Component mode synthesis

- Earlier (reduc.pdf)
  - Reduction principles
  - Reduction illustrations
- Now
  - coupling reduced models
  - Advanced reduction for coupling objectives

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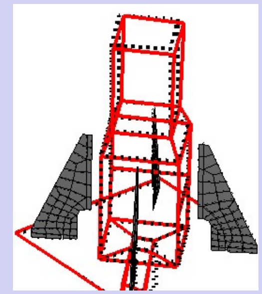
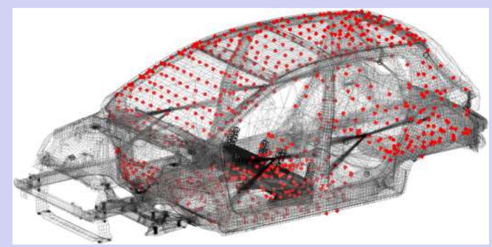
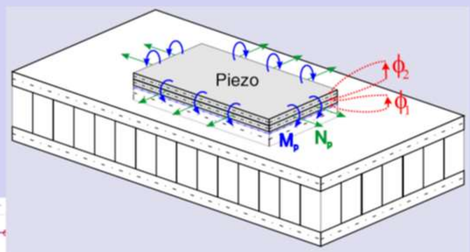
Moving complexity in the coupling part

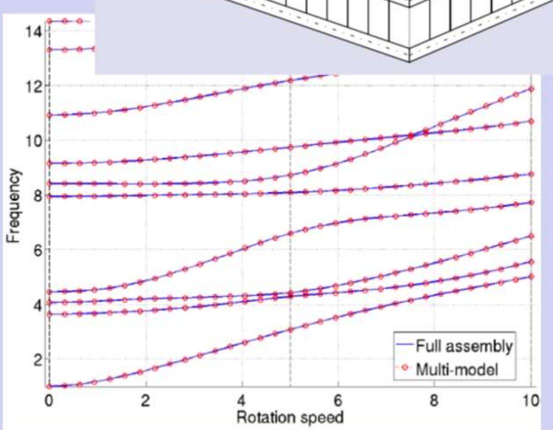
In → Reduced model → Sensors

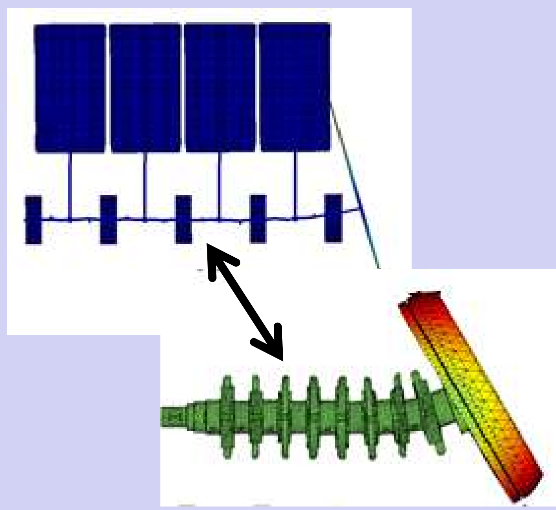
- Coupling : test/FEM, fluid/structure active control, …
- Local non-linearities : machining, bearings, contact/friction, …
- Optimization / uncertainty

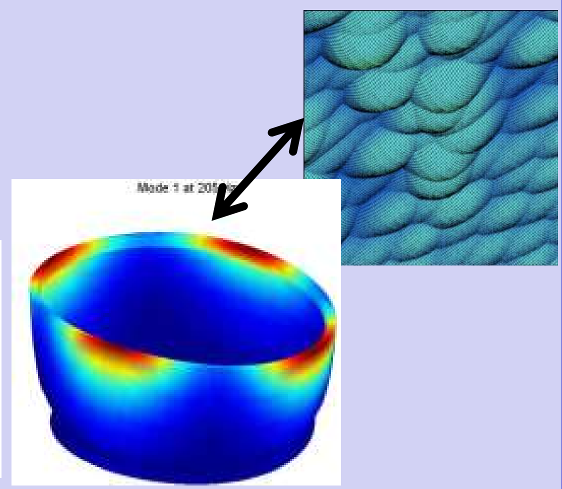






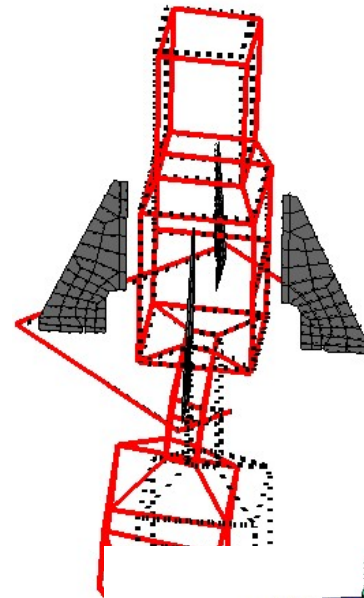


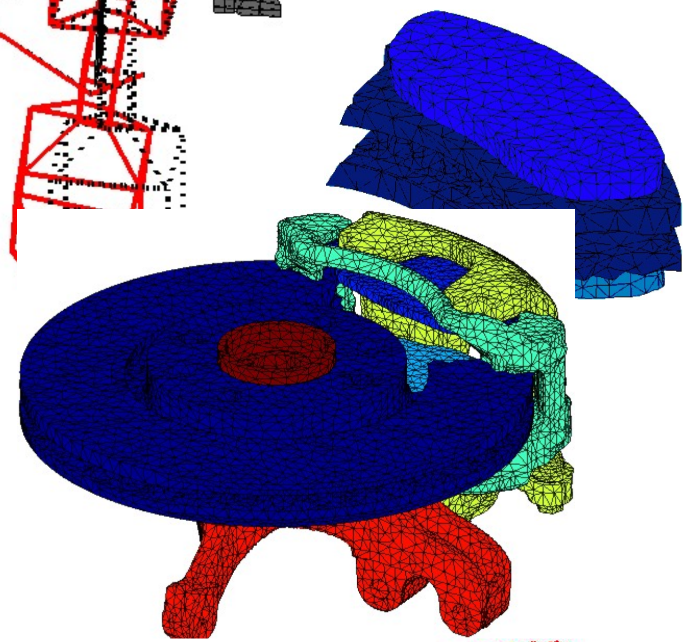


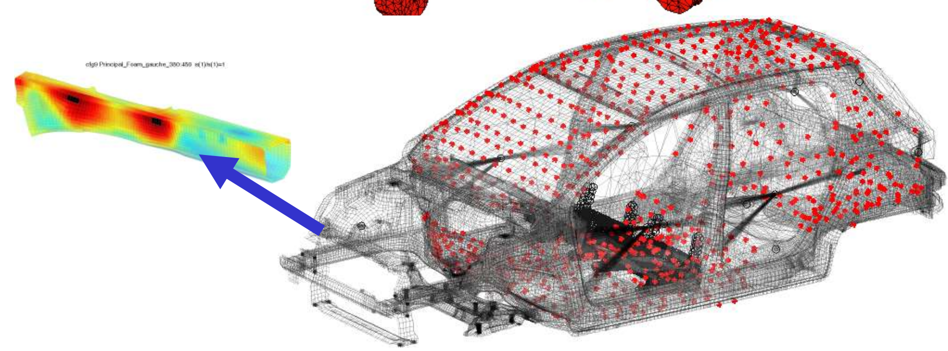


Sample CMS problems

1. Acoustic prediction from test shapes
2. Fluid structure interaction (in particular with heavy fluids)
3. Structural Dynamics Modification
4. Reduce a brake model while keeping
    - all elements of NL contact area
    - exact modes of linear model
5. Design of damping treatment for structure borne transfer
6. Non-linearity (contact on tip blade)

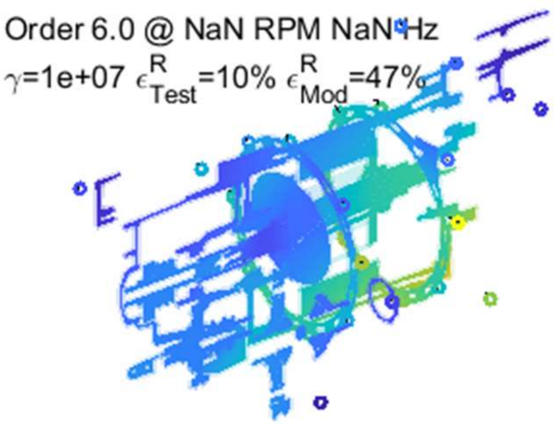






Why CMS ?

