Matériaux avancés
Advanced Materials

Fracture Mechanics
Mécanique de la rupture

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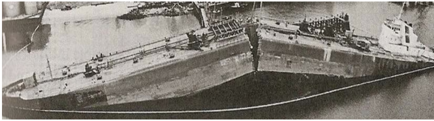
- **1919** –"Great Molasses Flood" Boston USA - A melasses tank[a] with a volume of 7500 m^3 breaks brutally and its contents are spread in the streets of the town making 12 deaths and 40 wounded people.
- **1924** – International Congress of Applied Mechanics - A.A Griffith presents a study on the glass behaviour with microcracks. His approach is basing on thermodynamical approach and at the time the glass is only used to manufacture bottles and glazes, his article is practically not noticed by the scientific community.
- **January 1943** – A little harbour in the east cost of US. Residents are woken in the morning by a huge explosion that makes them think in sabotage. In fact, the Schenectady tanker was suddenly broke in two by calm sea, inaugurating a long serie of disasters that affect some oil tankers and many Liberty Ships. These vessels were among the fisrt to be made using welding techniques...they have made charges



Fracture of « liberty ship » by great cold during the 1943 winter

[a] Melasses is a very viscous sirup from residue of the refining of sugar cane

- **50's** – Two Comet airplane, first commercial jets, broke during the fligh. Fractogarphic analysis show that the accident was the result of the propagation of fatigue cracks initiated close to the portholes in the aeras of stress concentration. Aeronautic engineers reproduce the phenomenon on a part of the fuselage in a vaccump chamber...

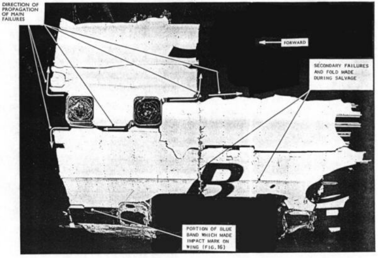
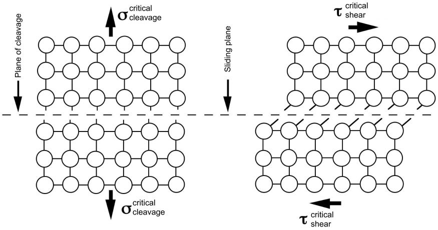


FIG. 12. PHOTOGRAPH OF WRECKAGE AROUND ADF AERIAL WINDOWS—G-ALYP.

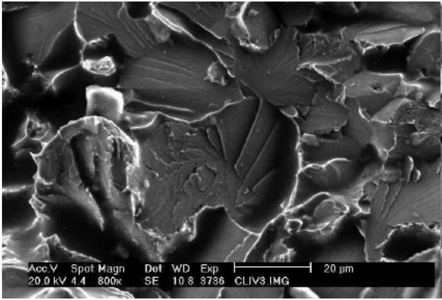
Reconstruction of the cabin of an Comet airplane, initiation of the crac

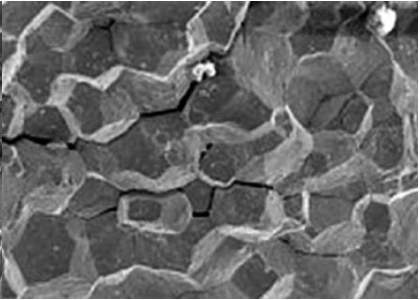
- Cleavage stress $\approx \frac{E}{10}$ (E Young modulus)
- Critical shear stress $\approx \frac{G}{10}$ (G shear modulus)
- Creation of crack = generation of new surfaces
- How is this possible ?
  - Local heterogeneities = stress concentrators
  - $\Rightarrow$ cleavage
  - Heterogeneities : microcracks, grain boundaries, exogenous inclusions, non deformable phases,...
- Macroscopic scale: fracture mechanics (fracture criterium)
- Micrococoscopic scale: damage mechanic



