Heat Transfer : Lecture notes

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Section 1

Introduction

Introduction

What is Heat Transfer?

Heat transfer is a science that seeks to predict the energy transfer that may take place between and inside materials as a result of temperature difference.

Remarks

- Thermodynamics deals with energy equilibrium and does not predict how fast the heat transfer will occur.
- In thermodynamics, you have learned two type of energy exchange by interaction of the system with surroundings : work and heat
- It is necessary to introduce physical rules in order to describe the heat transfer rate (in supplement of the two principles of thermodynamics.)

Understand heat transfer

Heat transfer properties

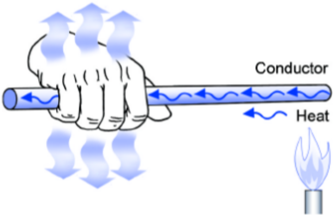
- Heat energy always moves from a warmer body to colder body
- Heat transfer will continue until both bodies have the same temperature

There is three modes of heat transfer

- Conduction
- Convection
- Radiation

Conduction

Exemple



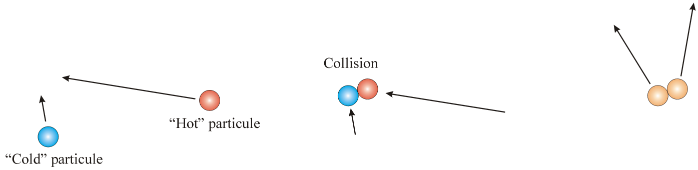
- The candle heat the edge of the bar.
- There is heat transfer in the bar and thus in your hand

Properties of thermal conduction

1. Conduction is a mode of energy transfer within and between bodies.
2. Conduction can occur in all state of matter: gases, liquids, solids or plasma.
3. The ability to transfer heat energy is quantify by the **Thermal Conductivity**
4. Conduction is preponderant in solids.

Conduction : physical mechanisms 1

- GAS or LIQUIDS: conduction is due to collisional and diffusive transfer of kinetics energy of molecule during their random motions. In liquids the conduction phenomenon is stronger because the distances between atoms are smaller (more collisions)



a) particle from the hot side migrate to the cold side

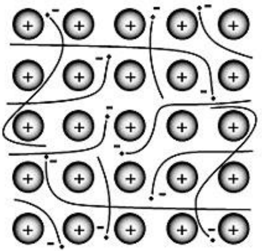
b) hot particle collides the cold particle

c) two particles with similar energy

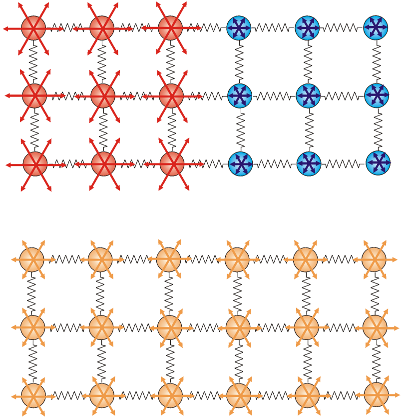
Conduction : physical mechanisms 2

- SOLIDS : for solids there is two mechanisms to explain heat transfer.

  - Free electrons effect (in metals): movement of free electrons in the lattice.

  - Lattice vibration : atoms in lattice vibrate, interact with their neighbors and transfer kinetics energy.

Convection

Definition of convection

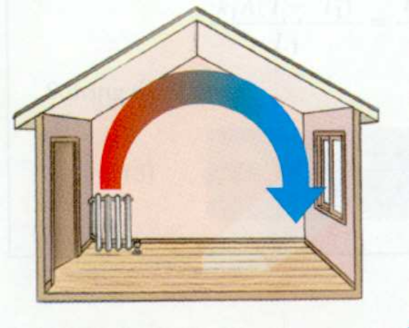
Convection is the transfer of heat by bulk motion (advection).

There are two main modes of convection:

- Natural convection : natural convection occurs due to temperature differences which affect the density of the fluid. Heavier (more dense) components will fall, while lighter (less dense) components rise, leading to bulk fluid movement.
- Forced convection : in the case of forced convection, fluid movement results from external surface forces

Natural convection

Case of gases : radiator in a room



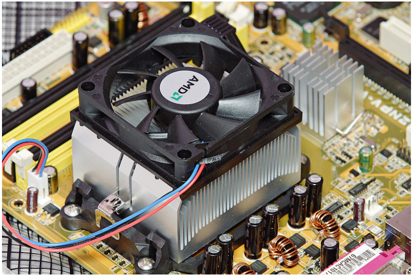
Heated air rises, cools and fall

Case of liquids : water in a pan

Hot water rises, cools and falls

Forced convection

Exemple: heat-sink and fan to cool a processor



- Heat-sink to increase the surface area which dissipates heat
- Fans to speed up the heat exchange.

Radiation

The radiation

Heat transfer by radiation is the transfer of through electromagnetic waves

Example: solar thermal power plant

- Concentrated solar power systems use mirrors to concentrate a large area of sunlight and thus solar thermal energy, onto a small area
- Electrical power is produced when the concentrated light is converted to heat, which drives a heat engine (usually a steam turbine) connected to an electrical power generator.

Contents of the lecture

Objectives

- To establish the physical rules in order to describe the heat transfer rate for conduction, convection and radiation.
- To highlight their consequences on the energy conservation (first law of thermodynamics)

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Section 2

Conduction

Subsection 1

Basis of thermodynamics

Basis of thermodynamics: internal energy

- The internal energy noted U is the total energy contained by a thermodynamic system
- Energy needed to create the system, but excludes
  - the energy to displace the system's surroundings
  - any energy associated with a move as a whole or due to external force fields
- Internal energy has two major components,
  - kinetic energy at microscopic scale: motion of particles (translations, rotations, vibrations),
  - potential energy at microscopic scale: interaction between particles.

Basis of thermodynamics: enthalpy

- The enthalpy is define as

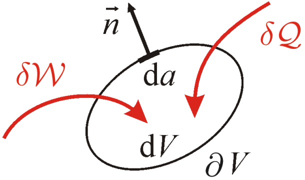
$$H = U + pV$$

- A small variation of enthalpy can also be express as:

$$\mathrm{d}H = \mathrm{d}U + p\mathrm{d}V + V\mathrm{d}p.$$

- Enthalpy includes
  - the internal energy, which is the energy required to create a system,
  - the amount of energy required to make room for it by displacing its environment and establishing its volume and pressure.

