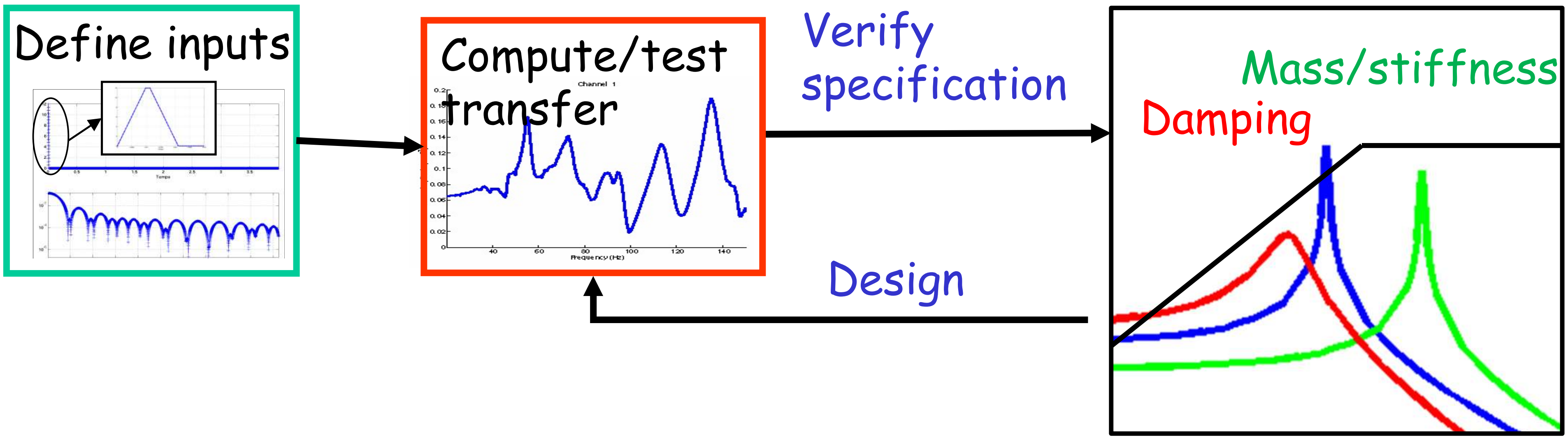
Damping and complex modes

1. Devices
2. Coupling/energy levels
3. Damping mechanisms, constitutive laws
4. Reduced models for damping: from material to structure

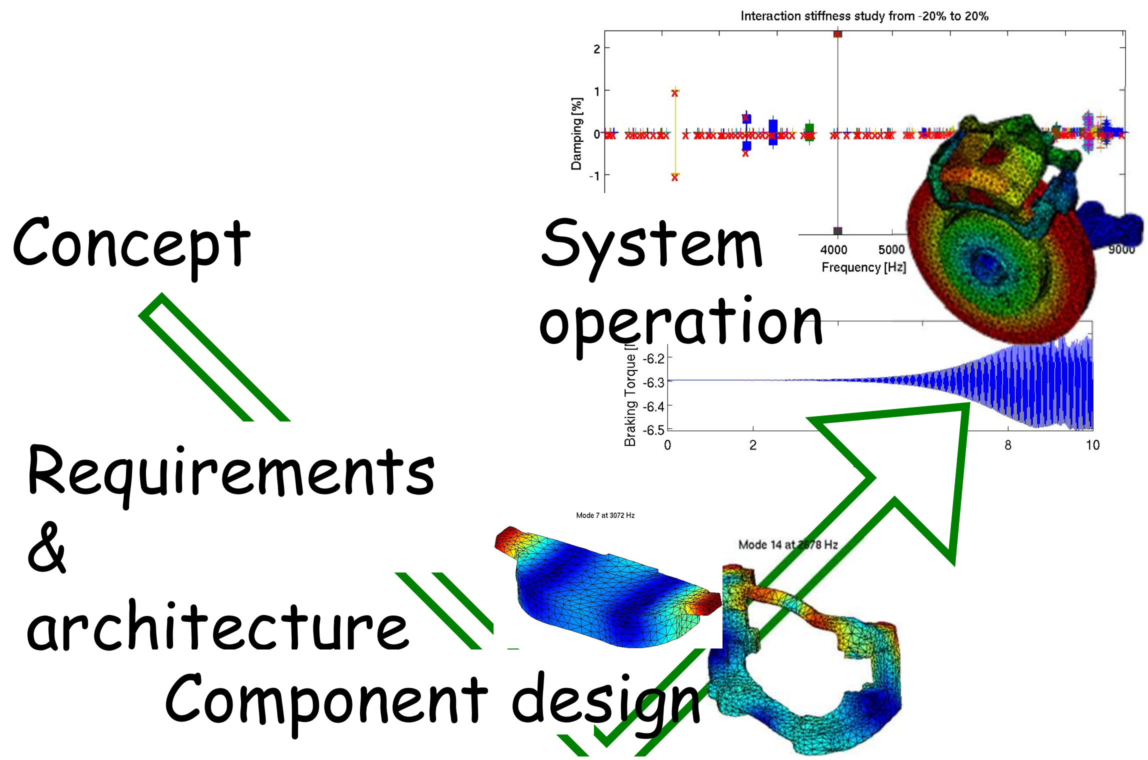
MS2SC http://savoir.ensam.eu/moodle/course/view.php?id=1874
PROVIR http://savoir.ensam.eu/moodle/course/view.php?id=490