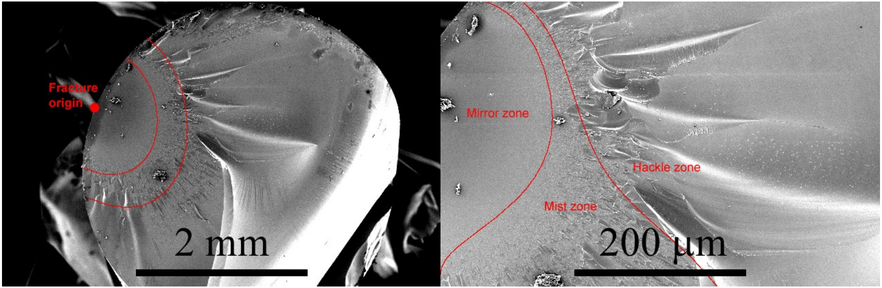
- Failure by rapid crack
  - **Brittle**: low energy consumption, brutal, high propagation speed ⇒ dangerous because the stress limitation in elastic behaviour of material serves no purpose, is the privileged field of Fracture Mechanics
  - **Ductile** : high plastic deformation, large quantities of energy consumed ⇒ less dangerous, taking into account easily by the criteria of traditional design and Fracture Mechanics
- Failure after progressive crack (followed by a rapid stage)
  - Under on cyclic loading
    - Fatigue
    - Thermal fatigue
  - Under static loading
    - Stress corrosion
    - Creep
  - Under complex loading
    - Fatigue-corrosion
    - Fatigue-creep

- Case of metals
  - Mean plane of crack normal to the direction of the higher component of the major tensile stress
  - Intergranular crack is possible if embrittlement occurs (corrosion, liquid metals embrittlement, ...)
  - Failure by cleavage → transgranular facets

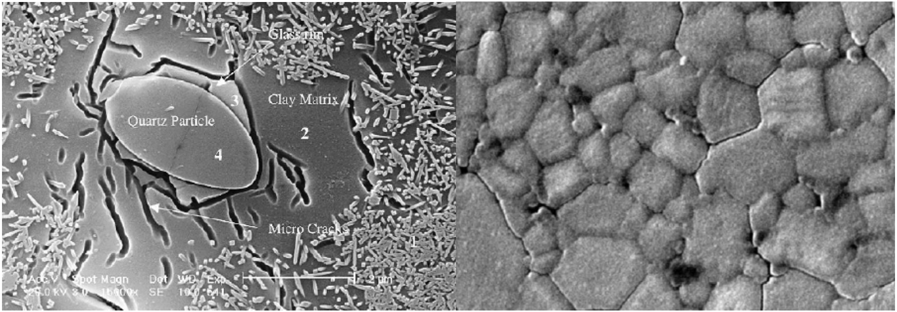




- Case of glasses
  - Crack close to the superficial defects (stripe, little impact)
  - 3 observable aeras
    - Mirror aera close to the starting crack
    - Transition aera where secondary cracks can exist
    - Final failure aera less plane
  - Propagation speed increase during the propagation → speed of sound in the glass is reached → dynamical phenomena

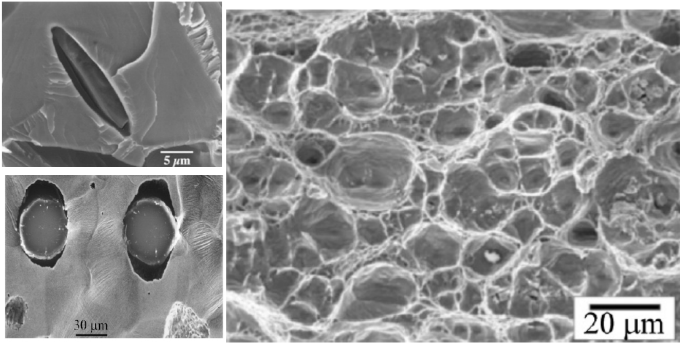


- Case of ceramics
  - Heterogeneous materials with many microcracks between constituents
  - Silmutaneous propagation of several cracks ⇒ fragmentation rather than fissuration
  - Planes of fissuration with different orientations no forcing perpendicular to the major stress tensile