Basis of thermodynamics: conservation of the energy



- For a system undergoing only thermodynamics processes, i.e. a closed system that can exchange only heat $\mathcal{Q}$ and work $\mathcal{W}$, the change in the internal energy dU is given by the first law of thermodynamics.

$$\mathrm{d}U = \delta\mathcal{W} + \delta\mathcal{Q}$$

Basis of thermodynamics: particular transformations

Two special cases can be considered:

- For a transformation at constant volume the work of the pressure force work $\delta\mathcal{W} = -p_{ext}\mathrm{d}V$ is equal to zero. We obtain

$$\begin{aligned} \mathrm{d}U &= \delta\mathcal{W} + \delta\mathcal{Q} && (1)\\ &= \delta\mathcal{Q} && (2) \end{aligned}$$

- For a transformation at constant pressure the equilibrium of pressure gives $p_{ext} = p$ and the first law of thermodynamics allows to express the enthalpy variation:

$$\begin{aligned} \mathrm{d}H &= \mathrm{d}U + p\mathrm{d}V + \underbrace{V\mathrm{d}p}_{=0} && (3)\\ &= \delta\mathcal{Q} && (4) \end{aligned}$$

Basis of thermodynamics: heat capacity and calorimetric coefficient

- Heat capacity per unit of mass at constant volume and at constant pressure are define respectively by

$$\frac{1}{m}\left(\frac{\partial U}{\partial T}\right)_V = c_V \qquad \text{and} \qquad \frac{1}{m}\left(\frac{\partial H}{\partial T}\right)_p = c_p \tag{5}$$

- The internal energy and the enthalpy can thus express as:

$$\mathrm{d}U = mc_v \mathrm{d}T + (\ell - p)\mathrm{d}V \quad \text{and} \quad \mathrm{d}H = mc_p \mathrm{d}T + (\bar{h} + V)\mathrm{d}p, \tag{6}$$

with ℓ and $\bar{h}$ two other calorimetric coefficients.

Basis of thermodynamics: heat capacity

- The heat capacity of various material is given in the table
- For a solids and liquids, which are slightly compressible, the heat capacity at constant volume or constant pressure are equal $:c_p = c_v = C$

Table: Heat capacity for various materials at ambient temperature and atmospheric pressure

	c_V in $Jm^{-1}kg{-1}$	c_p in $Jm^{-1}kg{-1}$
air	701	1005
water	4187	4187
steel	460	460

Subsection 2

Heat flux definition

Heat flux definition

- The rate at which heat is transferred across any surface at a point P per unit area per unit time is called the flux of heat at that point across that surface and is denote ϕ.
- This notion can be extended to a tri-dimensional problem. The heat flux vector $\vec{q}$ is then define as

$$\vec{q} = \begin{pmatrix} \phi_x \\ \phi_y \\ \phi_z \end{pmatrix} \tag{7}$$

where ϕ_x, ϕ_y and ϕ_z, are the rate of heat flow across a unit surface with a normal toward respectively x axis, y axis and z axis.

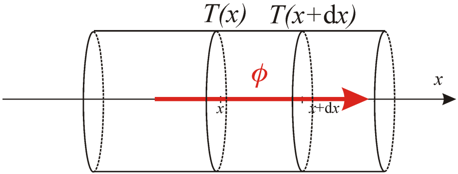
Heat flux properties

- The the heat flux across an infinitesimal surface dS with a normal $\vec{n}$ is $\phi = \vec{q}.\vec{n}\mathrm{d}S$
- The direction of the heat flux vector is the direction of the heat transfer
- The unit of heat flux is W/m^2

Subsection 3

Fourier'law

Fourier's law: 1D case



- Fourier's law of heat conduction is the basic law which says that the rate of heat flow across a unit area ϕ is proportional to the temperature gradient perpendicular to the area (x direction).

$$\phi = -k\frac{\mathrm{d}T}{\mathrm{d}x} \tag{8}$$

- k is called the thermal conductivity. This constant represent the ability to transfer heat through a material. Its unit is $\mathrm{W\,m^{-1}\,K^{-1}}$

Thermal conductivity

- The value thermal conductivity can vary from about 0.01 $Wm^{-1}K^{-1}$ for gases to 1000 $Wm^{-1}K^{-1}$ for pure metals. Following table gives some values of the thermal conductivity for various materials.

Table: Thermal conduction value for classical material at ambient temperature.

Material	Thermal conductivity [$Wm^{-1}K^{-1}$]
air	0.025
wood (white pin)	0.12
rubber	0.16
cement(Portland)	0.29
concrete	0.5
glass	1.1
water	0.58
soil	1.5
ice	2
steel	52
stainless steel	16
aluminium alloy	120-180
pure aluminium	237
copper	401
silver	429

Fourier's in an isotropic material: 3D generalization

- A generalization of the Fourier's law to a tri-dimensionnal isotropic material [1] gives:

$$\vec{q} = -k \, \overrightarrow{\text{grad}} \; T \tag{9}$$

- In the case of Cartesian coordinates:

$$\vec{q} = \begin{pmatrix} \phi_x \\ \phi_y \\ \phi_z \end{pmatrix} = -k \begin{pmatrix} \frac{\partial T}{\partial x} \\ \\ \frac{\partial T}{\partial y} \\ \\ \frac{\partial T}{\partial z} \end{pmatrix} \tag{10}$$

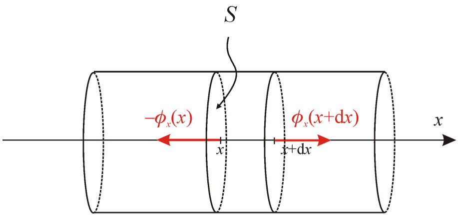
- With this definition the heat flux vector is normal to the isothermal surface.

