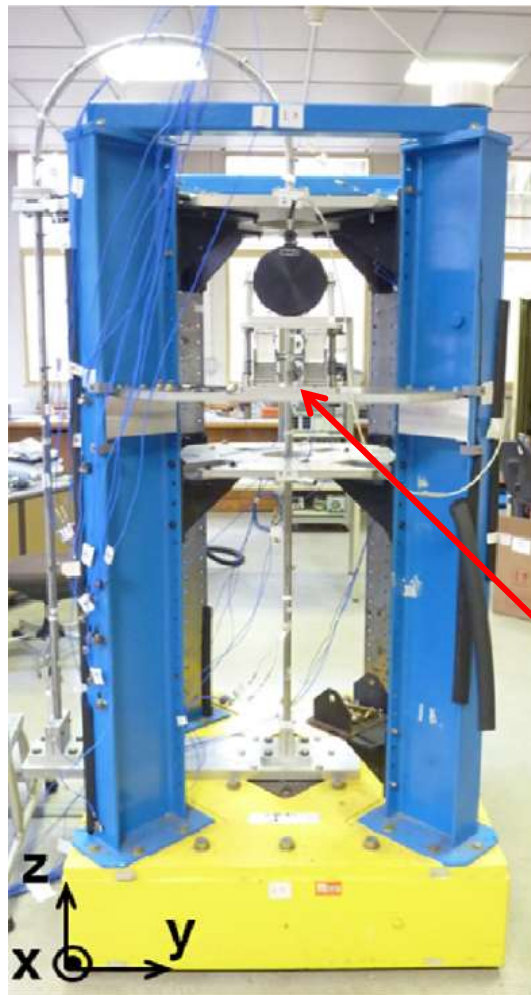
- DOE : basic edges, LHS
- Clustering & parameters



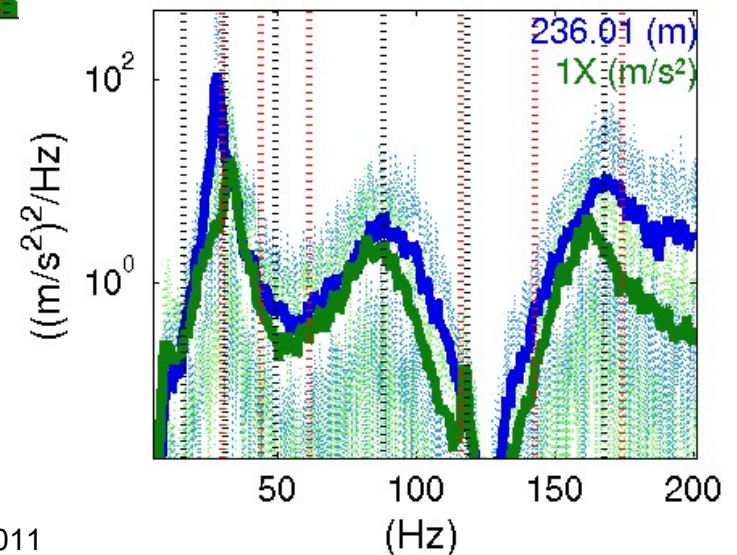
Energy coupling & NL system

Bending beam with gap & contact
Apparent damping with pure contact

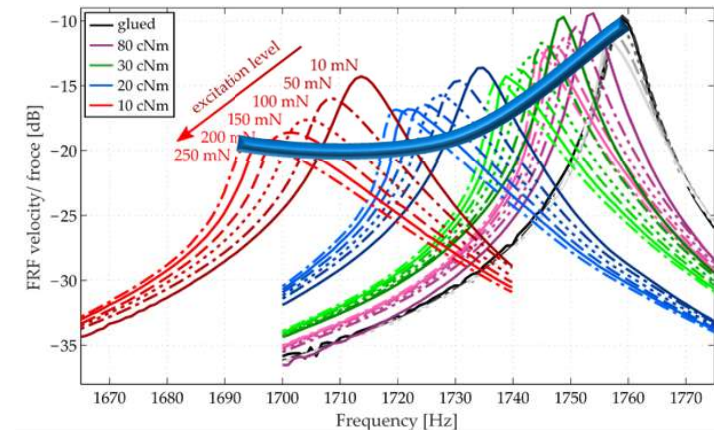
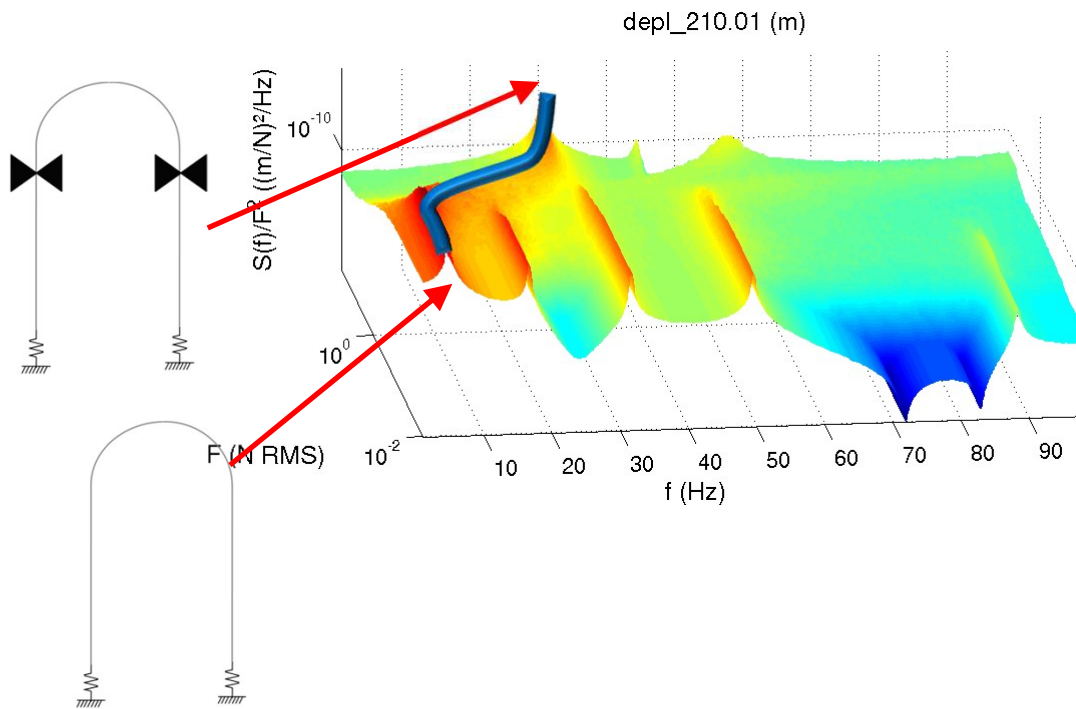
($F=0.33$ N RMS)