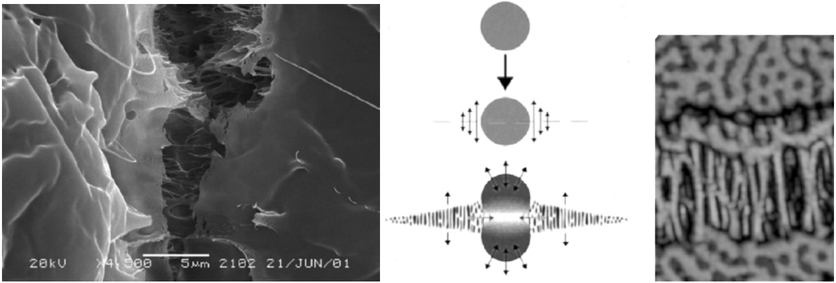
Industrial porcelain (quartz particles, mullite in clay matrix) – Ceramic composite Al_2O_3-SiC

- Case of metals
  - Macroscopic plasticity of material
  - Diffuse damage mechanism
  - Development of void on the defects (inclusions), precipitates, grain boundaries, intersection of sliding planes)
  - Coarsening, void coalescence → fractal appearance of the fracture surfaces



Voids around sulphurs (steel of Titanic), graphite nodules – Fracture topography

- Case of thermoplastic polymers
  - diffuse damage or crazing
  - Stress concentration around reinforcement particles ⇒ macromolecular chains under fibril form ⇒ craze formation



Craze, failure of a thermoplastic polymer – Craze around elastomer reinforcement / thermoplastic matrix – Crazing observed by AFM

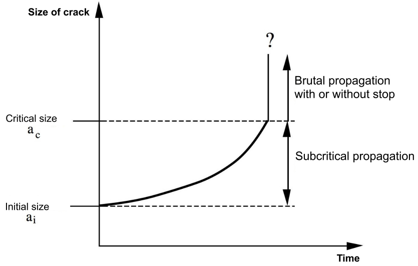
- Objectives
  - To develop a **macroscopic** failure criteria of a structure with a defect like crack type
  - Comparison between loading factor and a parameter characterising the resistance of material to the stable or unstable propagation of a crack
  - Cracks can be pre-existing (defects, ...) or can be coarsing in use (damage)
- *a* size cracks
  - Bidimensionnality matter discontinuity with sharp edge
  - Stable propagation or subcritique of crack with initial size a_0
    - Fluctuations of apllied loading (vibratory phenomena, stop and start stages,change of loading level in use, thermal transit,...)
    - Silmutaneous static loading and corrosive environement (stress-corrosion)
  - Instable or brutal propagation when crack reach a critical size a_c
    - Brutal failure of the part
    - Stop stage

- We must :
  - Have a relationship for the the subcritical propagation speed of the crack

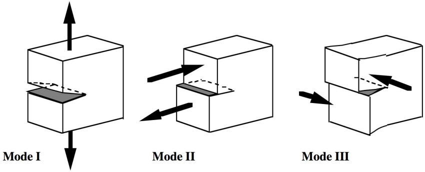
$$\frac{da}{dN} = f(\text{loading, environment, material,...})$$

  - Have a relationship of the size of the crack

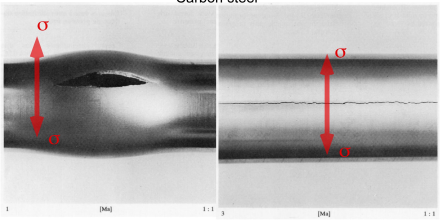
$$a_c = f(\text{maximum loading, material,...})$$