[1] In an isotopic material the thermal properties are the same in all the space directions

Subsection 4

The differential equation of conduction of heat in an isotropic solid: case of motionless solids

The differential equation of conduction of heat in an isotropic solid: case of motionless solids



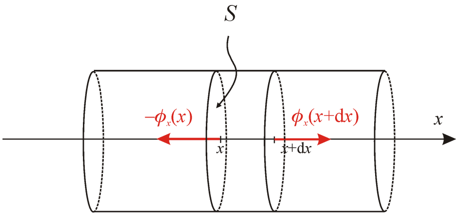
If we suppose a small volume $\mathrm{d}V = S\mathrm{d}x$ during a time dt, the first law of thermodynamics gives

$$\mathrm{d}H = \delta\mathcal{Q} \qquad \text{and then} \qquad \frac{\mathrm{d}H}{\mathrm{d}t} = \frac{\delta\mathcal{Q}}{\mathrm{d}t} \tag{11}$$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = S\mathrm{d}x\rho C\frac{\mathrm{d}T}{\mathrm{d}t} = \underbrace{-\phi_x(x+\mathrm{d}x)S + \phi_x(x)S}_{\text{Heat loss}} + \underbrace{r(x,t)S\mathrm{d}x}_{\text{Heat source}} \tag{12}$$

where $r(x,t)$ the heat source per unit of volume and per unit of time. Its unit is Wm^{-3}.

The differential equation of conduction of heat in an isotropic solid: case of motionless solids



$$\rho C \frac{\partial T}{\partial t} = \frac{-\phi_x(x+\mathrm{d}x) + \phi_x(x)}{\mathrm{d}x} + r(x,t) = -\frac{\partial \phi_x}{\partial x} + r(x,t) \qquad (13)$$

Taking into account of the Fourier's law $\phi_x = -k\frac{\partial T}{\partial x}$, we obtain:

$$\rho C \frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2} + r(x,t) \qquad (14)$$

Subsection 5

Boundary conditions

Boundary conditions

- $T(\ell) = T_0$: imposed temperature (Dirichlet condition)

- $\varphi(\ell) = \varphi_0 = -k \left.\frac{\partial T}{\partial x}\right|_\ell$: imposed flux (Neumann condition)

Boundary conditions

- $\varphi(\ell) = 0$: adiabatic condition

- $\varphi(\ell) = h(T(\ell) - T_a)$ convection condition with a fluid
  - h: convection coefficient($\mathrm{W\,m^{-2}\,K^{-1}}$)
  - T_a: fluid temperature

Boundary conditions

- Radiation: $\varphi(\ell) = \sigma F(T(\ell)^4 - T_p^4)$
  - σ Stefan Boltzmann constant
  - F view factor
  - temperature of the other surface

- Particular case of small temperature difference: $T(\ell) \approx T_p \approx T_m$:

$$\varphi = h_r(T(\ell) - T_p) \qquad \text{with} \qquad h_r = 4\sigma F T_m^3$$

- Particular case of high temperature T_p: $\varphi = -\sigma F T_p^4$ (Neumann condition)

Subsection 6

Particular thermal regimes

Particular thermal regimes

- Steady state regime: temperature field does not change any further:

$$\frac{\partial T}{\partial t} = 0, \qquad \frac{\partial \vec{\varphi}}{\partial t} = \vec{0}$$

$$T(M,t) = T(M)$$

- The periodical regime: the temperature oscillate independently of the initial conditions

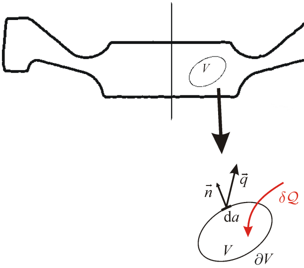
$$T(M,t) = T_0(M)\cos(\Omega t + \Phi)$$

and with a complex notation

$$\overline{T}(M,t) = T_0(M)e^{j\omega t}e^{j\Phi} \qquad \text{with} \qquad T(M,t) = \Re(\overline{T}(M,t))$$

The differential equation of conduction of heat in an isotropic solid: case of motionless solids

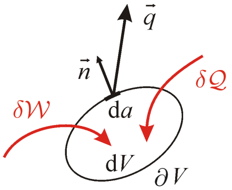
In the general 3D case, a small volume V is considered:



If we postulate the local equilibrium of this volume, the application of the first law of thermodynamics between time t and time $t + \mathrm{d}t$ for a transformation at pressure volume gives:

$$\mathrm{d}H = \delta\mathcal{Q} \tag{15}$$

The differential equation of conduction of heat in an isotropic solid: case of motionless solids



$$\frac{1}{\mathrm{d}t}\iiint_V \mathrm{d}h\mathrm{d}V = -\iint_{\partial V} \vec{q}.\vec{n}\mathrm{d}a + \iiint_V r(x,t)\mathrm{d}V \tag{16}$$

$$= \iiint_V \mathrm{div}\,\vec{q}\mathrm{d}V + \iiint_V r(x,t)\mathrm{d}V \tag{17}$$

with $\mathrm{d}h = \rho C \mathrm{d}T = \rho C \dot{T}\mathrm{d}t$ the variation of the specific enthalpy (enthalpy per unit of volume).

The differential equation of conduction of heat in an isotropic solid: case of motionless solids

The heat equation is then

$$\rho C \dot{T} = -\text{div } \vec{q} + r(x,t) \tag{18}$$

And with the Fourier's law the heat equation becomes

$$\rho C \dot{T} = \text{div} \left(k \overrightarrow{\text{grad}} \; T \right) + r(x,t) \tag{19}$$

If the heat conductivity is suppose to be independent of the space position

$$\rho C \dot{T} = k \Delta T + r(x,t) \tag{20}$$

with Δ the Laplacian operator.

Coordinate expressions of the Laplacian operator

Cartesian coordinates

$$\Delta f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} \tag{21}$$

Cylindrical coordinates

$$\Delta f = \frac{\partial^2 f}{\partial r^2} + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \vartheta^2} + \frac{\partial^2 f}{\partial z^2} \tag{22}$$

with $(x = r\cos\vartheta, y = r\sin\vartheta, z)$.

Spherical coordinates

$$\Delta f = \frac{\partial^2 f}{\partial r^2} + \frac{2}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \varphi^2} + \frac{1}{r^2\tan\varphi}\frac{\partial f}{\partial \varphi} + \frac{1}{r^2\sin^2\varphi}\frac{\partial^2 f}{\partial \vartheta^2} \tag{23}$$

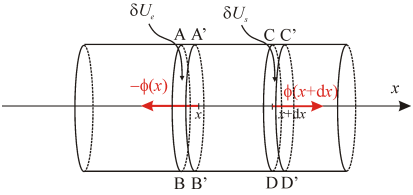
with $(x = r\sin\varphi\cos\vartheta, y = r\sin\varphi\sin\vartheta, z = r\cos\varphi)$.