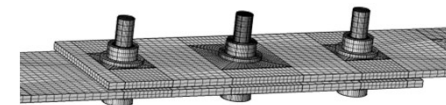
($F=2.52$ N RMS)



Damping occurs in the transition

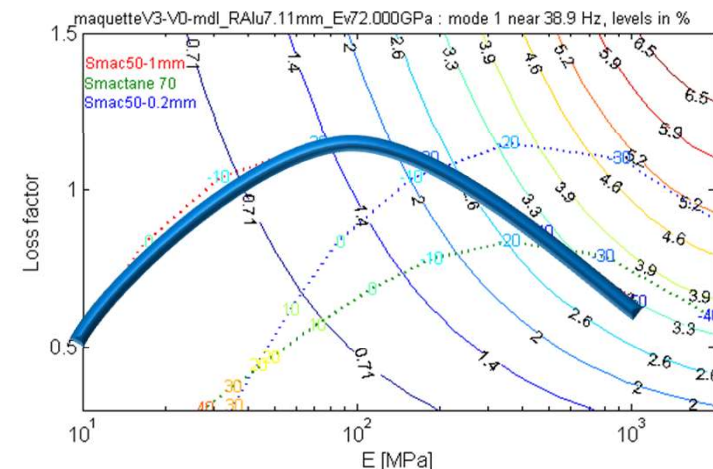


FEMTo-ST Orion Testbed



Transition

- Contact with broadband excitation **level**
- Friction : **amplitude/normal force** dependence
- Viscoelasticity : **temperature/frequency** dependence



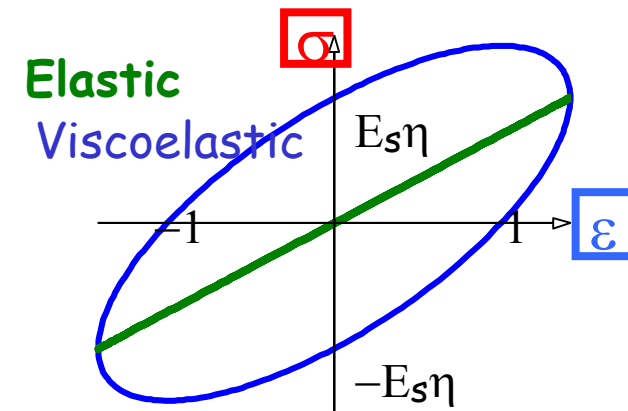
ENSAM/PhD. Hammami 2014
Hammami/Balmes/Guskov, MSSP 2015

Dissipation models : viscoelasticity

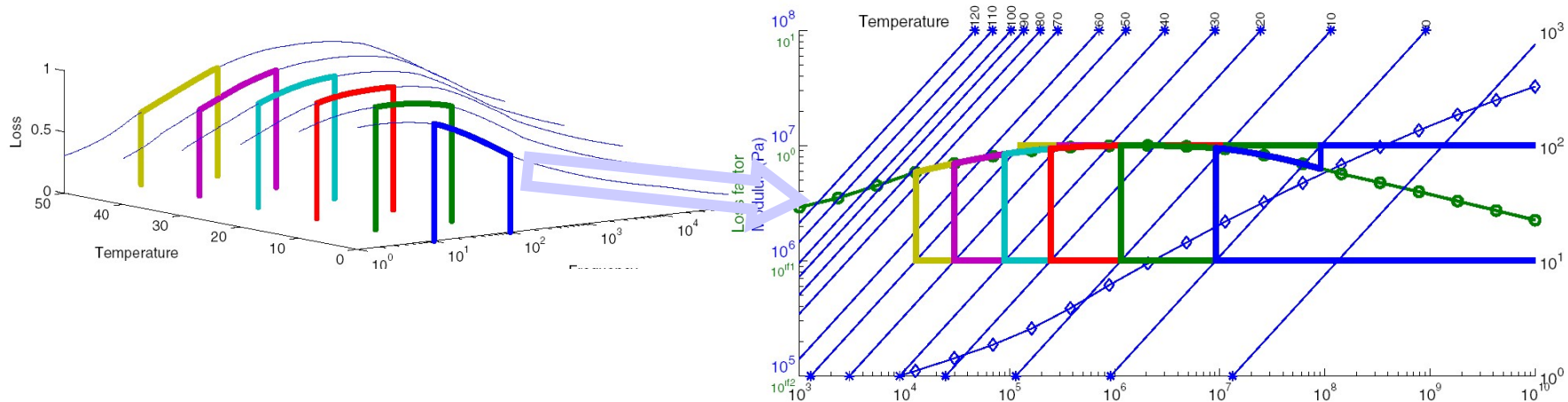
- Viscoelasticity (Bolzmannian material)
- Stress function of strain history
independent of amplitude
- Time convolution $\sigma(t) = \int_{-\infty}^t C(\tau)\epsilon(t - \tau)$
- Frequency complex modulus $\sigma = E(\omega, T, \epsilon_S)\epsilon$