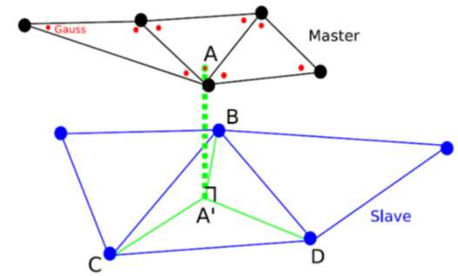
- A reason of procedure
  - Represent linear structural dynamics for coupling in another code (hybrid test/FEM, acoustics, multi-body dynamics, control, local non-linearity, …)
  - Transmit a compact/confidential model to another group/company
  - Understand effects of components
  - Reduced data output
- For computational cost objectives
  - One step approximations (low cost linear model)
  - Iterative (often parallel) solution of exact problem



Blackboard discussion

- Draw non conform contact case,
  gauss points (nodes special case)
  gap and sliding observation
  contact/friction constitutive law (surface laws)
  model loads
- Energy coupling (surface constitutive laws)
- Mathematically idealized bonding (constraints, Lagrange)
- $1-\epsilon$ compatibility
- Kinematic reduction for coupling
  - Remind McNeal & Craig-Bampton
  - Interface modes
  - Learning using exact solutions (CMT)

Incompatible mesh contact



- Gap (out of plane) incompatible
  - Define contact points matched
  - Match slave elements

$$g_g = [c_g]\{q\} = [N_{master}(r,s) - N_{slave}(r,s)]\{q\}$$

  - Associate integration rule and compute work

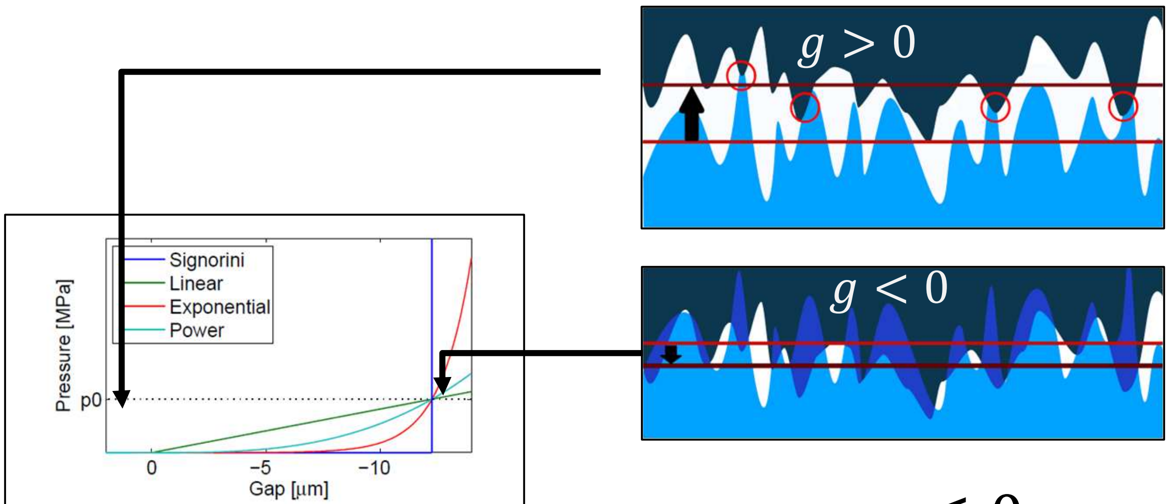
$$\{q^*\}^T\{P_{contact}\} = \sum_g \{q^*\}[c_g]^T w_g J_g P_g$$

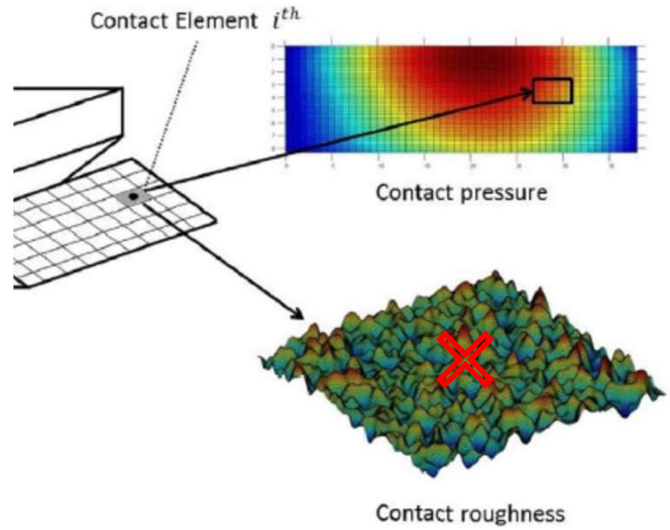
- Gap for compatible mesh
  - Use nodal displacement
  - Define gap at gauss points (zero thickness cohesive element)

- Extension in plane : adhesion/sliding/friction

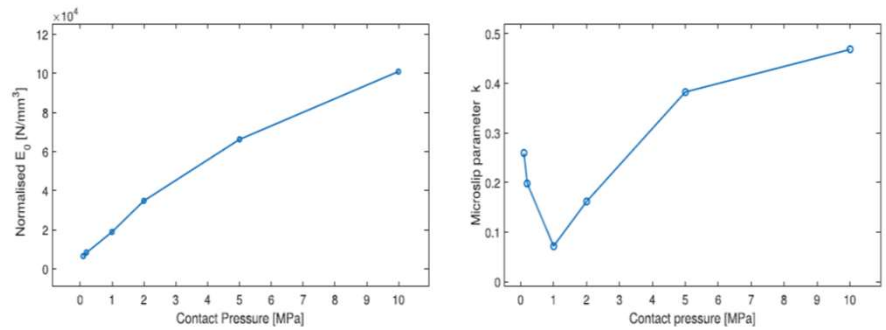
Contact constitutive law

- Macro-scale surface not flat (1 gauss)
- Macro load function of gauss strain





From : L. Pesaresi. JSV 2018



- Idealization Signorini

$$g \leq 0$$
$$p > 0$$
$$pg = 0$$

- Reality $p(\mathrm{g})$

- Friction : coulomb $\sigma_t = -\mu p \dfrac{v_t}{\|v_t\|}$

Stiffness/energy coupling

- Interface motion

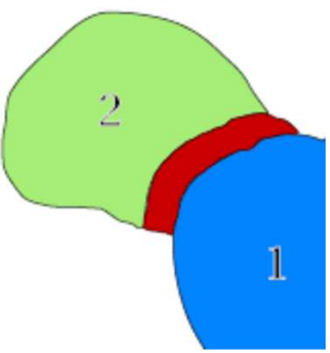
$$\{y_j(X,s)\} = [c_{j\,\mathrm{int}}(X)]\,\{q_j(s)\}$$

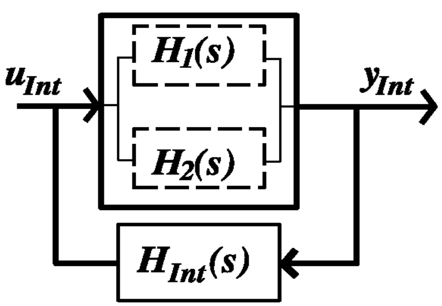
- Interface stiffness (cohesive elements)

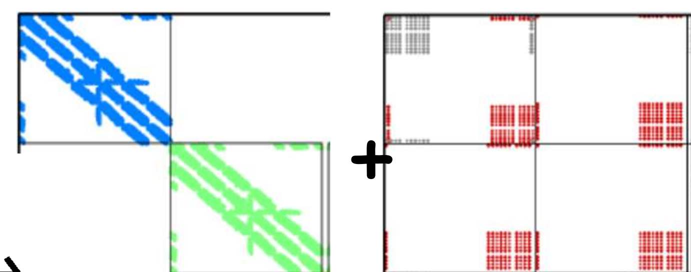
$$\begin{bmatrix} Z_{jj\,\mathrm{int}} & \cdots & Z_{cj\,\mathrm{int}} \\ \vdots & \ddots & \vdots \\ Z_{jc\,\mathrm{int}} & \cdots & Z_{cc\,\mathrm{int}} \end{bmatrix} \left\{ \begin{array}{c} \left\{ \begin{array}{c} [c_{\mathrm{int}}]\,\{q_j\} \\ \vdots \end{array} \right\} \\ \{q_{\mathrm{int}}\} \end{array} \right\} = \left\{ \begin{array}{c} F_{\mathrm{int}} \\ \vdots \\ \{0\} \end{array} \right\}$$

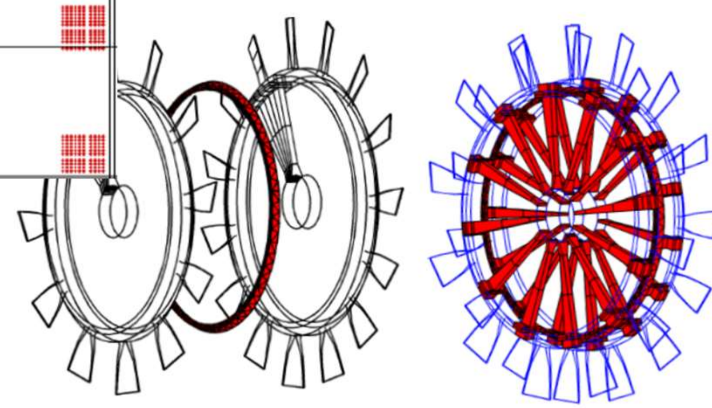
- Coupled equations (sum of energies)

$$\left(\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} + \begin{bmatrix} c_1^T & 0 \\ 0 & c_2^T \end{bmatrix} Z_{\mathrm{int}} \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \right) \left\{ \begin{array}{c} q_1 \\ q_2 \end{array} \right\} = [b]\,\{u(s)\}$$