- Crack propagation modes
  - Mode I: the more dangerous when the structure is under tensile loading. This mode is best known, large majority of prac tical problem
  - Mode II and III for compressive or shear loading
  - For complex loading the mode are a mixing of mode I, II and III



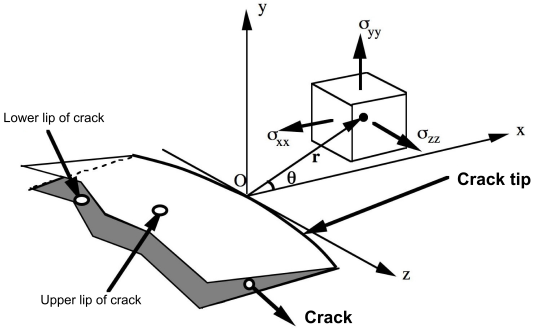
Carbon steel



Ductile failure　Brittle failure

Exemple of failure in mode I

- Base hypothesis
  - Linear elasticity can be applied
  - No plastic deformations
  - Perfect brittle failure type
  - Crack : 2D plane defect
  - Concepts of the lips of the crack and the crack tip



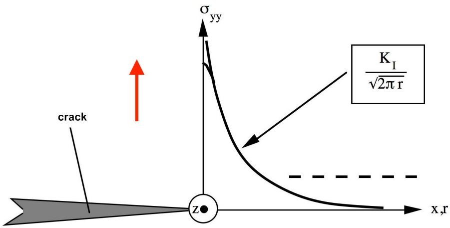
Crack sketch

- Stress field close to the crack in mode I (Irwin)

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) + C_{ij}(r^0) + C_{ij}(r^{1/2}) + O(r)$$

  - $O(r)$ terms of higher orders
  - $\frac{f_{ij}}{\sqrt{2\pi r}}$ shape function depending of θ and r giving the shape of stress distribution
  - K_I **stress intensity factor** fixing the level of stress distribution

- Significance
  All cracks loaded with mode I, in a given elastic material for a determined temperature and environment, will have a identical answer if they are subjected to the same stress intensity factor, independently to the shape and the size of the part
- It exists another expressions for the mode II and III
- Limitation
  - $r \to 0 \Rightarrow \sigma \to \infty$
  - Values of the stress are not infinite, curvature radius of the crack tip is not equal to none



- Case of a thick plate with W thickness (mode I)

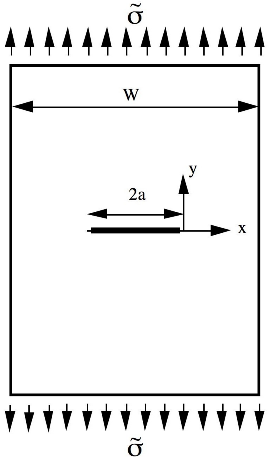
$$a \ll W \; \sigma_{yy} = \frac{\tilde{\sigma}\sqrt{\pi a}}{\sqrt{2\pi r}} \text{ d'où } K_I = \tilde{\sigma}\sqrt{\pi a}$$

  - crossing crack with the size $2a$
  - $\tilde{\sigma}$ normal stress to plane of the crack calculated supposing without it
  - K_I in MPa.m$^{1/2}$

- for a large number of structure loaded in mode I

$$K_I = \alpha\tilde{\sigma}\sqrt{\pi a}$$

α is a dimensionless geometrical factor depending of the part-crack configuration

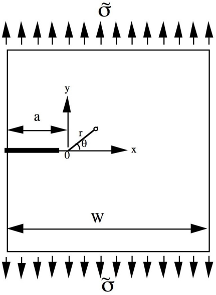


- Case of an emerging crack, thick plate with thickness W (mode I)

$$K_I = Y(a/W)\tilde{\sigma}\sqrt{\pi a}$$

$$Y(a/W) = \alpha = 5(20 - 13(a/W) - 7(a/W)^2)^{-1/2}$$

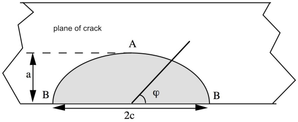
  - Crack with the size a
  - Rectilinear crack tip