$$E = E'(1 + i\eta)$$

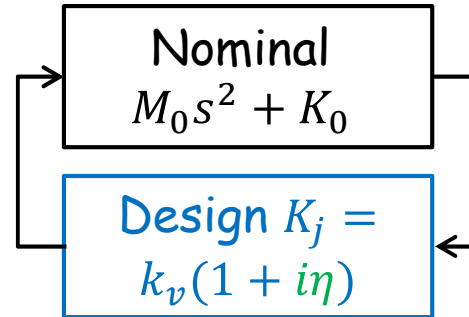
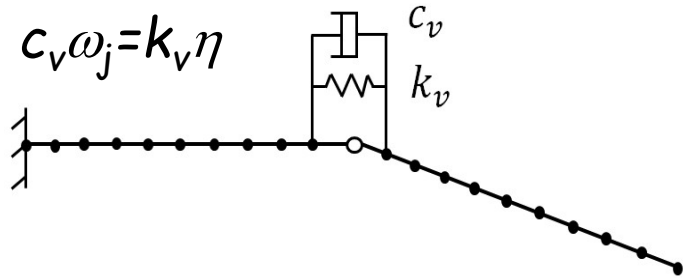
E' storage modulus, η loss factor



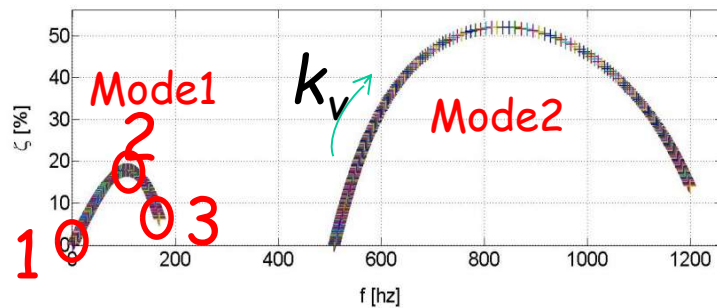
- Temperature/frequency equivalence
 $E(\omega, T, \epsilon_S) = E(\omega\alpha(T, \epsilon_S))$



A mechanical root locus (coupling)



k_v variable, $\eta=1$ constant loss
Output : $f(k_v, \eta) \zeta(k_v, \eta)$



- 1 ————— 1.113 Hz 0.28 % ————— Soft joint
- 2 ————— 98.06 Hz 17.79 % ————— Dissipative joint
- 3 ————— 181.2 Hz 0.11 % ————— Stiff joint

Damping can only exist
if joint is « working »



Energy fraction
in joint must be « sufficient »
(depends on k_v)

Related ideas : electro-mechanical coupling coefficient, pole/zero distance,
modal strain energy

On blackboard

- Parameter, nominal model selection
- b_p residual vector columns of ΔK or $\Delta K T_0$
- Complex modulus example
- Residue iteration
- Issues with reduction

Families of reduced models

\mathbf{p} physical parameters (geometry, constit.)

$Z(\mathbf{p}, s) = [M(\mathbf{p})s^2 + K(\mathbf{p})]$ finite element model

$Z_R(\mathbf{p}, s) = \mathbf{T}^T Z \mathbf{T}$ reduced dynamic model

$\mathbf{T}(\mathbf{p}) = [\phi_1 \dots \phi_{N_R} K^{-1} \mathbf{b}]$ modal model

$\{y\} = [H]\{u\}$ transfer functions



Kinematic reduction \Leftrightarrow choice fixed

Ritz basis $\{q\} = [T]\{q_R\}$

Response surface / reduction

All predict I/O relation but

Response surface methodologies

- no knowledge of internal state

PGD (higher dimension variable separation)