Subsection 7

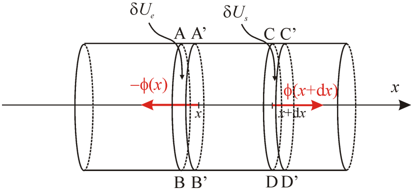
The differential equation of conduction of heat in an isotropic solid: case of moving solids

The differential equation of conduction of heat in an isotropic solid: case of moving solids

- Case of 1D problem:
  - We consider a solid medium moving with a velocity v_x along the x axis.
  - At time t we consider a volume of material contain in the domain ABCD. Between time t and $t + \mathrm{d}t$ the materials move from the domain ABCD to the domain A'B'C'D'.
  - $\mathrm{d}H$, δH_e and δH_s represent the enthalpy respectively in the A'B'CD, ABA'B' and CDC'D' domains.



The differential equation of conduction of heat in an isotropic solid: case of moving solids



- At time t the enthalpy contain in domain ABCD is

$$\mathrm{d}H(t) + \delta H_e \tag{24}$$

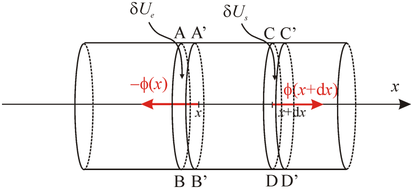
At time $t + \mathrm{d}t$ the material move in A'B'CD and the enthalpy in this domain is

$$\mathrm{d}H(t + \mathrm{d}t) + \delta H_s \tag{25}$$

- The first law of thermodynamics applied to the materials in the ABCD domain at time t and between t and $t + \mathrm{d}t$ is

$$\mathrm{d}H(t + \mathrm{d}t) + \delta H_s - \mathrm{d}H(t) - \delta H_e = \delta \mathcal{Q} \tag{26}$$

The differential equation of conduction of heat in an isotropic solid: case of moving solids



The different term of this last equation can be expressed as follow:

- $\mathrm{d}H(t+\mathrm{d}t) = S\mathrm{d}x\rho CT(t+\mathrm{d}t)$
- $\mathrm{d}H(t) = S\mathrm{d}x\rho CT(t)$
- $\delta H_s = Sv_x\mathrm{d}t\, T(x+\mathrm{d}x)$
- $\delta H_e = Sv_x\mathrm{d}t\, T(x)$

We then obtain

$$S\mathrm{d}x\rho CT(t+\mathrm{d}t) - S\mathrm{d}x\rho CT(t) + Sv_x\mathrm{d}t\, T(x+\mathrm{d}x) - Sv_x\mathrm{d}t\, T(x) = -S\phi_x(x+\mathrm{d}x) + S\phi_x(x) \quad (27)$$

and

$$\rho C\frac{\partial T}{\partial t}\mathrm{d}t\mathrm{d}x + \rho Cv_x\frac{\partial T}{\partial x}\mathrm{d}x\mathrm{d}t = -\frac{\partial \phi_x}{\partial x}\mathrm{d}x\mathrm{d}t \quad (28)$$

The differential equation of conduction of heat in an isotropic solid: case of moving solids

$$\rho C \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} \right) = -\frac{\partial \phi_x}{\partial x} \tag{29}$$

and with the Fourier's law and a constant heat conductivity

$$\rho C \frac{DT}{Dt} = \rho C \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} \right) = k \frac{\partial^2 T}{\partial x^2} \tag{30}$$

where $\frac{DT}{Dt}$ denotes the 'differentiation following the motion of the temperature with respect to the time.

The differential equation of conduction of heat in an isotropic solid: case of moving solids

The previous heat equation can be generalized to a three-dimensional problem:

$$\rho C \frac{DT}{Dt} = \rho C \left(\frac{\partial T}{\partial t} + v_x \frac{\partial T}{\partial x} + v_y \frac{\partial T}{\partial y} + v_z \frac{\partial T}{\partial z} \right) = \rho C \left(\frac{\partial T}{\partial t} + (\vec{v}.\overrightarrow{\text{grad}})\ T \right)$$
$$= k \Delta T \qquad (31)$$

Section 3

Convection

Subsection 1

Introduction on convection

Introduction to convection

- **Convection**: heat transfer in a fluid or between a fluid and a solid due to *bulk motion* (advection)
- Two main types of thermal convection:
  - **Natural convection** (or free convection): the fluid motion is due to body force which acts on a fluid where there is *density gradient*. The net effect is a *buoyancy force* which induces fluid movement. In the most common case, the density gradient is do to *temperature gradient*.
  - **Forced convection**: the relative motion in the fluid or between the fluid and the surface is maintained by *external means* such as a fan or a pump.

Since free convection flow velocities are generally much smaller than those associated with forced convection, the corresponding convection transfer rates are also smaller.

Subsection 2

Convection heat transfer coefficient

Convection heat transfer coefficient

- Estimation of the heat flux due to convection on a fluid surface interface:

Typical value of the convection heat transfer coefficient

Process	h in $\mathrm{W\,m^{-2}\,K^{-1}}$
Free convection in gases	2 - 25
Free convection in fluids	50 - 1000
Forced convection in gases	25 - 250
Forced convection in fluids	50 - 20000
Convection with phase change (Boiling or condensation)	25000 - 100000

How to estimate the convection heat transfer coefficient in a given configuration ?

$$h = f(C, k_f, \rho, \mu, \alpha, L, V, T...)$$

Subsection 3

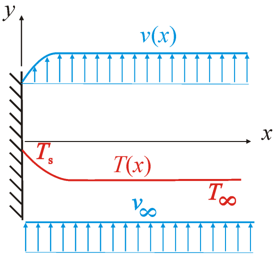
Forced convection

Forced convection : associated equations

Hypothesis :

- steady state regime
- incompressible fluid: $\rho = \text{Cst}$

The momentum equation is (Navier Stockes equation):