Design process

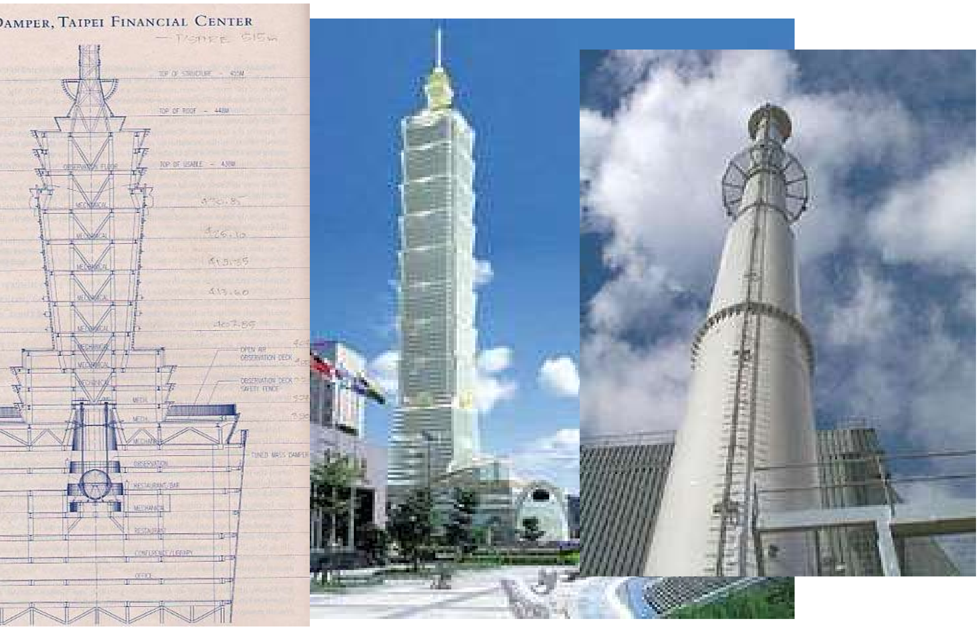


Designing for damping is difficult

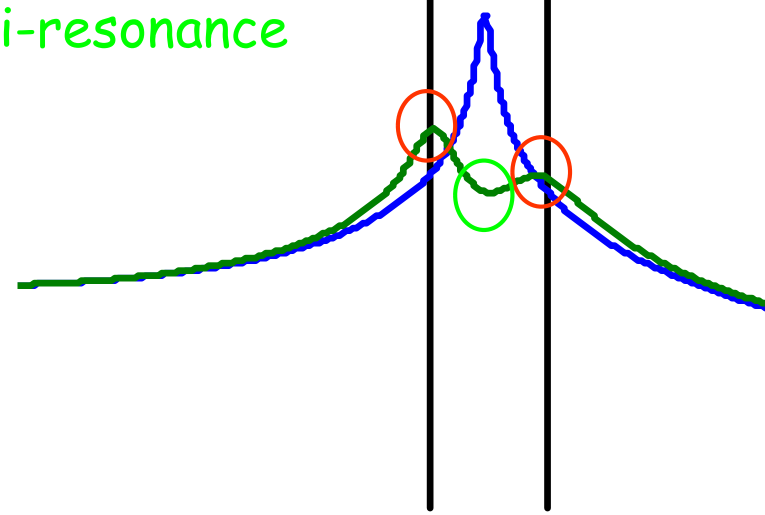
- Physical damping models are computer intensive
- NVH response typically evaluated at system level

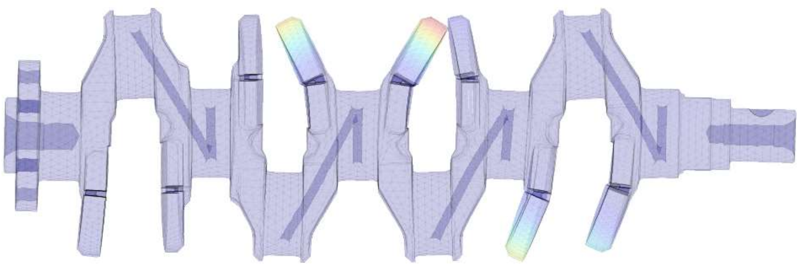


Tuned mass dampers / vibration absorber



- Resonance splitting