- time and space are coupled

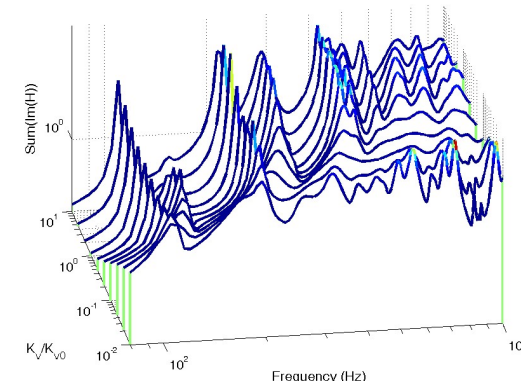
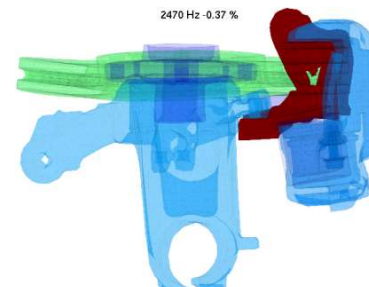
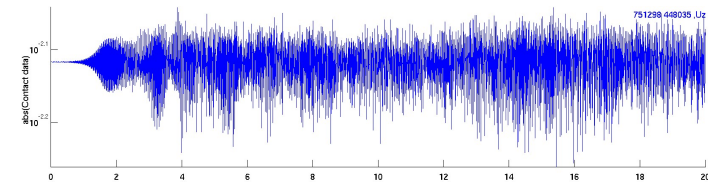
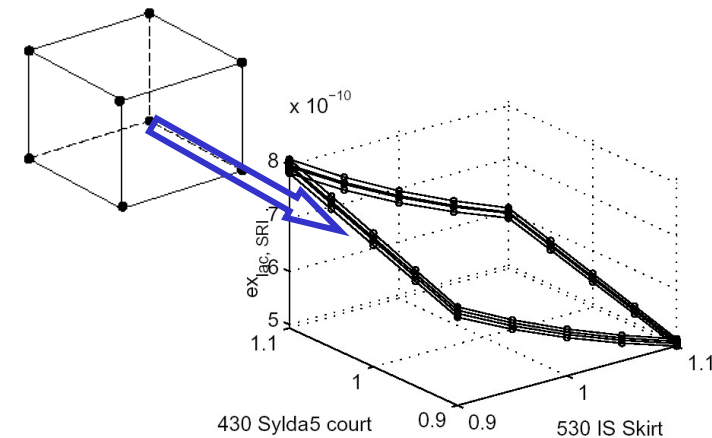
Fixed basis **reanalysis**

- Response surface for system matrices

$$T^T Z(p) T \approx f(p, T^T M_i T)$$

But

- still **dynamic model**
- **restitution** $\{q\} = [T]\{q_R\}$
knowledge of all internal states



First order enhancement to MSE

$[Z(E(s),s)] \{q\} = \{F\}$ Damped viscoelastic resp. rewritten as

$$[Z(E_o,s)] \{q\} = \{F\} - \left[\sum (E(s) - E_o) / E_o [\text{Im}(Z - Z_o)] \right] \{q\}$$

Tangent system, internal loads

Basis contains

- Modes to represent nominal resonances
- Flexibility to viscoelastic loads associated with nominal modes

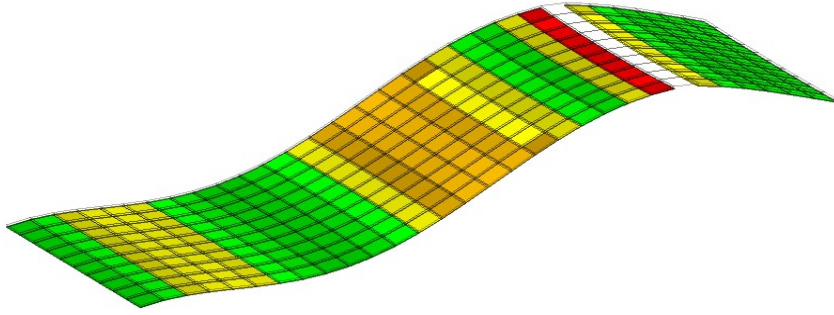
$$T = \left[\phi_{1:NM} \quad K_o^{-1} [\text{Im}(Z - Z_o)] \phi_{1:NM} \right]$$

Modes MSE static response to unit load

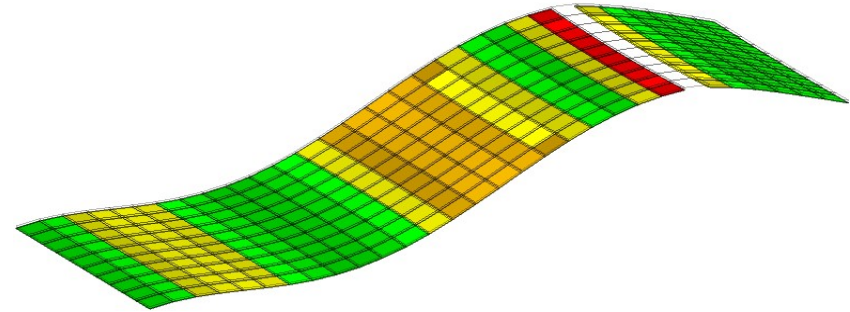
Principle of reduction
(assumptions on excitation space & freq) unchanged