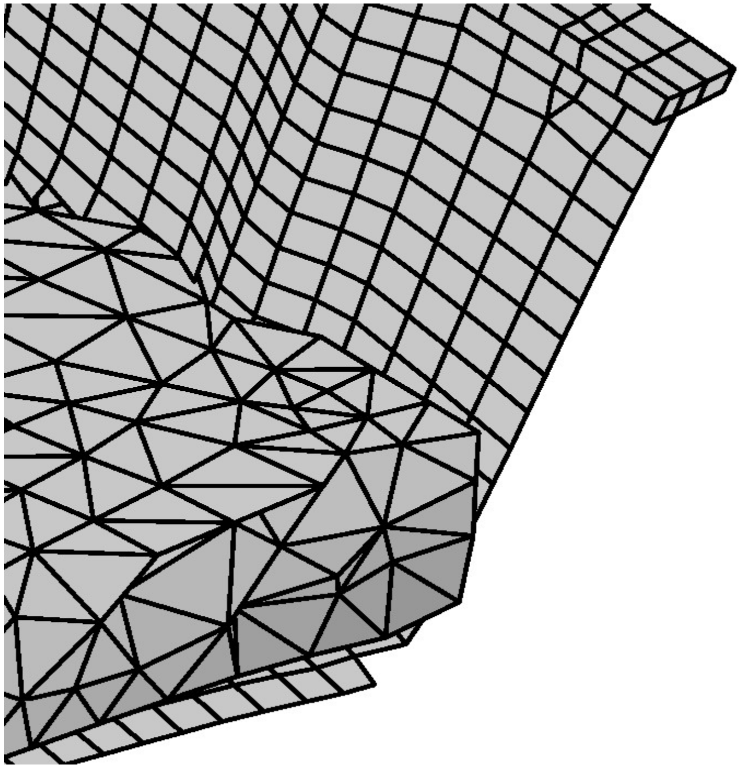




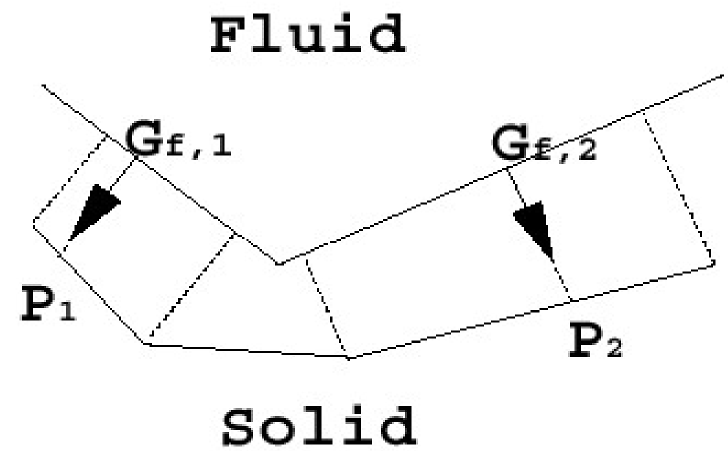


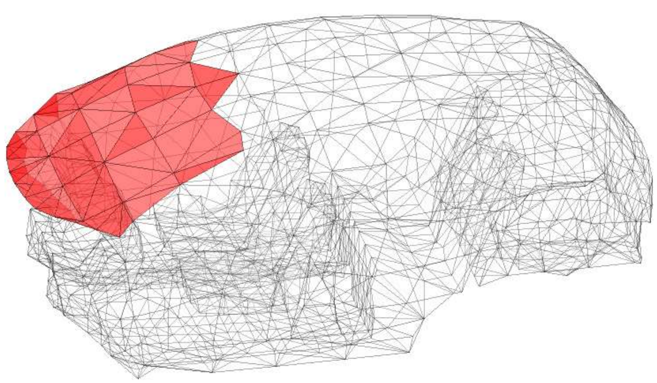


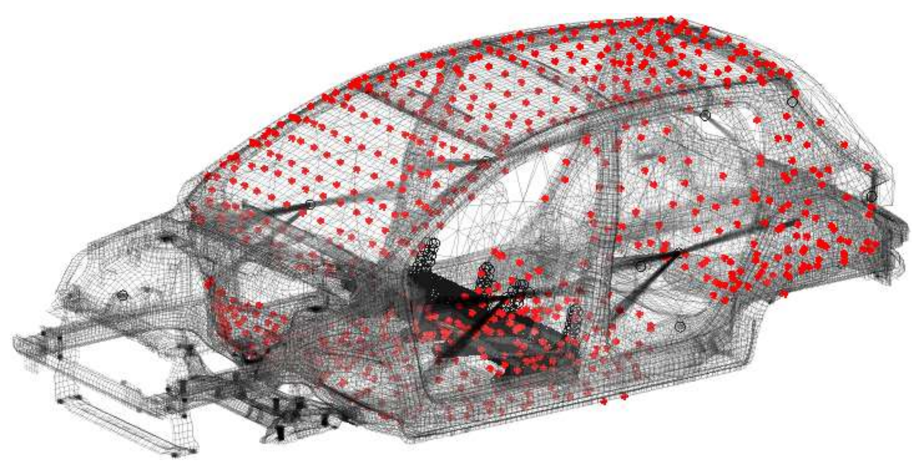
Incompatible mesh : fluid/structure



$$\begin{bmatrix} K & -C \\ 0 & F \end{bmatrix} \begin{Bmatrix} U \\ p \end{Bmatrix} - \omega^2 \begin{bmatrix} M & 0 \\ C^T & K_p \end{bmatrix} \begin{Bmatrix} U \\ p \end{Bmatrix} = \begin{Bmatrix} F^{ext} \\ 0 \end{Bmatrix}$$







Limiting case : continuity

Solve with zero relative interface motion

$$\{y_{1Int} - y_{2Int}\} = [c_1 \quad -c_2]_{Ng\times(N_1+N_2)} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0$$

- Classical : eliminate constraint (T kernel)

$$[T] \text{ with } [c_{Int}]\,[T] = 0$$

$$\{q\} = [T]\,\{q_R\}$$

$$\left[T^T Z T\right]\{q_R\} = \left[T^T b\right]\{u\}$$

- Lagrange multiplier solution
- Penalize (approximate energy)

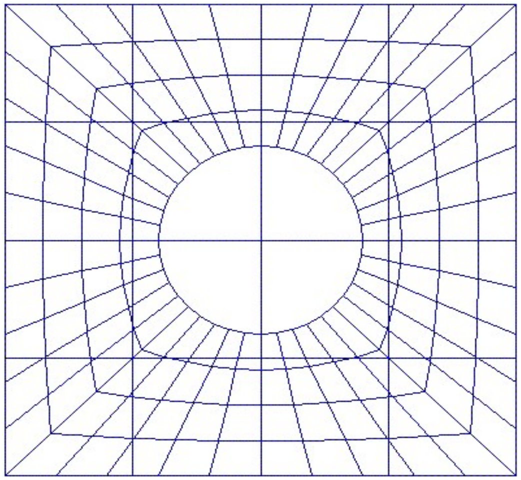
$$\begin{bmatrix} Z(s) & c_{Int}^T \\ c_{Int} & 0 \end{bmatrix} \begin{Bmatrix} q \\ \lambda \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

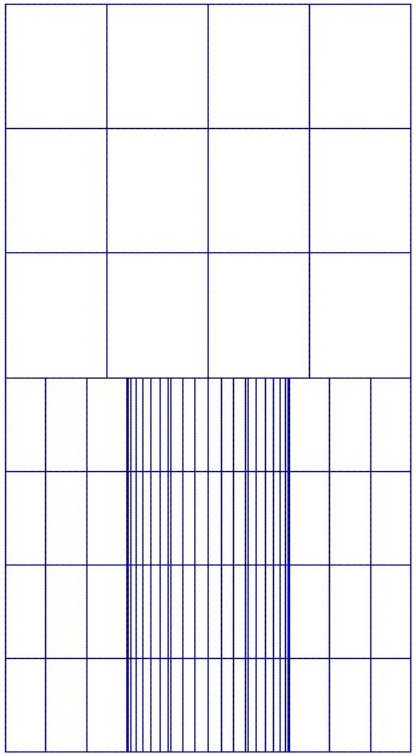
$$\left(\begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} + \begin{bmatrix} c_1^T \\ -c_2^T \end{bmatrix}\left[\frac{I}{\epsilon}\right][c_1 \quad -c_2]\right)\begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [b]\,\{u(s)\}$$

- Other approach : continuity enforced over volume (Ben Dhia, Arlequin)

Incompatible meshes

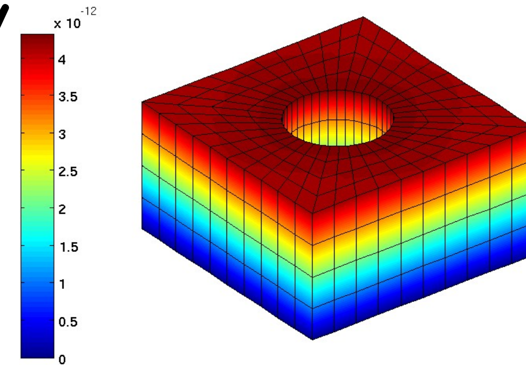
- Occur regularly
  - Result of automated meshing (conform mesh generation can be very difficult)
  - Contact problems
- Test case : compression of 2 cubes
  - Cube over drilled cube
  - Coarse upper cube
  - Refined lower cube
  - Master upper cube