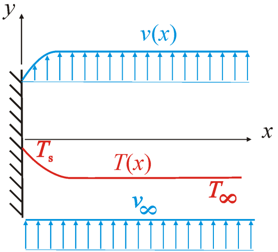


$$\rho \frac{\mathrm{d}\vec{v}}{\mathrm{d}f} = \rho \vec{g} - \overrightarrow{\text{grad}}\, p + \mu \vec{\Delta} \vec{v}$$

Projection along y axis:

$$\rho \left(u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) + \underbrace{\rho \frac{\partial v}{\partial t}}_{=0} = \rho g - \frac{\partial p}{\partial y} + \mu \left(\frac{\partial^2 v}{\partial x^2} + \underbrace{\frac{\partial^2 v}{\partial y^2}}_{\approx 0} \right)$$

Forced convection : associated equations



Boundary conditions :

$$u(0,y) = 0; v(0,y) = 0; v(\infty, y) = v_\infty$$

Mass equation in steady state regime:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Heat equation:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} + \underbrace{\frac{\partial T}{\partial t}}_{=0} = \frac{k_f}{\rho C}\left(\frac{\partial^2 T}{\partial x^2} + \underbrace{\frac{\partial^2 T}{\partial y^2}}_{\approx 0}\right)$$

with $a = \frac{k_f}{\rho C}$ the heat diffusivity in $m^2 s^{-1}$
Boundary conditions :

$$T(0,y) = T_s; T(\infty, y) = T_\infty$$

Forced convection : dimensionless variables and equations

Definition of dimensionless variables:

- $x^* = \frac{x}{L}$, $y^* = \frac{y}{L}$, where L is a characteristic length for the problem (i.e. length of the plate),
- $u^* = \frac{u}{V}$, $v^* = \frac{v}{V}$, where V is the velocity upstream of the surface,
- $T^* = \frac{T - T_\infty}{T_s - T_\infty}$, the dimensionless temperture,
- $p^* = \frac{p}{\rho V^2}$, the dimensionless pressure

Dimensionless momentum equation:

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = \frac{\partial p^*}{\partial y^*} + \underbrace{\mu \frac{L}{\rho V} \frac{V}{L^2}}_{= \frac{\mu}{\rho L V} = \frac{1}{Re}} \frac{\partial^2 v^*}{\partial {x^*}^2}$$

with Re the reynolds number : $Re = \frac{\text{Inertia forces}}{\text{Viscous forces}} \approx \frac{\frac{\rho V^2}{L}}{\frac{\mu V^2}{L^2}}$

Forced convection: dimensionless variables and equations

Dimensionless heat equation equation:

$$u^* \frac{\partial T^*}{\partial x^*} + v^* \frac{\partial T^*}{\partial y^*} = \frac{a}{VL} \frac{\partial^2 T^*}{\partial {x^*}^2}$$

with

$$\frac{a}{VL} = \frac{k_f}{\rho C} \frac{1}{VL} = \frac{1}{Re} \frac{\rho}{\mu} \frac{k_f}{\rho C} = \frac{1}{Re} \underbrace{\frac{k_f}{\mu C}}_{\frac{1}{Pr}}$$

Pr the Prandt number : $Pr = \frac{\text{Momentum diffusion}}{\text{Heat diffusion}} \approx \frac{\frac{\mu}{\rho}}{a}$

with $\frac{\mu}{\rho}$ the kinematic viscosity in $\mathrm{m^2 s^{-1}}$

Forced convection: dimensionless variables and equations

Boundary condition of the heat transfer problem:

$$\varphi = h(T_S - T_\infty)$$

and

$$\varphi = -k_f \left.\frac{\partial T}{\partial x}\right|_{x=0} = -k_f \left.\frac{\partial T^*}{\partial x^*}\right|_{x^*=0} \frac{T_\infty - T_s}{L}$$

thus

$$h = \frac{k_f}{L} \left.\frac{\partial T^*}{\partial x^*}\right|_{x^*=0}$$

and the Nusselt number is defined as

$$Nu = \left.\frac{\partial T^*}{\partial x^*}\right|_{x^*=0} = \frac{hL}{k_f}$$

The Nusselt number represents the ratio of the heat flux due to convection to the heat flux:

$$Nu = \frac{\varphi_{\text{convection}}}{\varphi_{\text{conduction}}} = \frac{h\Delta T}{k_f \frac{\Delta T}{L}} = \frac{hL}{k_f}$$

Correlation in forced convection

The equilibrium equations prompt us to expect heat transfer correlation of the form:

$$g(Nu, Re, Pr) = 0$$

or

$$Nu = f(Re, Pr)$$

Example of correlation in forced convection

- Laminar flow on a plate: $Re < 3 \times 10^5$ and $Pr > 0.5$

$$Nu = 0.664 Re^{1/2} Pr^{1/3}$$

- Turbulent flow on a plate: $Re > 5 \times 10^5$ and $Pr > 0.5$

$$Nu = 0.035 Re^{4/5} Pr^{1/3}$$

Example of correlation in forced convection

- Laminar flow in a pipe: $Re < 2300$

$$Nu = 3.66$$

- Turbulent flow in a pipe: $Re > 10^4$

$$Nu = 0.023 Re^{4/5} Pr^{0.4}$$

Subsection 4

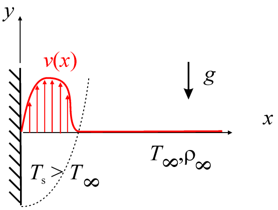
Natural convection

Natural convection

hypothesis :

- steady state regime
- gravity force is taken into account

The momentum equation is



$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} - g + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial x^2}$$

Far from the surface, a quasistatic pressure is supposed:

$$\frac{\partial p}{\partial y} = -\rho_\infty g$$

and thus

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \frac{g}{\rho}\left(\rho_\infty - \rho\right) + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial x^2}$$

Natural convection

The volumetric thermal expansion coefficient

$$\beta = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_p = -\frac{1}{\rho}\left(\frac{\partial \rho}{\partial T}\right)_p$$

and thus

$$\beta \approx -\frac{1}{\rho}\frac{\rho_\infty - \rho}{T_\infty - T}$$

Density difference is thus

$$\rho_\infty - \rho \approx \rho\beta(T - T_\infty)$$

The momentum equation becomes:

$$u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = \underbrace{g\beta(T - T_\infty)}_{\text{Buoyancy force}} + \frac{\mu}{\rho}\frac{\partial^2 v}{\partial x^2}$$

The heat equation remains unchanged:

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \frac{k}{\rho C}\frac{\partial^2 T}{\partial x^2}$$

Normalized equations of natural convection

Normalization of variables: $x^* = \frac{x}{L}$, $y^* = \frac{y}{L}$, $u^* = \frac{u}{V}$, $v^* = \frac{v}{V}$ and $T^* = \frac{T - T_\infty}{T_s - T_\infty}$
where L is a characteristic length and V an arbitrary reference velocity.
The momentum equation is reduced to:

$$u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = \frac{g\beta(T_s - T_\infty)L}{V^2} T^* + \frac{1}{Re} \frac{\partial^2 v^*}{\partial {x^*}^2}$$

and

$$\frac{g\beta(T_s - T_\infty)L}{V^2} = \underbrace{\frac{g\beta(T_s - T_\infty)L^3}{\nu^2}}_{Gr} \underbrace{\left(\frac{\nu}{VL}\right)^2}_{1/Re^2}$$

where Gr is the Grashof number which indicates the ratio of buoyancy force to the viscous force acting in the fluid.

Correlation in natural convection

The equilibrium equations prompt us to expect heat transfer correlation of the form:

$$Nu = f(Re, Gr, Pr)$$

This type of correlations are pertinent when forced and free occur. Generally $\frac{Gr}{Re^2} >> 1$ and force convection effects may be neglected and:

$$Nu = f(Gr, Pr)$$

In this case the natural convection flow is induced solely by buoyancy forces.

Example of correlation in natural convection

- Vertical plate in laminar flow if $10^4 < Ra = GrPr < 10^9$ and $T_s = Cst$:
$$Nu = 0.59 Ra^{1/4}$$
- Verticale plate in turbulent flow ($10^9 < Ra < 10^{12}$)
$$Nu = 0.13 Ra^{1/3}$$
- Vertical cylinder (length L):
  - if $\frac{D}{L} \geq 35 Gr^{-1/4}$ plate correlation
  - if $\frac{D}{L} \geq 35 Gr^{-1/4}$:
$$Nu \exp\left(\frac{-2}{Nu}\right) = 0.6 \left(\frac{D}{L}\right)^{1/4} Ra^{1/4}$$

Section 4

Thermal radiation

Subsection 1

The nature of thermal radiation

The nature of thermal radiation

Thermal radiation can be viewed as consisting of:

- *Electromagnetic waves:* as predicted by electromagnetic wave theory (Maxwell's equation)
- *Massless energy parcels called photons:* as predicted by quantum mechnics

Remarks: neither point of view is able to describe completely all radiative phenomena that have been observed (both concepts will be used)

Properties of electromagnetic waves and photons

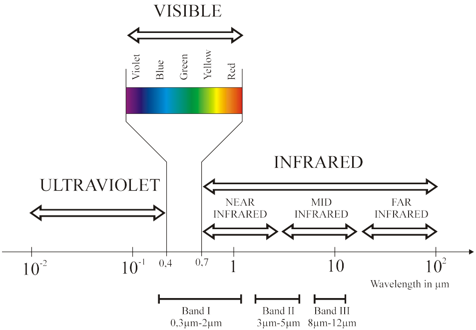
- Electromagnetic waves or photons propagate through any medium at a velocity $c = \frac{c_0}{n}$ where :
  - c_0 the **speed of light** in vacuum ($c_0 = 2.998 \times 10^{-8}$ m/s)
  - n the **refractive index** of the medium (for the air at room temperature and atmospheric presssure, $n = 1.00029$
- a wave or a photon can be chatacterized by:
  - its **frequency**: ν (unit:Hz)
  - its **wavelength**: λ (unit: m
  - its **angular frequency**: ω (unit: rad/s)
  - the **photon energy**: $\mathcal{E} = h\nu$ where $h = 6.626 \times 10^{-34}$ J s is the planck's constant

These quantities are related to one another through the formula:

$$\nu = \frac{\omega}{2\pi} = \frac{c}{\lambda}$$

Spectral range of thermal radiation

- Visible range: $\lambda = 0.5\,\mu\text{m}$
  - $\nu = 6.0 \times 10^{14}$ Hz
  - $\omega = 9.5 \times 10^{13}$ rad/s
  - $\mathcal{E} = 3.0 \times 10^{-19}\,\text{J} = 2.5\,\text{eV}$
- Near infrared range: $\lambda = 5\,\mu\text{m}$
  - $\nu = 6.0 \times 10^{13}$ Hz
  - $\omega = 9.5 \times 10^{12}$ rad/s
  - $\mathcal{E} = 3.0 \times 10^{-20}\,\text{J} = 0.25\,\text{eV}$



Thermal radiation is defined to be those electromagnetic waves which are emitted by a medium due solely to its temperature. The associated spectral tange i between 0.1 µm and more than 100 µm.

Subsection 2

Radiation quantities

The total emissive power

- The **total emissive power** noted E is the energy emitted by an object per unit of time and per unit object surface

$$E = \frac{\mathcal{E}}{S\Delta T}$$

with

- E: emissive power in $\mathrm{W\,m^{-2}}$
- $\mathcal{E}$: emitted energy in J
- S: object surface in m^2
- Δt: time in s

thus

$$\mathrm{d}\mathcal{E} = E\mathrm{d}t\mathrm{d}S$$

E depends on the **local temperature** of the surface and on the **physical surface properties**.

Spectral emissive power

- The **spectral emissive power** noted E_λ is the energy emitted by an object per unit of time, per unit object surface and per unit spectral range

$$E_\lambda = \frac{\mathrm{d}E}{\mathrm{d}\lambda}$$

- properties:

$$dE = E_\lambda \mathrm{d}\lambda$$

$$E = \int_0^\infty E_\lambda \mathrm{d}\lambda$$

$$\mathrm{d}\mathcal{E} = E_\lambda \mathrm{d}t \mathrm{d}S \mathrm{d}\lambda$$

The solid angle

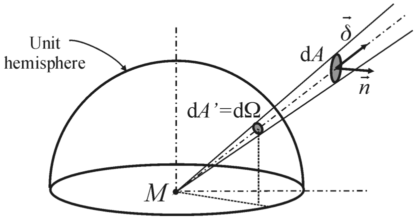
- The solid angle noted Ω is a measure of the amount of the field of view from one particular point M that a given object or surface covers.

dA' is the associated surface of dA on the unit hemisphere and called the solid angle $d\Omega$:

$$dA' = d\Omega$$

The solid angle has no unit. It is expressed in steradian (sr).