  - Two lower resonances or
  - One anti-resonance

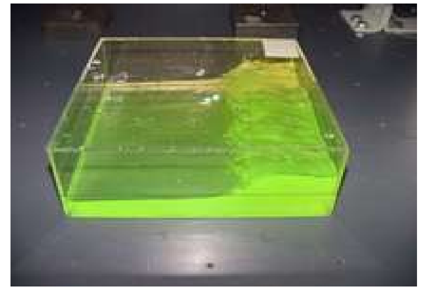


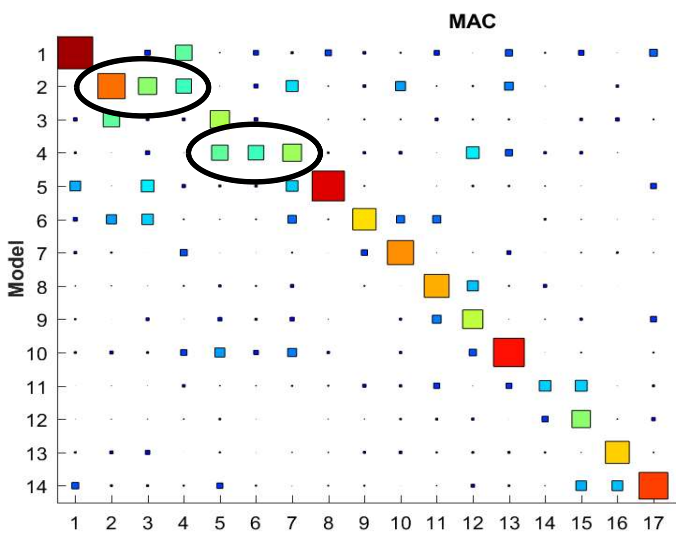


- Countless applications : helicopters, buildings, lamp posts, cars, …
- Current trend : non-linear absorber (self tuning)

Sample dampers

- Fluid sloshing : No moving parts
- Rings, cables, side masses, …
- Electrical wires