- Cas of a emerging elliptic crack

$$K_I = Y((a/W),(a/c),\phi)$$

  - Representative of a fatigue crack initiated on the surface by only one site
  - K_I varies along the crack tip



- General case
  - Stress intensity factor = severity of the crack/mechanical loading couple for a given structure
  - Many configurations are already studied
    - Catalogue or form (ASME code, Rooke et Cartwrig form,...)
    - Software (ASME, CETIM,...)
    - Direct methods (finite element method, integral equations,...) in order to calculate stress and strain fields

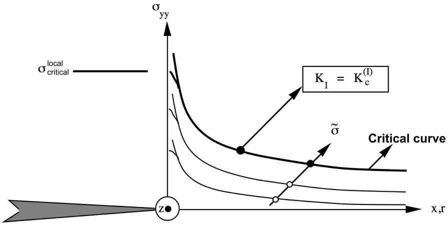
- Si $\sigma^{\text{local}} > \sigma^{\text{local}}_{\text{critique}}$ crack propagation occurs with

$$K_I = K_{I_C}$$

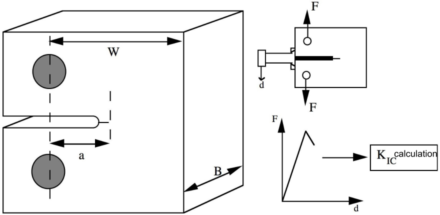
then

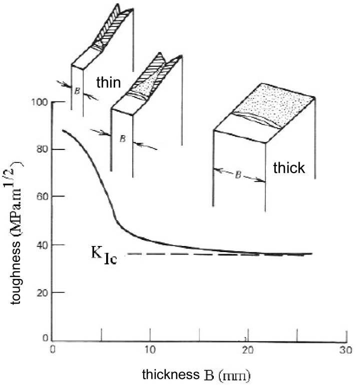
$$\alpha\tilde{\sigma}\sqrt{\pi a_C} = K_{I_C} \rightarrow a_C = \frac{K_{I_C}^2}{\pi\alpha^2\tilde{\sigma}^2}$$

  - K_I is calculated
  - K_{I_C} is a characteristic of material and can be measured
- K_{I_C} is the critical stress intensity factor or fracture toughness

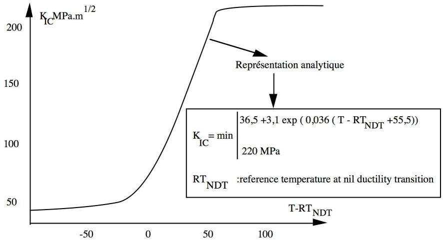


- Détermination of K_{I_C}
  - For steels ⇒ NF-A-03-80
  - Normalized test samples, crack initiated at the bottom of a machined notch
  - Failure using tensile tester, measure of the load / displacement couple ⇒ calculation of K_{I_C}
- Influence of B thickness of the test sample
  - $B \nearrow \rightarrow K_{I_C} \searrow$
  - Plane stress (thin test sample), plane strain (thick test sample)
  - $B \searrow \rightarrow$ plastified aera $\nearrow$
  - Asymptotic value of K_{I_C} $\rightarrow$ containment of the plasticity for $B = 2,5\left(\frac{K_{I_C}}{\sigma y}\right)^2$
- K_{I_C} : critical stress intensity factor in mode I and for plane deformation or **fracture toughness** (*tenacité à la rupture*)
  - Plan strain state: most unfavourable case, conservative calculation
  - Validity of LEFM ⇒ $B \geq 2,5\left(\frac{K_{I_C}}{\sigma y}\right)^2$ (not often carried out)





- K_{I_C} function of
  - the nature of material (processing, forming, microstructure)
  - the temperature (little variation for stainless steels and high limit elastic (HLE) steels)
  - the speed of application of the loading (notion of dynamic K_{I_d})