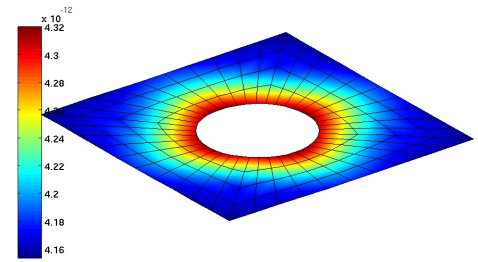


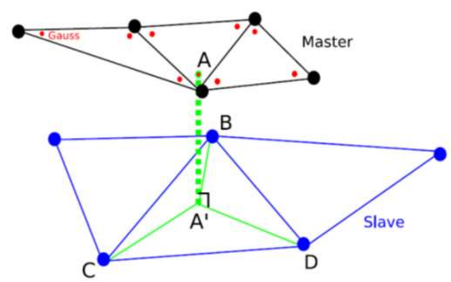


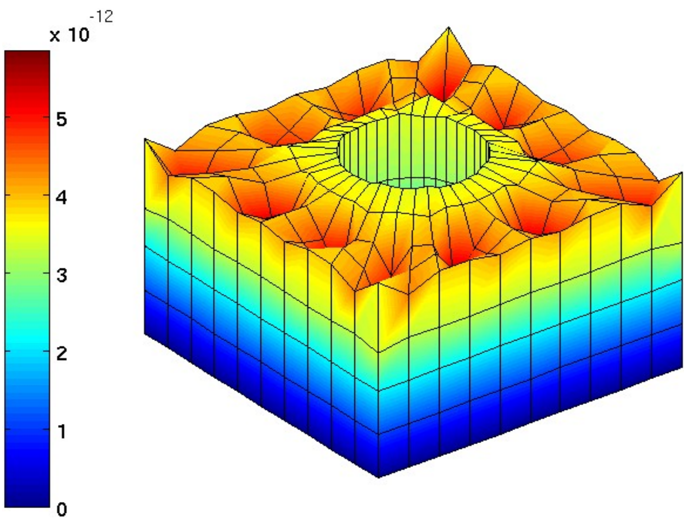
Incompatible mesh issues

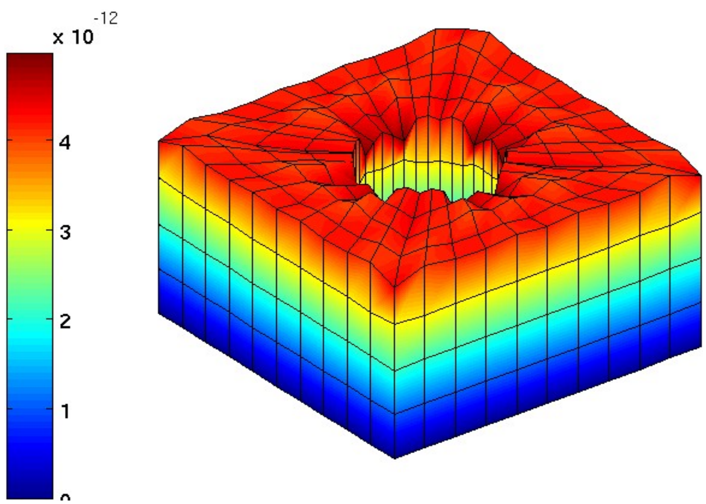
- Solution depends of interpolation strategy
  - Number of contact points matched
  - Number of slave elements matched
- Poor results when using coarse mesh as master

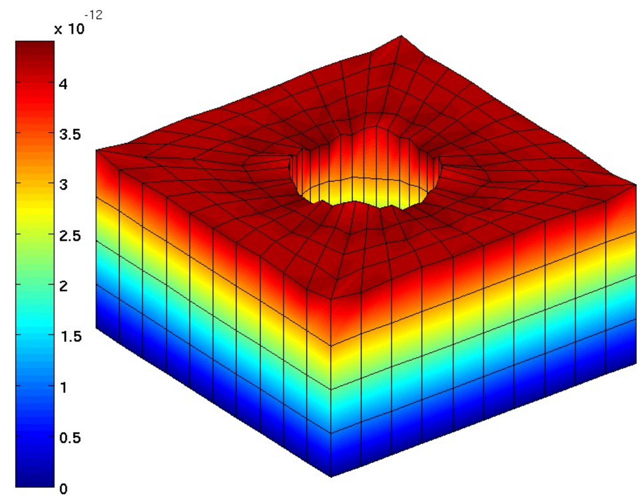








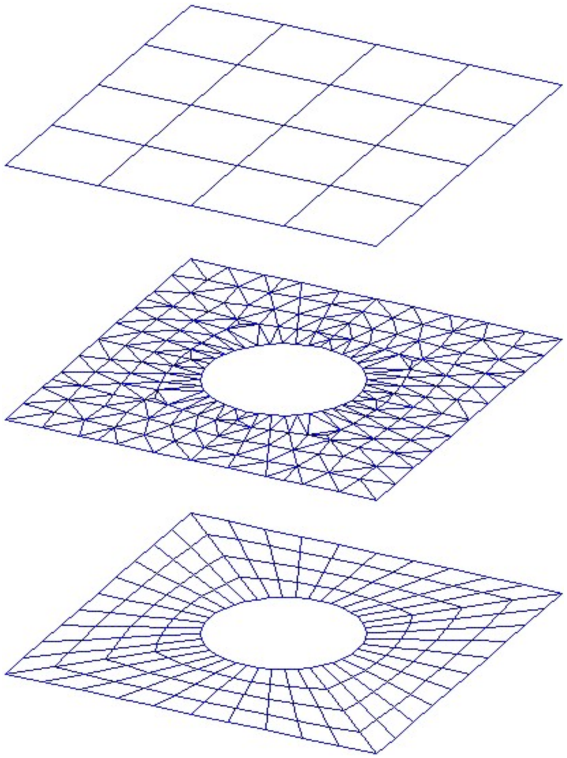




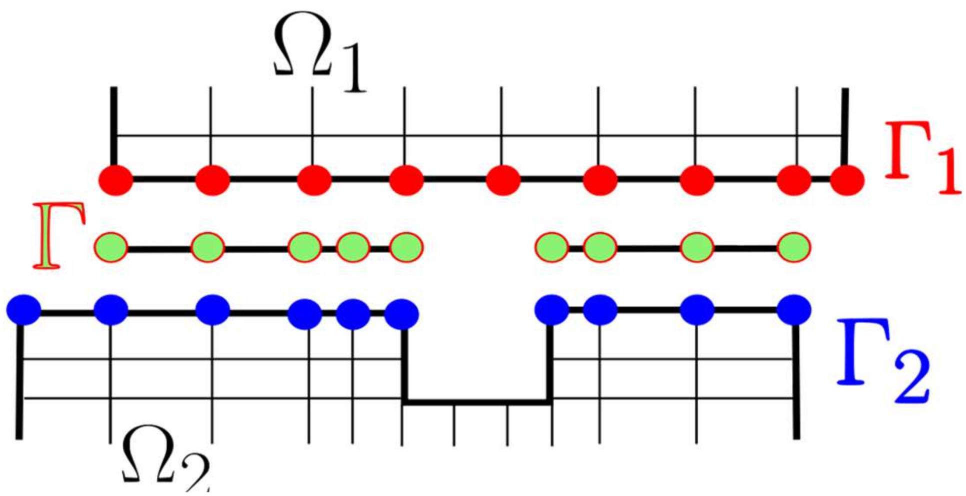
Discontinuity : numerical implementation

- Construction of a third interface
  - Domain intersection
  - Nodes of both surfaces
  - Delaunay triangulation
- Gap observation at nodes or Gauss points Γ
  - Projection for Γ_1 and Γ_2
  - Cross product operator

$$[A] = [C_{NOR}]\,[C_{NOR}]^T$$

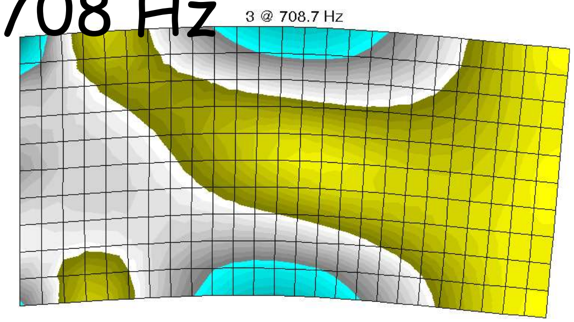


- [A] ill conditioned if
  - Under integration
  - Master points not matched

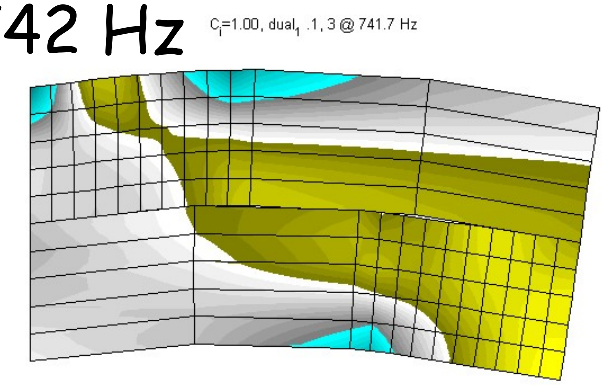


Incompatibility and locking

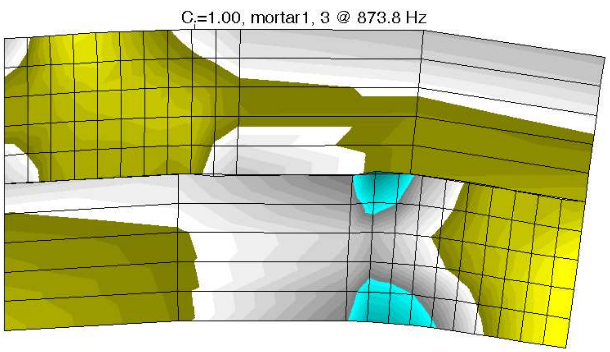
708 Hz



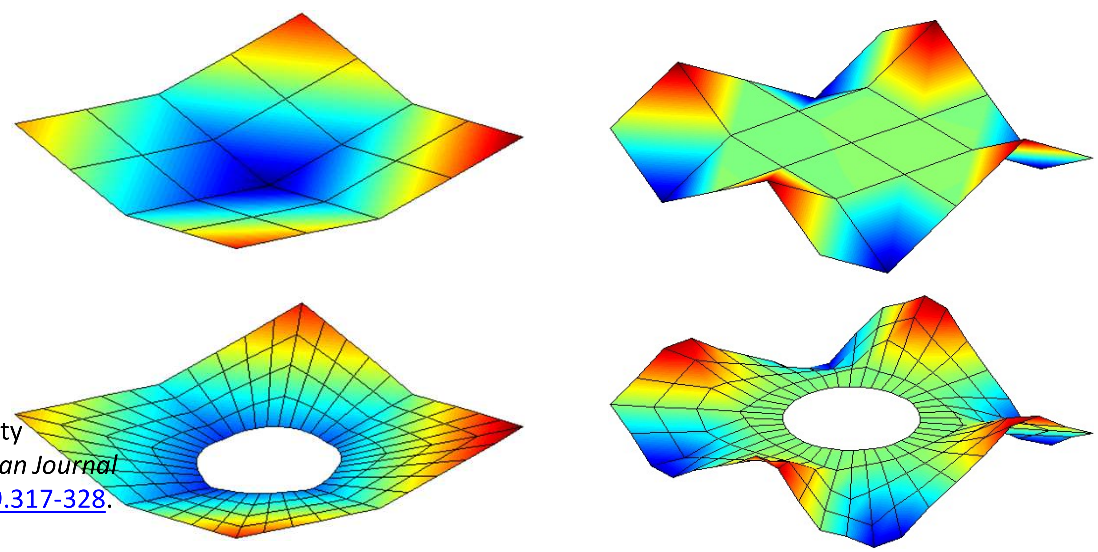
742 Hz



873 Hz=locking



- Strong continuity = locking
- Weak sense for continuity needed [1]



[Skip to Vector sets and bases](#)

[1] G. Vermot Des Roches, E. Balmes, H. Ben Dhia, and R. Lemaire, “Compatibility measure and penalized contact resolution for incompatible interfaces,” *European Journal Of Computational Mechanics*, vol. 19, pp. 317–329, 2010, doi: [10.3166/ejcm.19.317-328](https://doi.org/10.3166/ejcm.19.317-328).