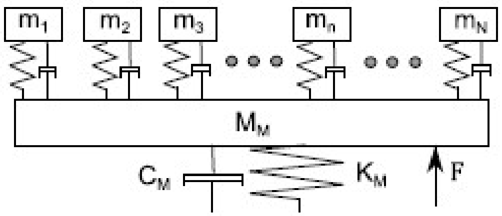


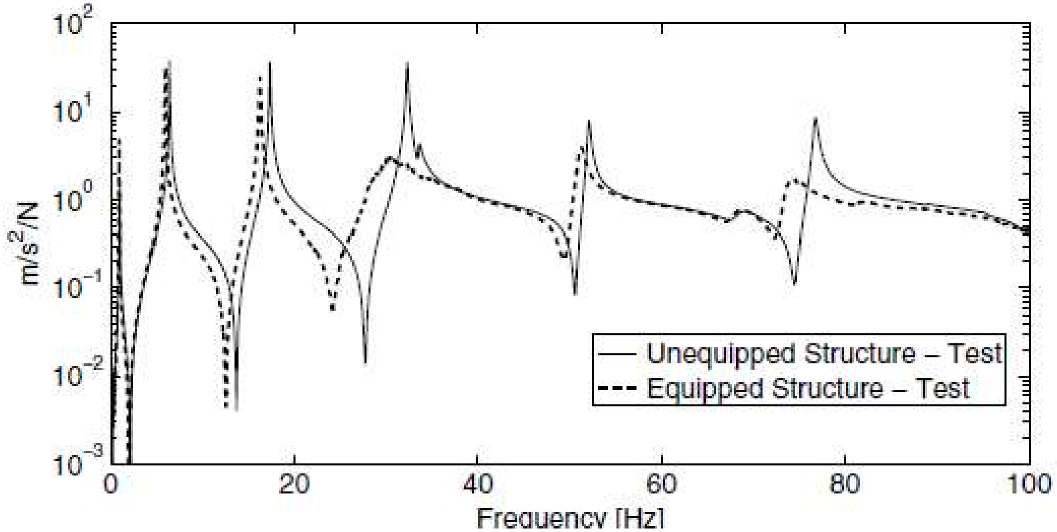


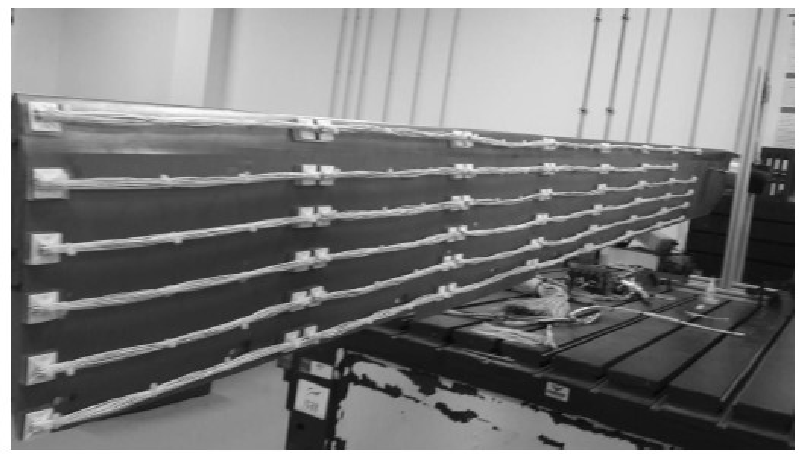
World record

www.multitech-fr.com

Damping due to local modes



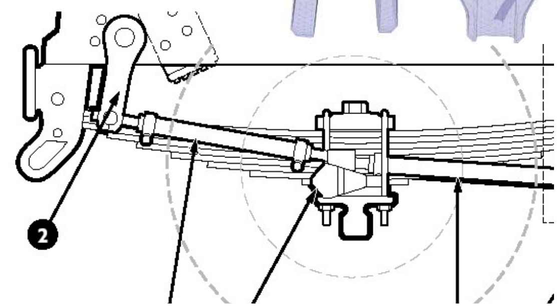


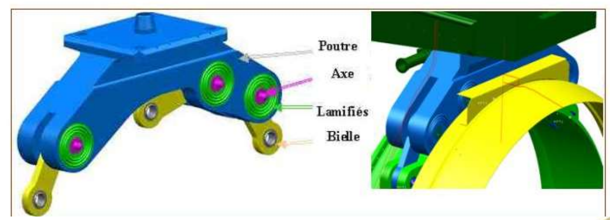


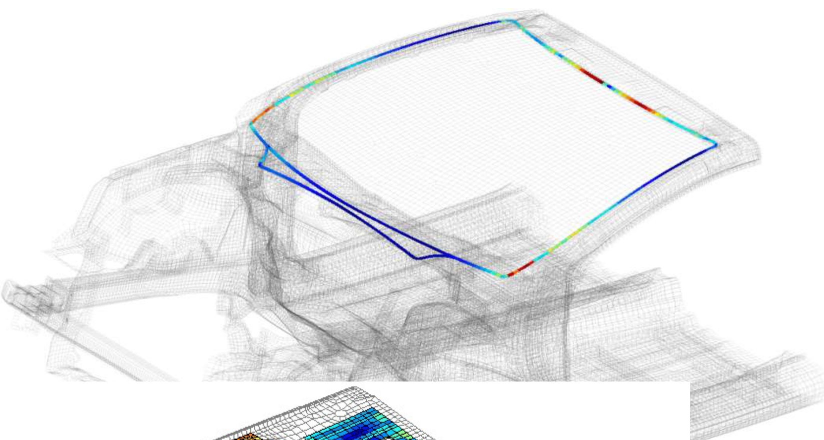