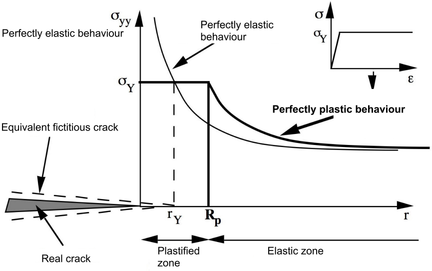


16MnNiMo5 steel quenched and tempered (vessel wall of nuclear reactors)

- but also...
  - Rate of irradiation (Neutron flux effect $\Rightarrow$ creation of voids)
    - Displacement of the curve towards the high temperature
    - For "nuclear" steels:

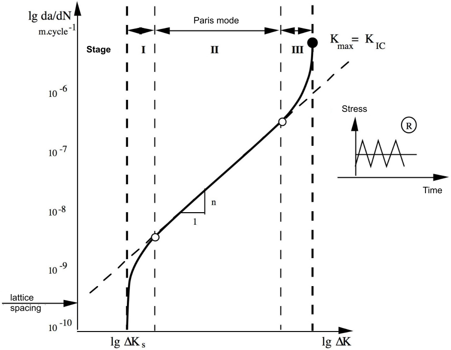
      $$\Delta RT_{\text{NDT}} = 8 + (24 + 1537(\%P - 0,008) + 238(\%Cu - 0,08 + 191\%Ni^2\%Cu))\left(\frac{f(n.cm^2)}{10^{19}}\right)^{0,35}$$
    - $K_{I_C} = 200\text{MPa.m}^{1/2}$ @ the startup, @ the end of 40 years $K_{I_C} = 50\text{MPa.m}^{1/2}$, that is to say a factor 16 on the critical size of cracks !!!

- Plastique aera
  - In front of the crack
  - Of size $R_P = \frac{1}{\beta\pi}\left(\frac{K_I}{\sigma_y}\right)^2$ with $\beta = 1$ (plane stress), $\beta = 3$ (plane deformation)
  - Containment effect for the tick structures
    - For example $K_I = 30$ MPa.m$^{1/2}$ $\begin{cases} R_P = 40\ \mu\text{m si } \sigma_y = 1500 \text{ MPa} \\ R_P = 1 \text{ mm si } \sigma_y = 300 \text{ MPa} \end{cases}$
  - Calculation is more complex for the real materials (finite elements approach)
  - FELM can be applied using fictitious dimension $a^* = a + \beta r_y$ with $r_y \approx \frac{R_P}{2}$ and $1 < \beta < 1,5$ (calculus code)
  - If $r_y \nless a \Rightarrow$ more complex approach must be used (J Rice integral,...)



- $a < a_c$ no brutal crack but...
- The subcritic crack can increase by size if there is
  - a variable mechanical loading in time : progressive cracking by **fatigue**
  - a static mechanical loading associated with a chemical phenomenon: progressive cracking by **stress corrosion**
  - a static loading @ high temperature: **creep**
  - a combination of the previous effect (creep-fatigue, fatigue-corrosion,...)
- $\rightarrow$ no dangerous cracks can become it
- for the continuation, one will consider (fundamental postulate) that
  - cracks with known size are localized in the part (NDT, crack-test,...),
  - the initiation stage or starting stage is not the object of the fracture mechanics