Quality measurement (1-ε)-compatibility

- Measure the norm difference between the basis vectors of Γ_1 and their projection on Γ_2

$$C_2^1(\{q_1\}) = \frac{\|\pi_2^1\{q_1\}\|}{\|\{q_1\}\|}$$

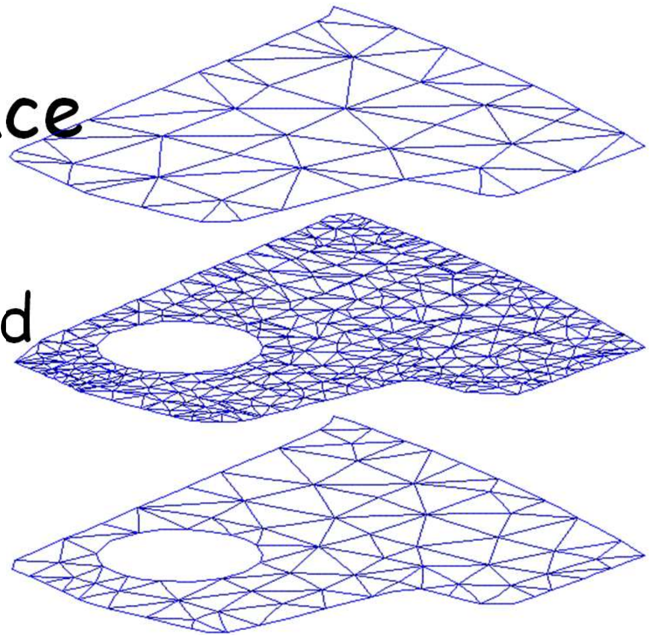
- Realize this leads to an eigenvalue problem

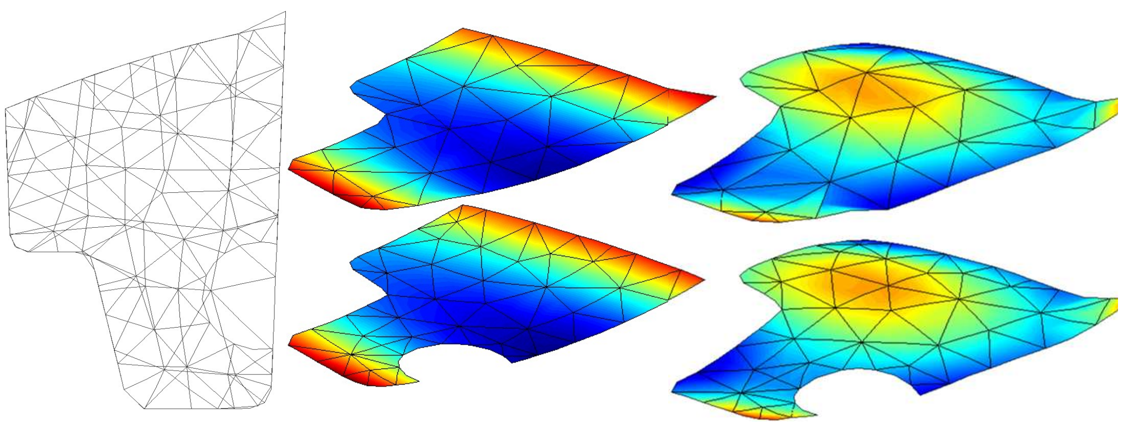
$$C_2^1(\{q_1\})^2 = \frac{\{q_1\}^T[A_{21}]^T[A_{22}]^{-1}[A_{21}]\{q_1\}}{\{q_1\}^T[A_{11}]\{q_1\}}$$

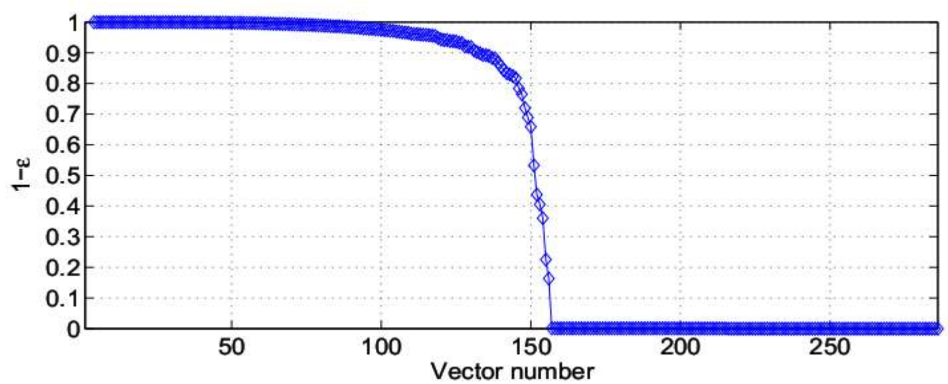
- Use of an inner product with mechanical meaning (pressure load with surface stiffness density)

Illustration on a brake model

- Master/Slave strategy not obvious
- Mesh refinement differences
- Application to the pad/caliper interface
- Compatibility issues
  - Spurious movements for partially matched contact elements
  - Movement over drilled parts







Classical reduction bases + variants

CMS = coupling + reduction

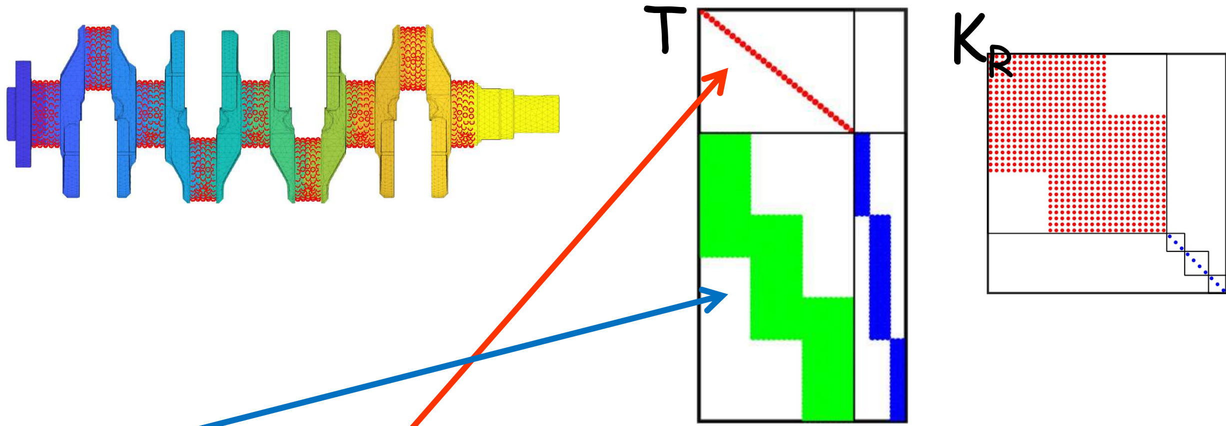
- Static condensation + fixed interface modes = Craig-Bampton
- Free modes + attachment modes (static correction)
- … + residual vectors for parametric changes

Discuss now :

- … + interface modes
- CMT : Trace of assembled modes
- … + component modes
- ODS, POD, Snapshot POD, … (see Avanded_Modal_Periodic.pdf)

Interface reduction / model size / sparsity

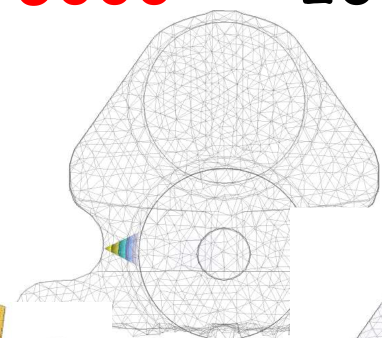
- Craig-Bampton often sub-performant because of interfaces

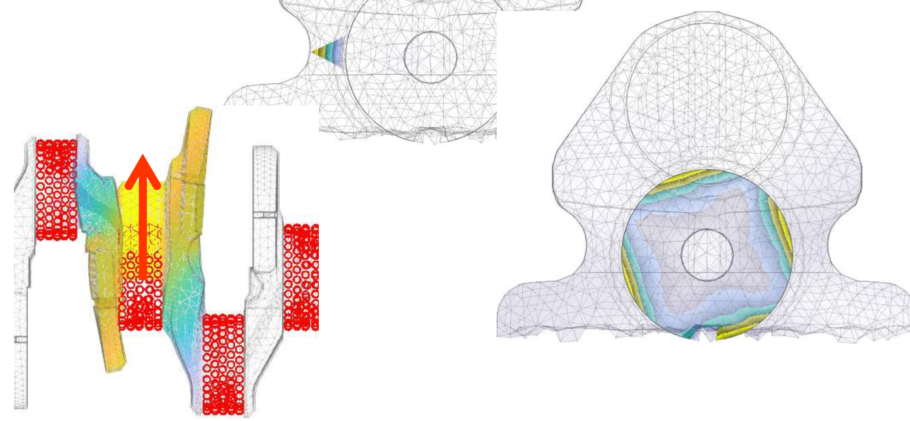


5000^2 = 200 MB

2e6 rest x 5000 Int = 74GB

- Unit motion can be redefined : interface modes
  Fourier, analytic polynomials, local eigenvalue
  5000 -> 500 interface DOFs.
- Disjoint internal DOF subsets