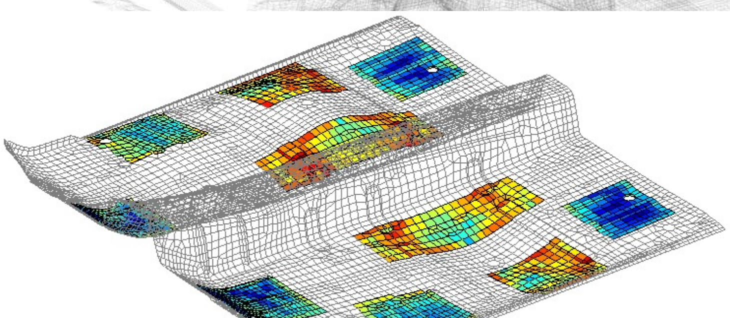
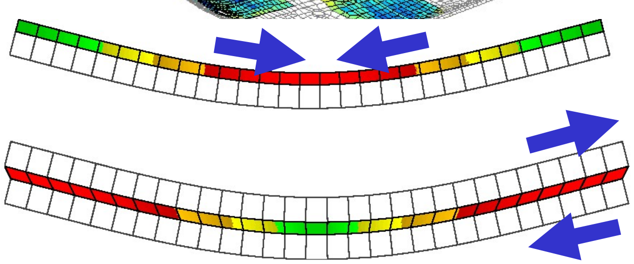
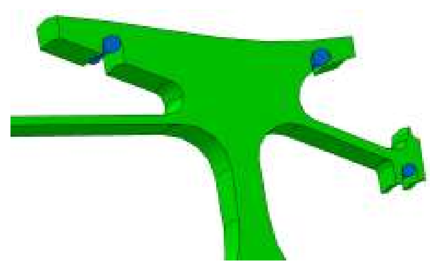
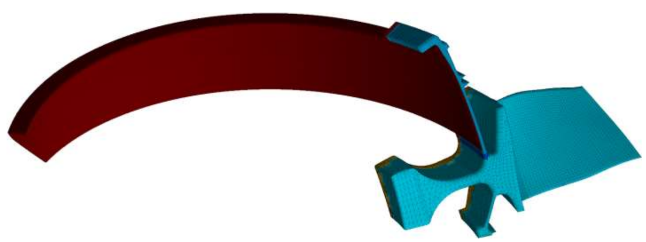
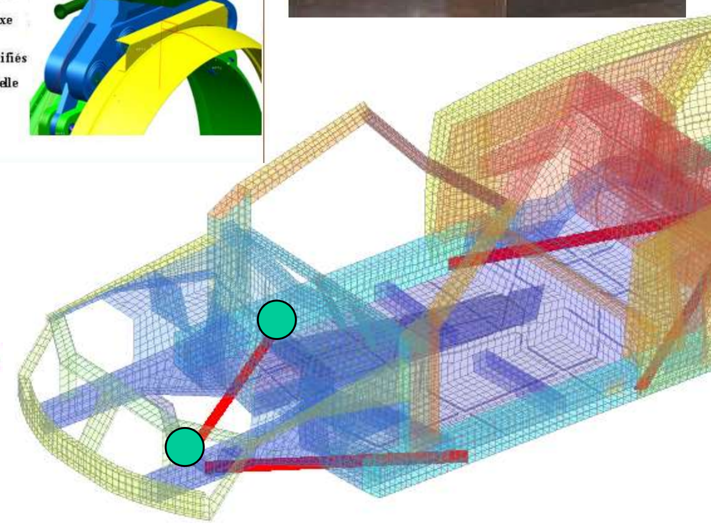
Soize "structural fuzzy" 1987
Arnoux, A. Batou, C. Soize, L. Gagliardini. JSV 2013
Loukota, Passieux, Michon. Journal of Aircraft 2017