- Case of periodic mechanical loading with constant amplitude
  - Speed of the propagation function of ΔK_I and $R = \frac{K_{I_{min}}}{K_{I_{Max}}}$
  - $\Delta K_I = \begin{cases} K_{I_{Max}} - K_{I_{min}} = K_{I_{Max}}(1-R) = \\ \quad \alpha(\sigma_{Max} - \sigma_{min})\sqrt{\pi a} = \\ \quad \alpha\Delta\sigma\sqrt{\pi a} \text{ pour } R \geq 0 \\ K_{I_{Max}} = \alpha\sigma_{Max}\sqrt{\pi a} \text{ for } R < 0 \end{cases}$
  - Representation using bilogarithmic coordinates
  - $\frac{da}{dN}$ in m.cycle^{-1}
  - Low threshold $\Delta K = \Delta K_s$: propagation limit
  - High threshold $K_{I_{Max}} = K_{I_c}$ or $\Delta K = K_{I_{Max}}(1-R)$: failure
  - 3 stages
    - Stage I : large variation of ΔK, below ΔK_s no detection of a notable evolution of the size of the crack, unfortunately $\Delta K_s \approx$ some % of K_{I_c}
    - Stade II : Paris mode
    - Stade III : high variation of the crack size $\Rightarrow$ failure
  - Influence of R



- Crack opening: $\Delta\sigma \Rightarrow \Delta K \Rightarrow \Delta a$

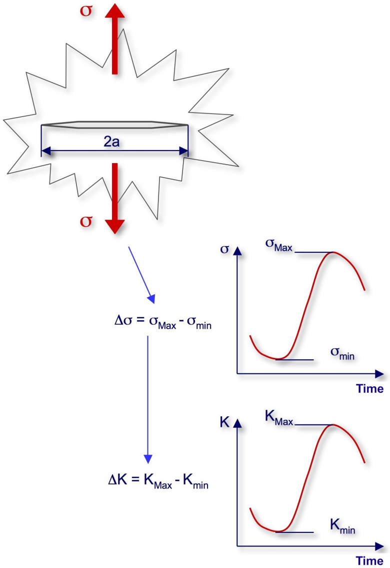
$$\frac{da}{dN} = C\Delta K^m$$

$\frac{da}{dN}$ in mm/cycle, ΔK in MPa.m$^{1/2}$, $2 < m < 6$ and C function of material and the environment

- Other form of Paris law (in order to take into account the average of stress)

$$\frac{da}{dN} = C\frac{\Delta K^m}{1 - R/2} \text{ with } R = \frac{K_{\text{min}}}{K_{\text{Max}}}$$

- Remark
  - In mode I $\Delta K = \Delta K_{\text{I}}$
  - In mixed mode $\Delta K = f(\Delta K_{\text{I}}, \Delta K_{\text{II}}, \Delta K_{\text{III}})$
  - The mode I ended up being dominating



- Example : value of da/dN for a same material according to the environment
  - In the air

$$\frac{da}{dN} = 2,6.10^{-8}\Delta K^{2,7}$$

  - In the PWR water

$$\frac{da}{dN} = 1,9.10^{-5}\Delta K^{1,4}$$

  - For $\Delta K = 40\text{MPa.m}^{1/2}$
    - In the air : $da/dN = 4,8.10^{-4}$ mm.cycle^{-1}
    - In the PWR water : $da/dN = 3,5.10^{-3}$ mm.cycle^{-1}

- Limitation
  - $\frac{da}{dN} = C\Delta K^n = C\pi^{n/2}a^{n/2}\Delta\sigma^n \rightarrow d\left(\frac{da}{dN}\right) = \frac{dC}{C} + \frac{n}{2}\frac{da}{a} + n\frac{\Delta\sigma}{\Delta\sigma}$
  - For a crack size a the error on the propagation speed is n time more important than the error on the stress evaluation
  - Metallic materials ($2 < n < 4$) : 5% of error on $\sigma \rightarrow$ 10 to 20% of error on $\frac{da}{dN}$ – acceptable
  - Ceramic ($n \rightarrow 20$) : 5% of error on $\sigma \rightarrow$ 100% on $\frac{da}{dN}$ – inadmissible
  - For materials with high value of n : not good approach
- Two philosophies :
  - **Fracture mechanics** : the structure contains defects initially immediately propageables (one has just seen it)
  - The initial structure is supposed to be free from defects immediately propageables: it is necessary to carry out tests with not cracked test samples (**Wöhler curves**)