Separate requirements for learning shapes :

bandwidth, inputs external & parameter
truncation, sparsity

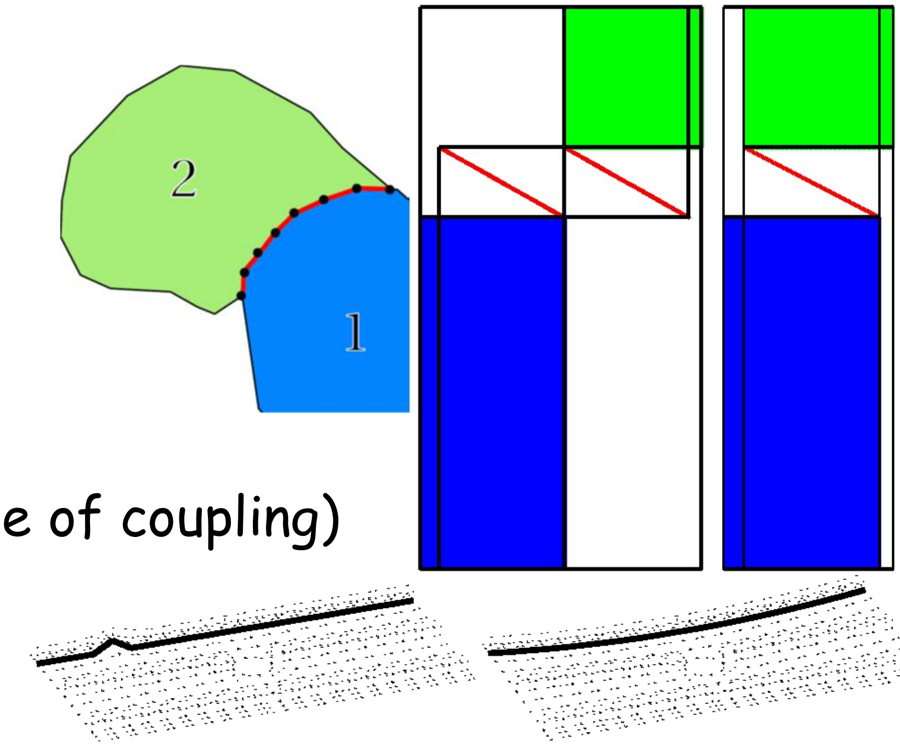
Interfaces for coupling

Classical CMS : continuity coupling

- Reduced independently
- All interface motion (or interface modes)
- Assembly by continuity

Difficulties

- Mesh incompatibility
- Large interfaces
- Strong coupling (reduction requires knowledge of coupling)

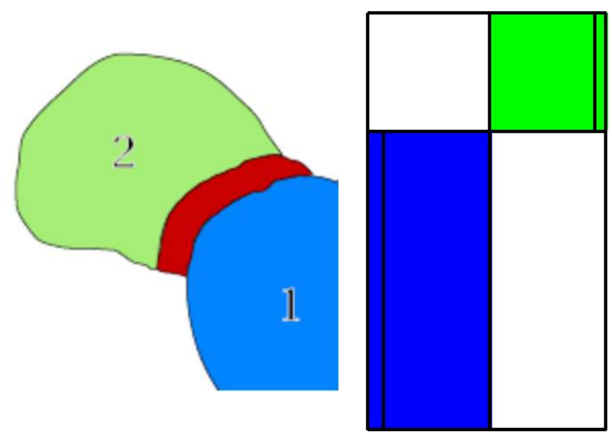


Disjoint components : energy coupling

- Assembly by computation of interface energy (example Arlequin)

Difficulties

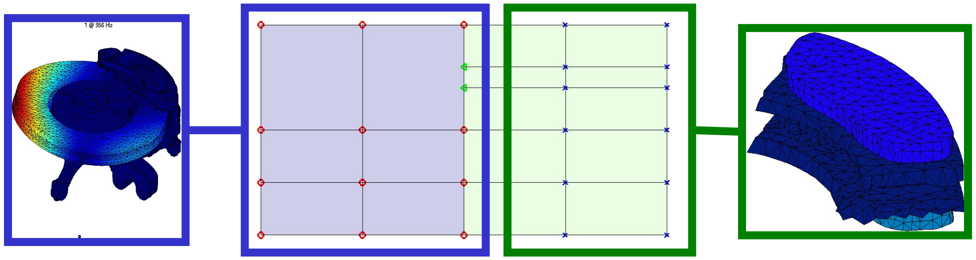
- Use better bases than independent reduction



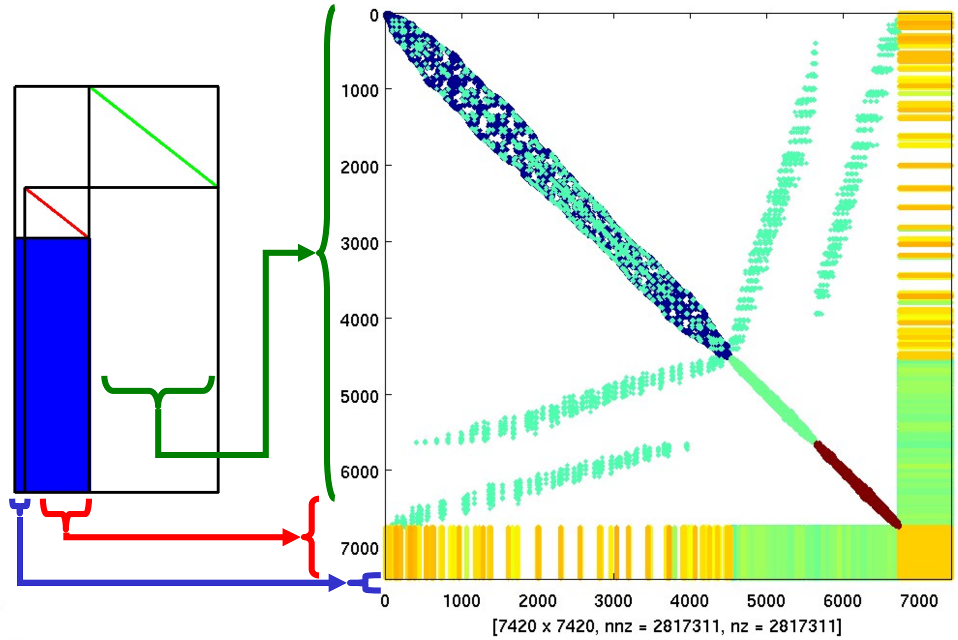
Revised notion of interface

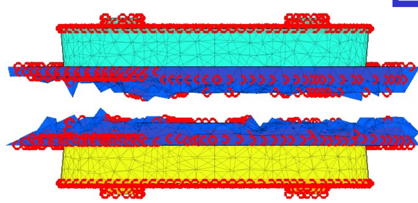
Classical CMS (Craig-Bampton)

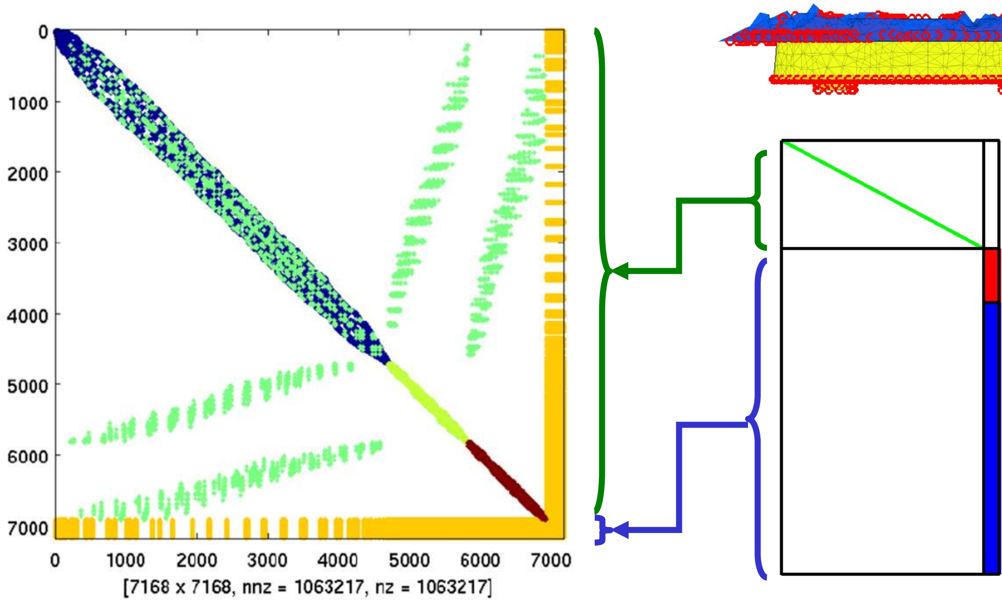
- System is brake without contact area



- Reduction : modes of system and interface loads
- Many interface DOFs needed heavily populated matrix