Non proportional damping model

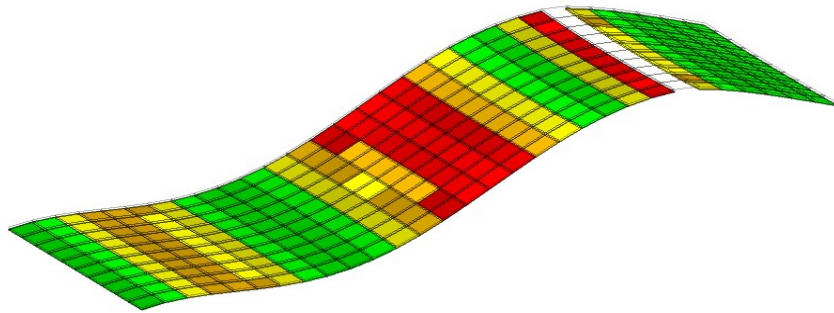
Modal



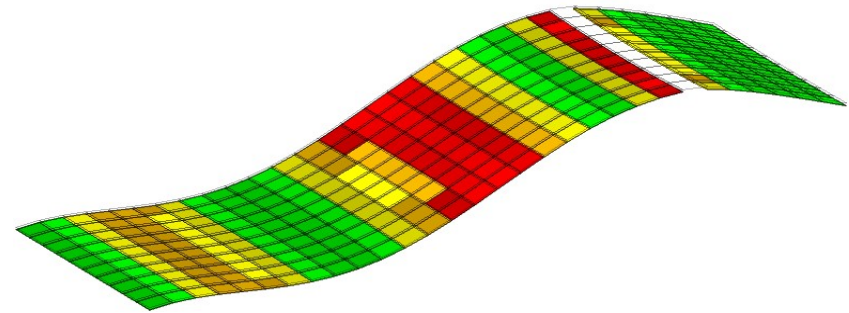
MSE



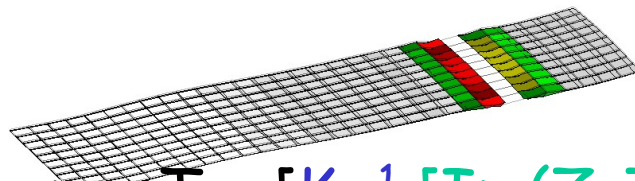
First order



Exact



• Forced response around a resonance



First order shape : $T = [K_o^{-1} [\text{Im}(Z-Z_o)] \phi_4]_{\text{orth}}$

Reanalysis

Reanalysis

Multi-model bases

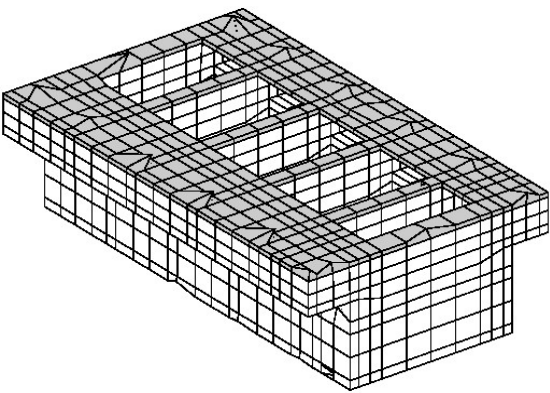
- Modes
- Multi-model
- Modes + sensitivities

$$T = [\Phi(p_0)]$$

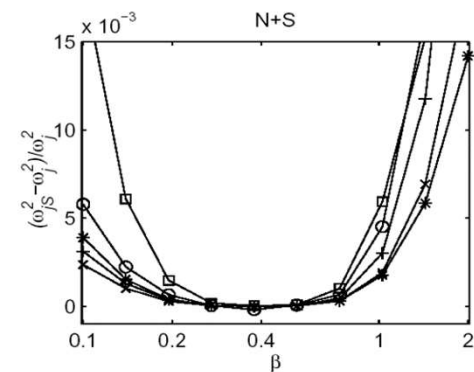
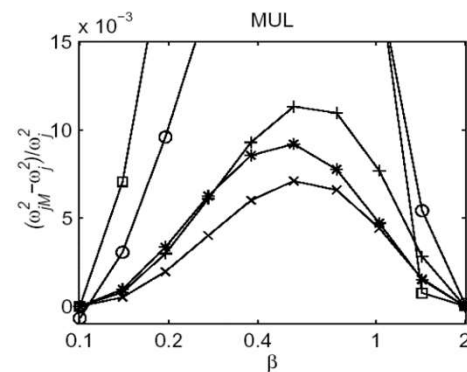
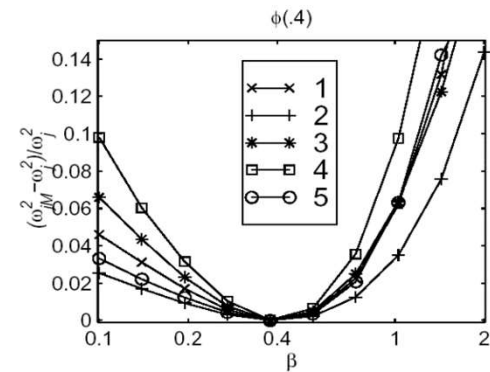
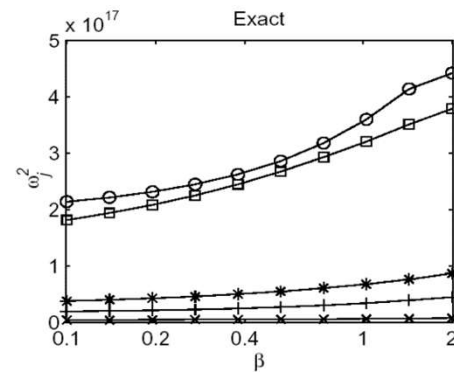
$$T = [\Phi(p_1) \ \Phi(p_2)]$$

$$T = [\Phi(p_0) \ \partial\Phi(p_0)/\partial p]$$

$$\left[\left[T^T K(p) T \right] - \omega_{jR}^2(p) \left[T^T M(p) T \right] \right] \{ \phi_{jR}(p) \} = \{ 0 \}$$