The solid angle of a sphere and an hemisphere are respectively 4π sr and 2π sr.



The solid angle

- Expression of the solid angle of a small surface $\mathrm{d}A$ in P with a normal $\vec{n}$ from the point M.

$\mathrm{d}A_p$ is the projection of $\mathrm{d}A$ on the plane with the normal $\vec{\delta}$.

$$\mathrm{d}A_p = \vec{\delta}.\vec{n}\,\mathrm{d}A = \cos\theta\,\mathrm{d}A$$

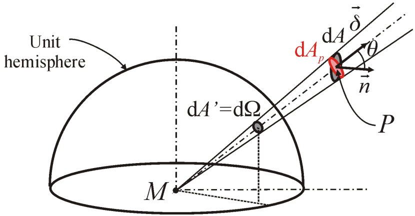
The solid angle is then:

$$\mathrm{d}\Omega = \mathrm{d}A' = \frac{\mathrm{d}A_p}{r^2}$$

where $r = MP$.

And thus:

$$\mathrm{d}\Omega = \frac{\cos\theta \mathrm{d}A}{r^2} = \frac{\vec{\delta}.\vec{n}\mathrm{d}A}{r^2}$$



Solid angle

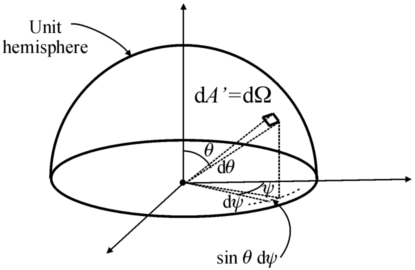
- Properties of the solid angle

- Small solid angle in spherical coordinates:

$$d\Omega = \sin\theta \, d\psi \, d\theta$$

- For all the hemisphere surface

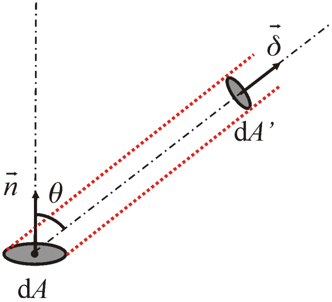
$$\Omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} \sin\theta \, d\theta \, d\psi = 2\pi$$



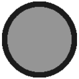
Radiative intensity

Estimation of the energy radiated by a surface dA in a direction $\vec{\delta}$

- Definition of the area normal to rays dA'



$\theta = 0^\circ$



$\mathrm{d}A' = \mathrm{d}A$

$0^\circ < \theta < 90^\circ$

$\mathrm{d}A' = \cos\theta\,\mathrm{d}A$

$\mathrm{d}A' = \vec{\delta}.\vec{n}\mathrm{d}A$

$\theta = 90^\circ$

$\mathrm{d}A' = 0$

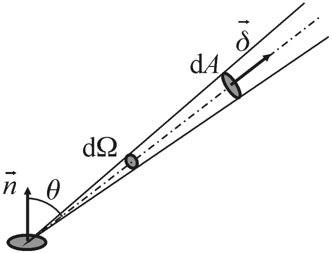
Radiative intensity

- The **radiative intensity** noted I is en energy emitted by a surface with an unit surface normal to the rays, per unit of time and solid angle

$$I(\vec{\delta}) = \frac{\mathrm{d}\mathcal{E}}{\mathrm{d}A'\mathrm{d}\Omega\mathrm{d}t}$$

or

$$I(\vec{\delta}) = \frac{\mathrm{d}\mathcal{E}}{\vec{\delta}.\vec{n}\mathrm{d}A\mathrm{d}\Omega\mathrm{d}t}$$



- The **radiative spectral intensity** noted I_λ is the energy emitted by a surface per unit surface normal to the rays, per unit of time, per unit of solid angle and per unit of spectral range

$$I_\lambda(\vec{\delta}, \lambda) = \frac{\mathrm{d}\mathcal{E}}{\vec{\delta}.\vec{n}\mathrm{d}A\mathrm{d}\Omega\mathrm{d}t}\mathrm{d}\lambda$$

Relationship between radiative intensity and emissive power

If the radiative intensity does not depend on $\vec{\delta}$:

$$E = \int_{hemisphere} I \vec{\delta}.\vec{n} \mathrm{d}\Omega = \int_0^{2\pi} \int_0^{\frac{\pi}{2}} I \cos\theta \sin\theta \mathrm{d}\theta \mathrm{d}\psi$$

after calculations

$$E = \pi I$$

and we have the same type of equation with the spectral quantities

$$E_\lambda = \pi I_\lambda$$

Subsection 3

Basic law of thermal radiation

Surface properties

Lets us consider an incident wave witch will interact with a medium

- (i): incident wave (energy rate: q_i)
- (r): reflected wave (energy rate: q_r)
- (a): absorbed wave (energy rate: q_a)
- (t): transmitted wave (energy rate: q_t)

It allows to define the following coefficient:

- $\rho = \frac{q_r}{q_i}$ the reflectance,
- $\alpha = \frac{q_a}{q_i}$ the absoptance,
- $\tau = \frac{q_t}{q_i}$ the transmittantce.

Surface properties

- Energy balance for the incident wave:
- case of tranparent medium:
- case of semi-transparent:
- opaque medium

Perfect absorber definition

- DEFINITION: a perfect absorber (also called a black surface) is an opaque medium that does not reflect any radiation:

$$q_r = 0 \qquad \text{and} \qquad q_t = 0$$

- PROPERTIES:
  - $q_i = q_a$ and thus $\alpha = 1$ (all the incident energy is absobed by the surface)
  - a black surface emits a maximum amount of radiated energy i.e. more than any other surface at the same temperature

Radiative intensity of the black surface

The spectral intensity of the black surface is given by the Planck's law:

$$I_\lambda^0(\lambda, T) = \frac{2hc^2\lambda^{-5}}{\exp\left(\frac{hc}{k\lambda T}\right) - 1}$$

where

- k=$1.380\,662 \times 10^{-23}\,\mathrm{J\,K^{-1}}$ the Boltzmann's constant,
- h=$6.626\,176 \times 10^{-34}$ Js the Planck's constant
- c=$2.998 \times 10^{8}\,\mathrm{ms^{-1}}$ the light speed in vacuum