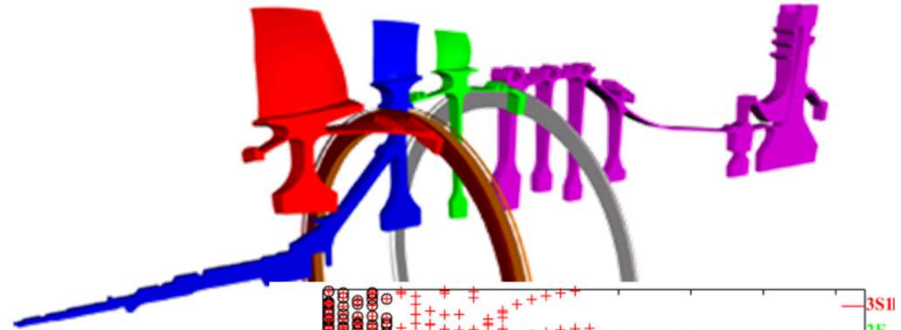
Disjoint component with exact modes

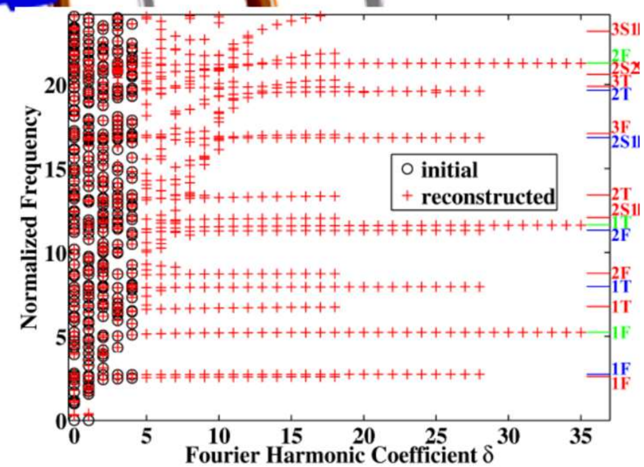
- No reduction of DOFs internal to contact area
- Reduction : trace of full brake modes on reduced area *(no need for static response at interface)*

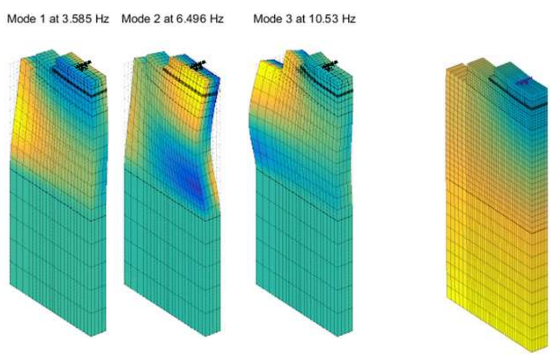
Interface reduction : wave/cyclic

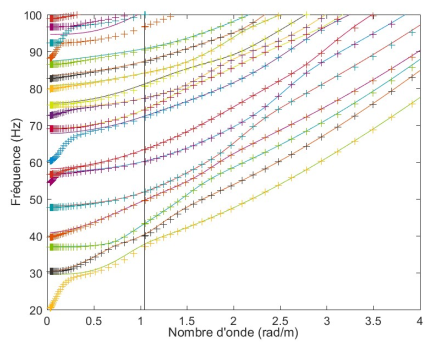
Best interface reduction = learn from full system modes

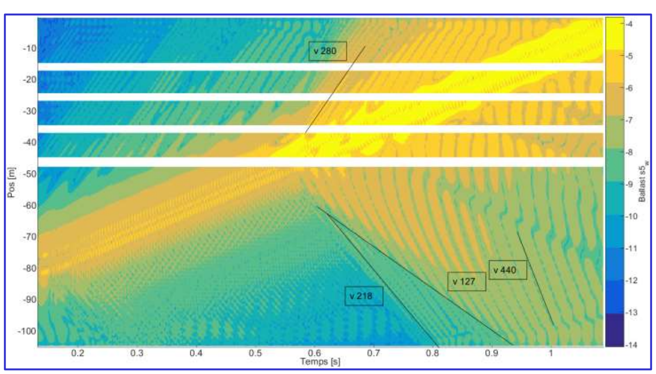
1. Learn using wave (Floquet)/cyclic solutions
2. Build basis with left/right compatibility
3. Assemble reduced model

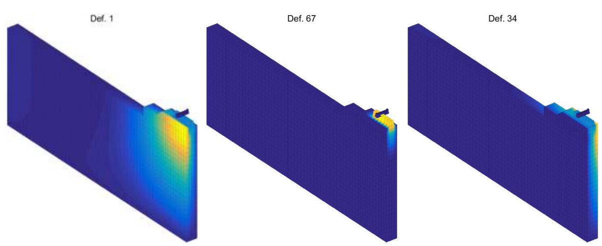


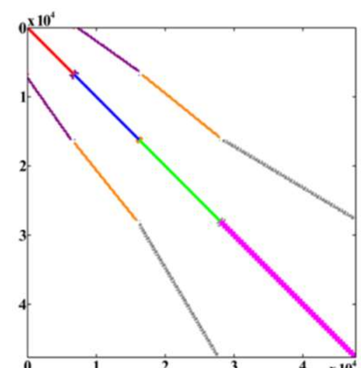












Arlaud, 2016 https://pastel.archives-ouvertes.fr/tel-01455077

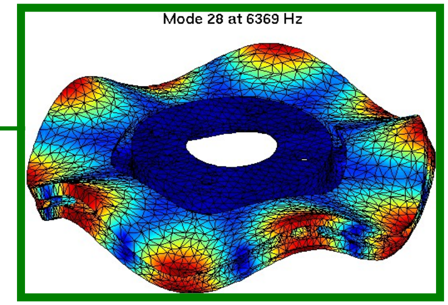
Pinault 2020 https://pastel.archives-ouvertes.fr/tel-03131802

Sternschuss 2008 https://tel.archives-ouvertes.fr/tel-00366252

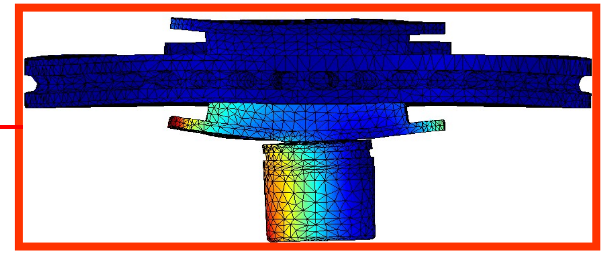
Open issues : nominally exact reduced model

1980 : interest large linear solution
2017 : enhanced coupling

- Component Mode Tuning method
  - free/free real modes (explicit DOFs)
  - trace of the assembled modes on the component

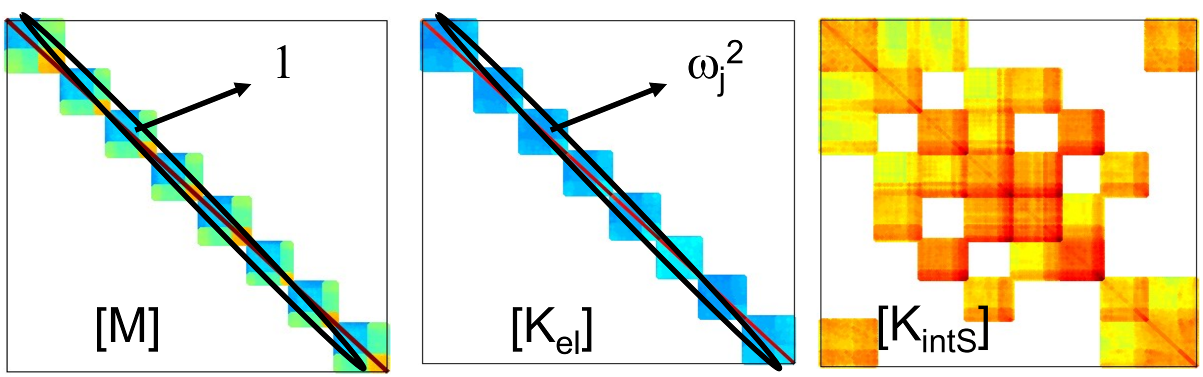


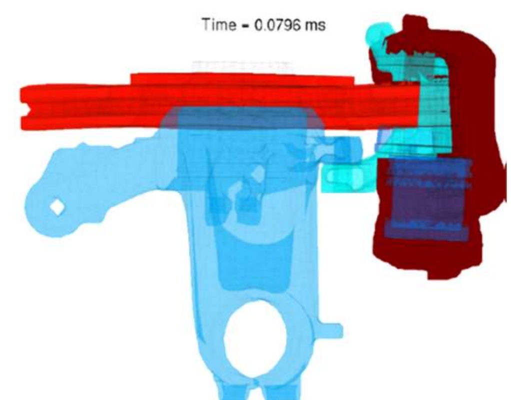
$$[T_{ci}] = \left[[\phi_{ci}] \quad [\Phi_{|ci}]\right]_{Orth.}$$



  - Reduced model is sparse
  - Free mode amplitudes are DOFs
  - Reduced model has exact nominal modes