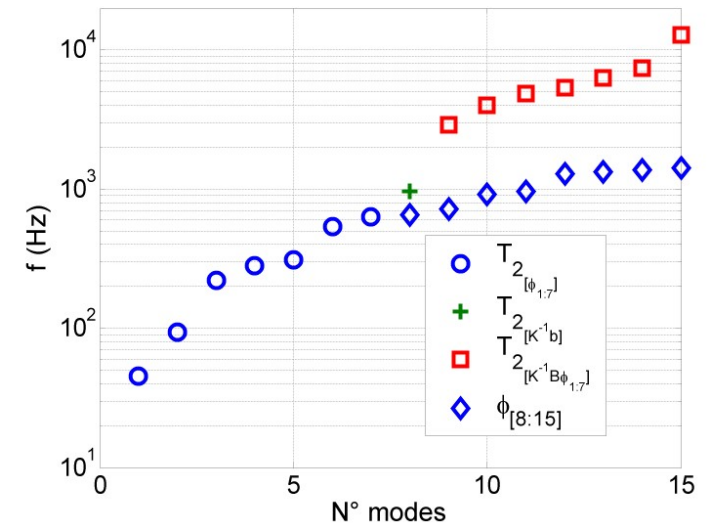


Frequencies

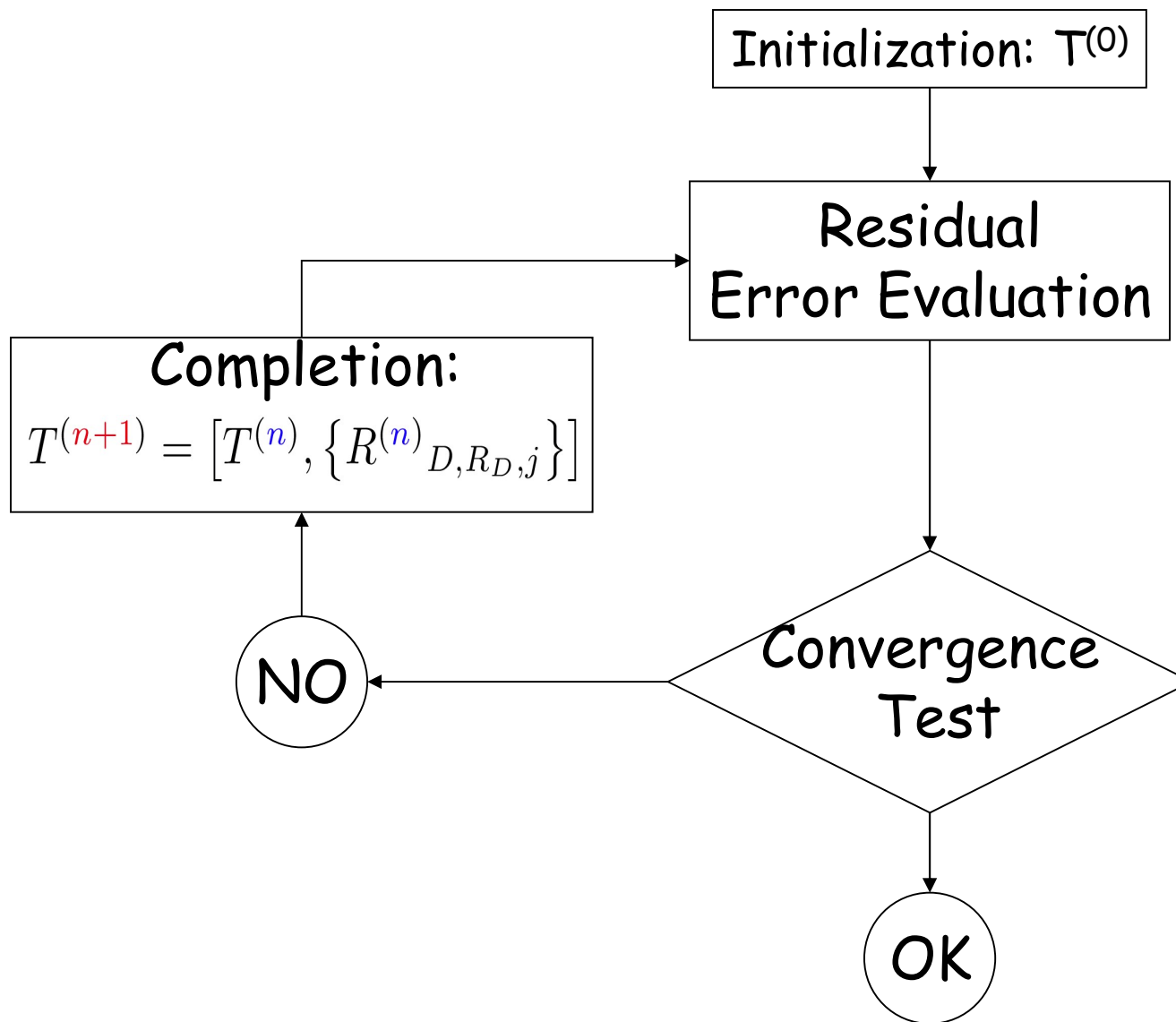


Reduction Bases/Starting Point

- Modal $T = [\Phi(p_0)]$
- Modal + static Responses to input
 $T = [\Phi(p_0) \ K^{-1}b]$
- Modal + static responses
 to representative loads
 $T = [\Phi(p_0) \ K^{-1}[b \ R_j] \ \Phi(p_k)]$
- Frequencies of corrections \square
 differ from modes \blacklozenge : the added
 information is VERY different