And thus

$$E_\lambda^0(\lambda, T) = \frac{2\pi hc^2\lambda^{-5}}{\exp\left(\frac{hc}{k\lambda T}\right) - 1}$$

It is customary to introduce:

- $C_1 = 2\pi hc^2 = 3.7418 \times 10^{-16}\,\mathrm{Wm^2}$
- $C_2 = \frac{hc}{k} = 143\,883.7418 \times 10^{-16}\,\mu\mathrm{mK}$

Properties of the Planck's law

- I_λ^0 is independent of the considered direction $\vec{\delta}$
- Wien's displacement law:

$$\lambda_{max} = \frac{2898\,\mu\mathrm{mK}}{T}$$

- Total emissive power of a black surface:

$$E^0(T) = \int_0^\infty E_\infty^0(T,\lambda)\mathrm{d}\lambda = \sigma T^4$$

where
$\sigma = \frac{2\pi^5 k^4}{15 h^3 c^2} = 5.67 \times 10^{-8}\,\mathrm{Wm^{-2}K^{-4}}$
the Stefan-Boltzmann constant

Emissivity

- To characterize the radiation properties of a real surface, its radiation is compared with the radiation of a black surface at the same temperature.
- Several emissivity can be defined:
  - The hemispherical emissivity:

$$\varepsilon = \frac{E}{E^0}$$

  - The spectral hemispherical emissivity:

$$\varepsilon_\lambda(\lambda) = \frac{E_\lambda(\lambda)}{E^0_\lambda}$$

  - The directional emissivity:

$$\varepsilon_{\vec{\delta}}(\vec{\delta}) = \frac{I(\vec{\delta})}{I^0}$$

  - The spectral directional emissivity:

$$\varepsilon_{\lambda,\vec{\delta}}(\lambda, \vec{\delta}) = \frac{I_\lambda(\lambda, \vec{\delta})}{I^0_\lambda}$$

Emissivity

- PROPERTIES:
  - An emissivity is between 0 and 1
  - A black surface has an emissivity of 1
  - A grey surface is defined by

    $$\alpha = \varepsilon$$

    and in this case $1 = \alpha + \rho$ and thus

    $$\varepsilon = 1 - \rho$$

Subsection 4

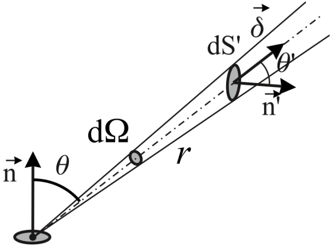
Heat exchange by radiation between two surfaces

Case of infinitesimal surfaces

Heat transfer rate from dS to dS' :

$$q_{\mathrm{d}S\to\mathrm{d}S'} = I(\vec{\delta})\mathrm{d}S\cos\theta\,\mathrm{d}\Omega$$

$$= I(\vec{\delta})\mathrm{d}S\cos\theta\,\frac{\cos\theta'\mathrm{d}S'}{r^2}$$



This transfer can be compared to the total radiated pover leaving dS:

$$q_{tot} = E\mathrm{d}S = \pi I\mathrm{d}S$$

if I is independent of $\vec{\delta}$

Definition of the view factor:

$$F_{\mathrm{d}S\to\mathrm{d}S'} = \frac{q_{\mathrm{d}S\to\mathrm{d}S'}}{q_{tot}} = \frac{I(\vec{\delta})\mathrm{d}S\cos\theta\frac{\cos\theta'\mathrm{d}S'}{r^2}}{\pi I\mathrm{d}S} = \frac{\cos\theta\cos\theta'\mathrm{d}S'}{\pi r^2}$$

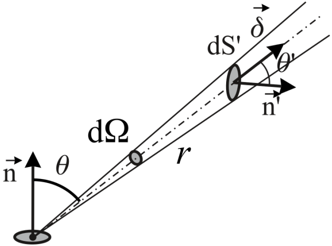
Case of infinitesimal surfaces

Heat transfer rate from $\mathrm{d}S$ to $\mathrm{d}S'$:

$$q_{\mathrm{d}S\to\mathrm{d}S'} = F_{\mathrm{d}S\to\mathrm{d}S'} E \mathrm{d}S$$

with

- $F_{\mathrm{d}S\to\mathrm{d}S'}$ the view factor
- E the missive power
- $\mathrm{d}S$ the surface



Case of finite surfaces

Heat transfer rate from S to S' :

$$q_{S\to S'} = \int_{S'}\int_{S} I(\vec{\delta})\cos\theta\frac{\cos\theta'}{r^2}\mathrm{d}S\mathrm{d}S'$$

Expression of the view factor:

$$F_{S\to S'} = \frac{\int_{S'}\int_{S} I(\vec{\delta})\cos\theta\frac{\cos\theta'}{r^2}\mathrm{d}S\mathrm{d}S'}{\pi\int_{S} I(\vec{\delta})\mathrm{d}S}$$

If I does not depend on $\vec{\delta}$ and does not vary across the surface:

$$F_{S\to S'} = \frac{1}{S}\int_{S'}\int_{S}\frac{\cos\theta\cos\theta'}{r^2}\mathrm{d}S\mathrm{d}S'$$

And thus:

$$q_{S\to S'} = F_{S\to S'}E\mathrm{d}S$$

Algebra of the view factors

- Reciprocity: $SF_{S \to S'} = S' F_{S' \to S}$
- Concave surface
- Convexe surface
- Enclosure of N surfaces:

$$\sum_{i=1}^{N} F_{S_j \to S_i} \quad \text{for} \quad j \in [\![1; N]\!]$$

Algebra of the view factors

- Between two planes with an angle α

Net heat energy rate exchange by a grey surface in an enclosure

The net radiative energy rate leaving the surface:

$$\begin{aligned}\varphi^{net} &= S(\varepsilon E^0 - \alpha H) = \varepsilon S(E^0 - H) \\ &= S(J - H) \\ &= \frac{\varepsilon S}{1-\varepsilon}(E^0 - J)\end{aligned}$$

- H irradiation onto S
- ρH reflection
- $E = \varepsilon E^0$ radiation
- $J = E + \rho H = \varepsilon E^0 + (1-\varepsilon)H$

Section 5

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