Disc
OuterPad
Inner Pad
Anchor
Caliper
Piston
Knuckle
Hub

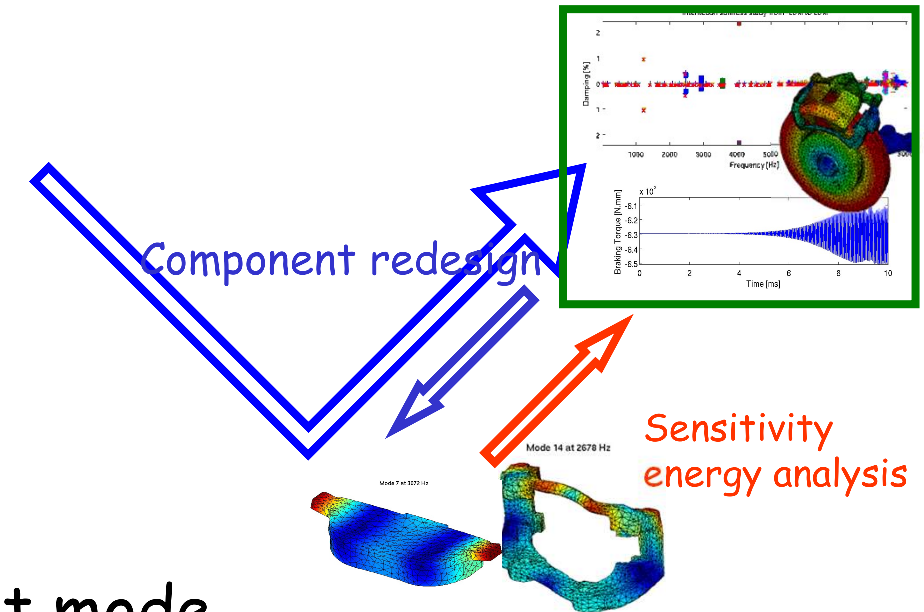




PhD Vermot des Roches 2010
https://tel.archives-ouvertes.fr/tel-00589951

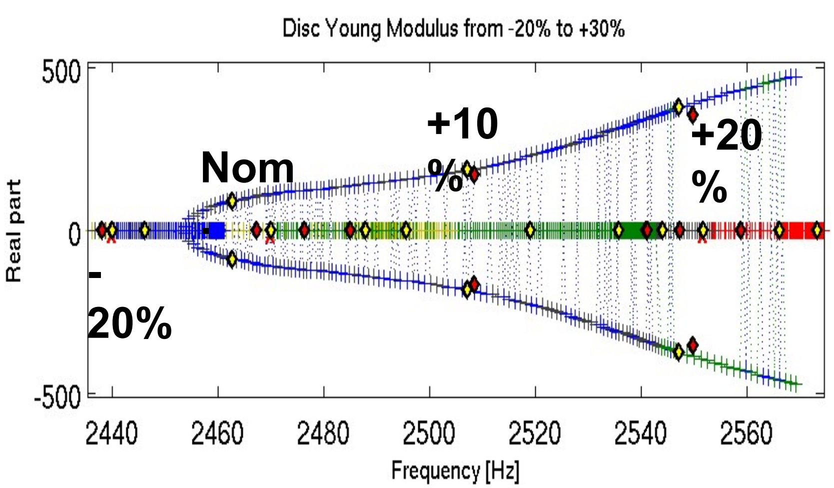
CMT & design studies

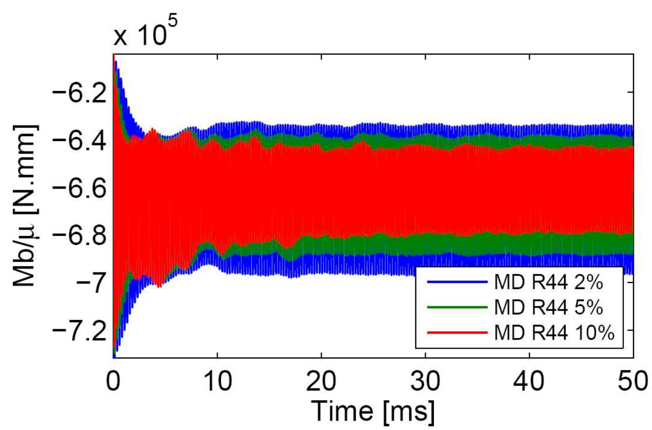
- One reduced model / multiple designs

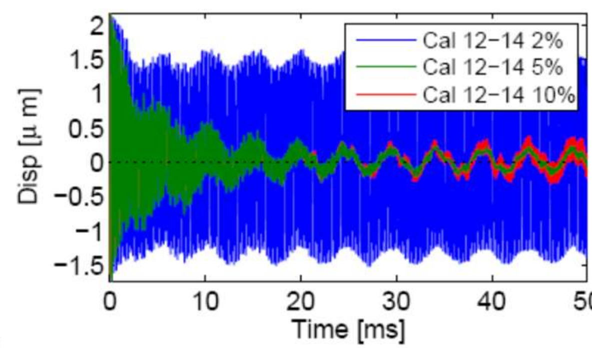


Examples

- impact of modulus change
- damping real system or component mode

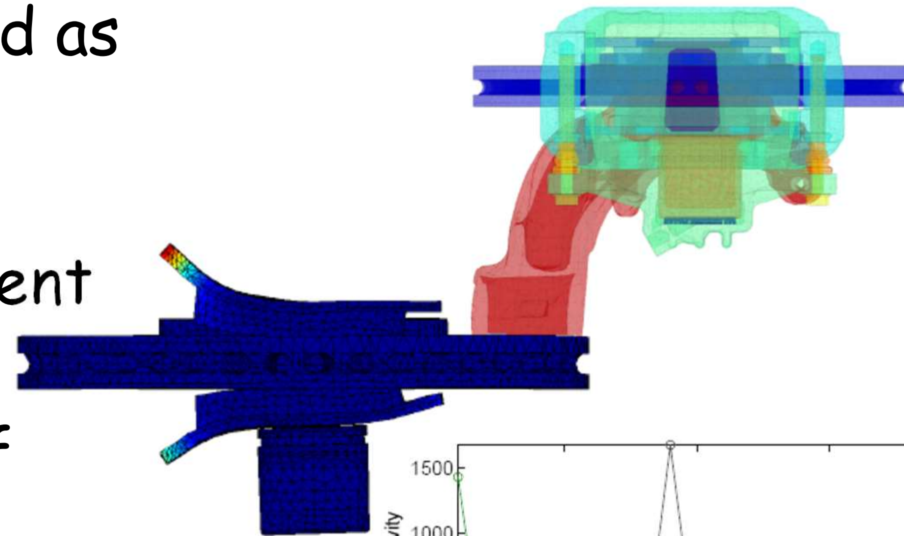


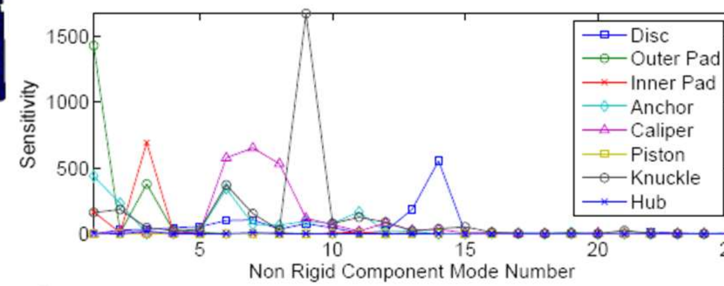


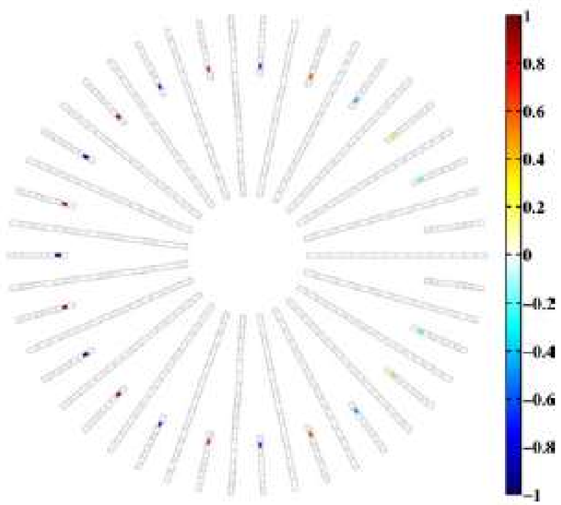


Component modes as design parameters

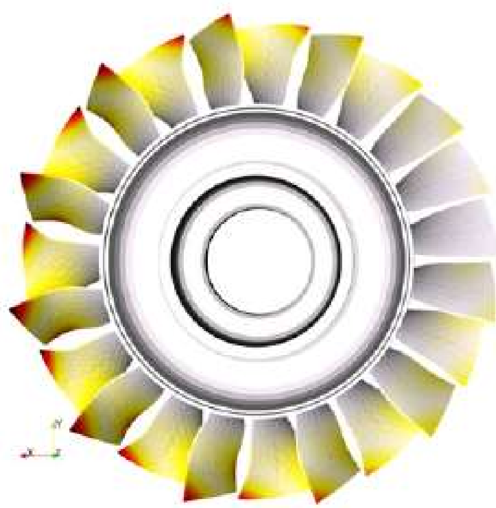
- Component modes can be used as explicit reduced DOFs
- Brake application : which mode of which component should be modified
- Engine application : effect of blade mistuning

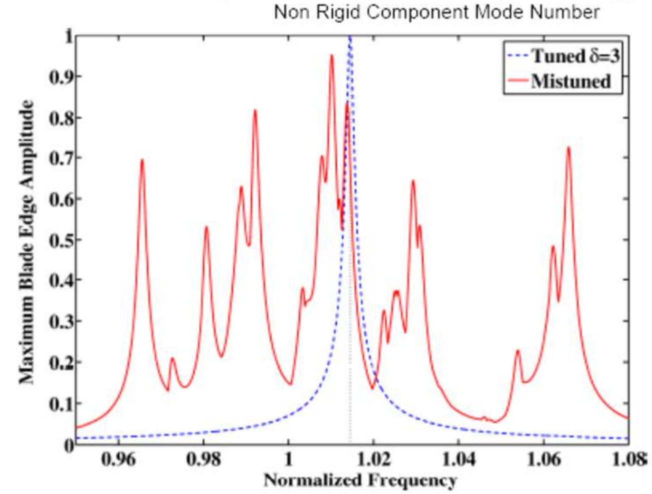






(c)





Réponse forcée