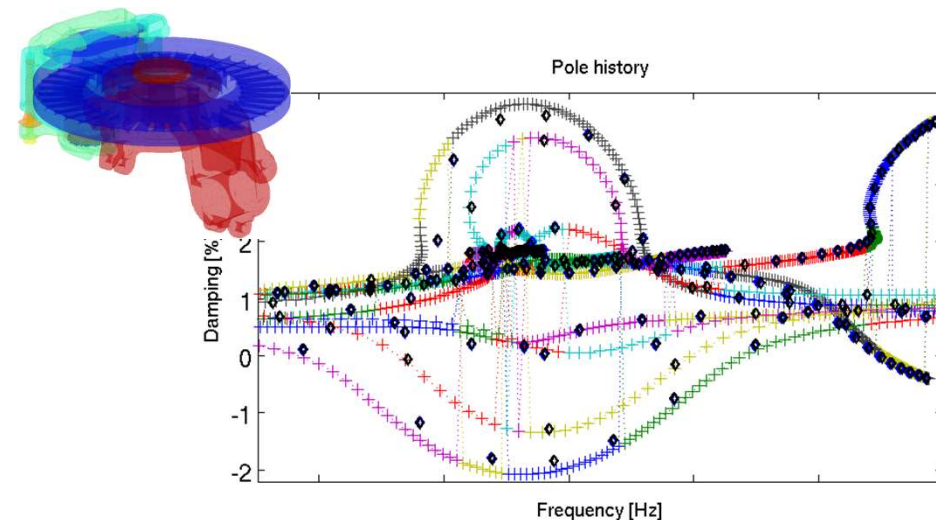
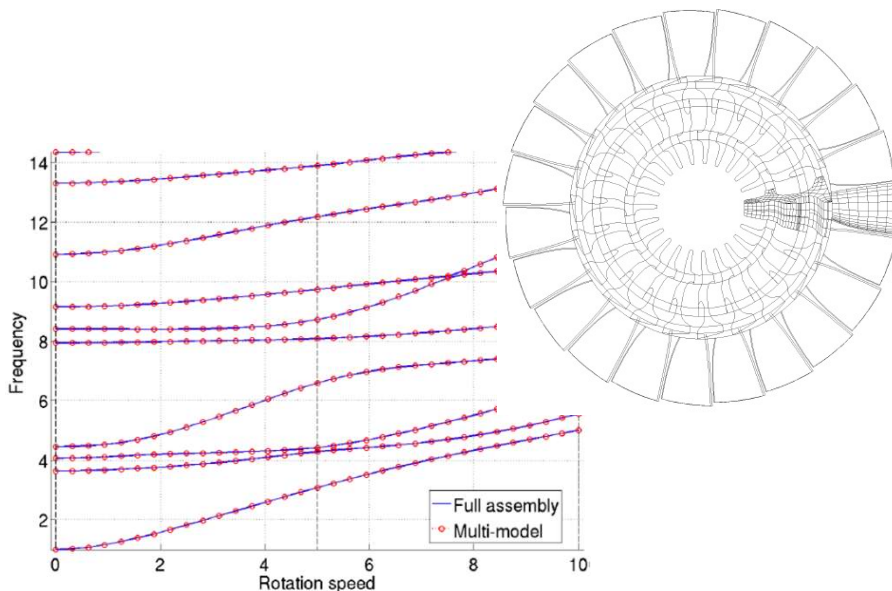
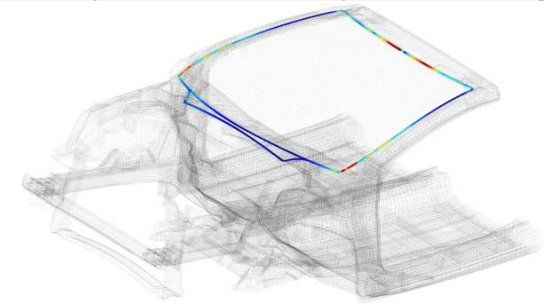
Iterative Procedure



Fixed basis : enormous cost reduction

- **Windshield joint complex modes** at 500 design points for $\frac{1}{2}$ cost of direct solver
- **Campbell diagram** : 200 rotations speeds for the cost of 4.
- **Squeal instabilities as function of pressure** : few pressures sufficient for interpolation

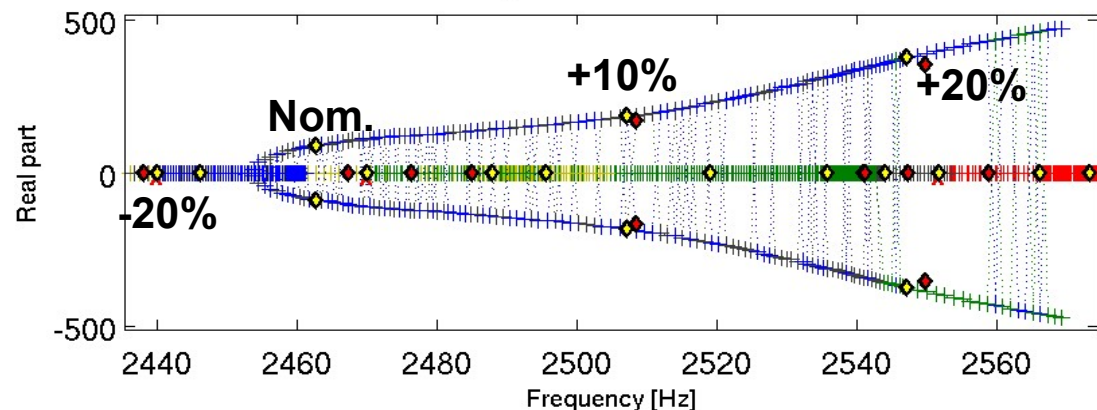
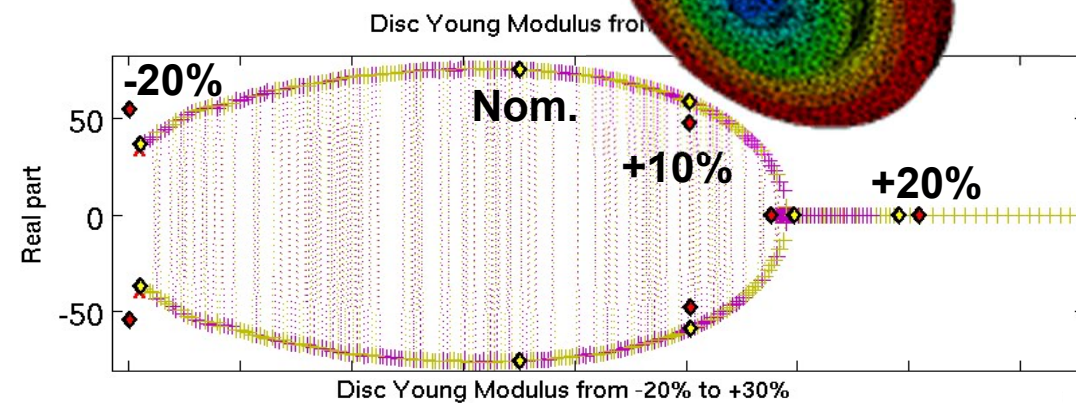
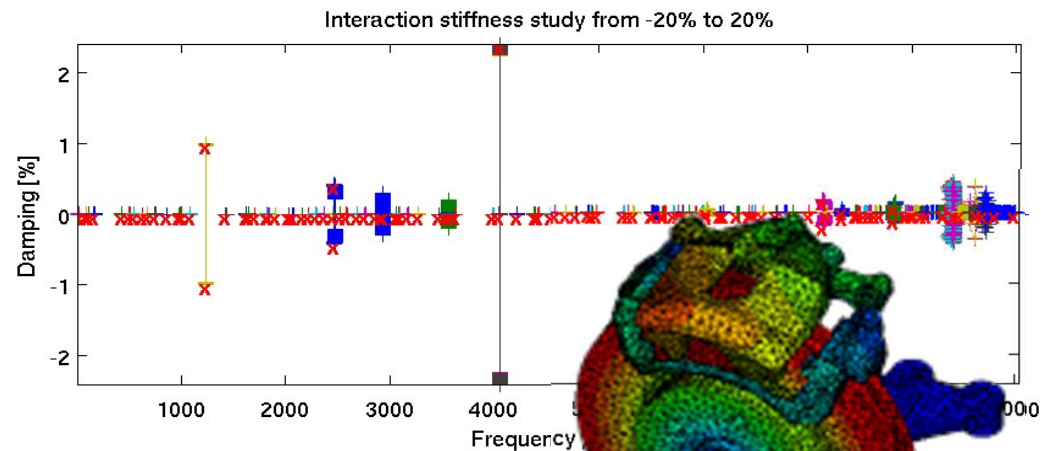
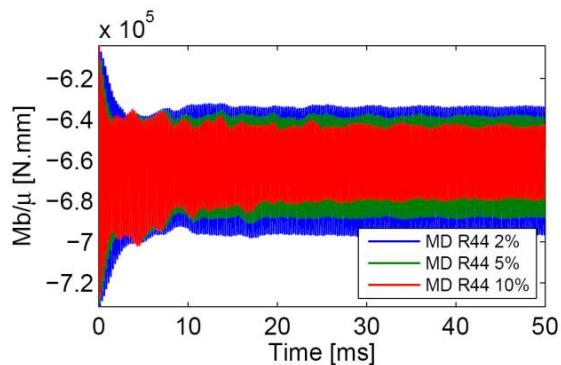
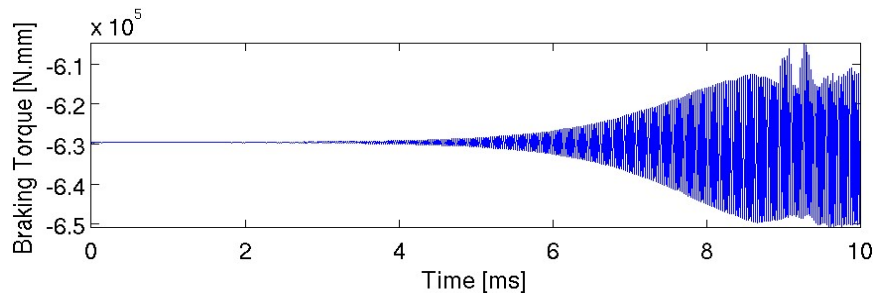
Ψ, λ	SOL107	2200s
Φ, ω	SOL103	300s
Ψ, λ Reduced	First order Error <4%	490s
$\Psi, \lambda(500 \times T)$	SOL107	~ 12 days
$\Psi, \lambda(500 \times T)$ reduced	First order Error small	~1000s



Reanalysis : squeal example

Objectives:

- complex mode stability
- Transient simulation



Other applications

- Multi-stage cyclic symmetry (SNECMA).
 - Which stage, which diameter, ...
 - Mistuning (which blade)
- Damping design (PSA)
 - Fixed system modes, component redesign