Damping devices

- Resonant mass
- Struts, point connections
- Line/surface treatments

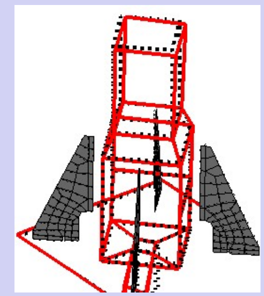


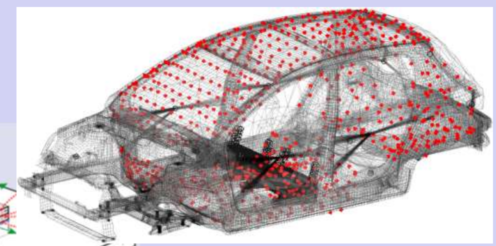







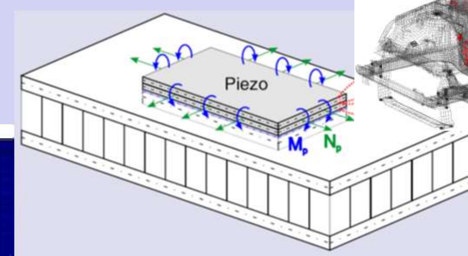
Q2 : undamped nominal + coupling

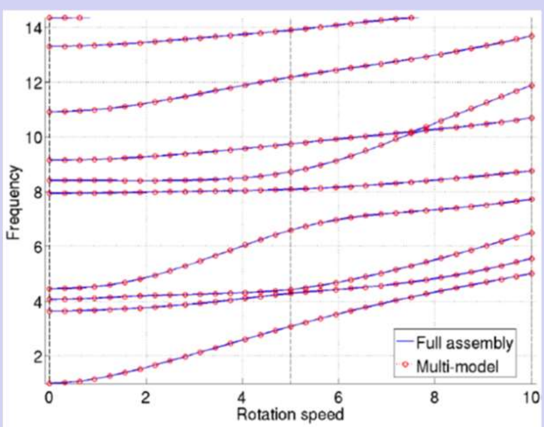
In → Undamped reduced model → Sensors

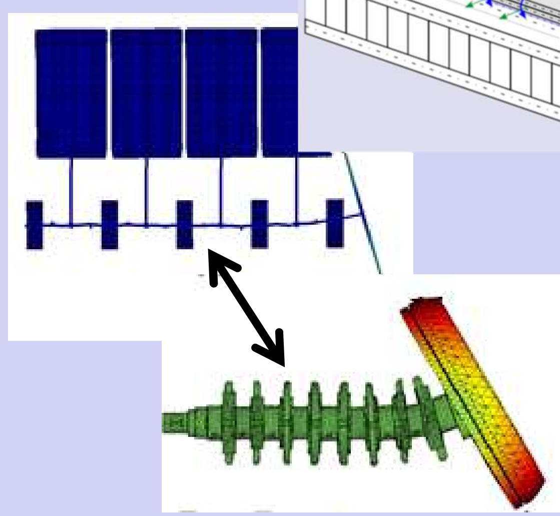
- Coupling : test/FEM, fluid/structure active control, …
- Local non-linearities : machining, bearings, contact/friction, …
- Environment/design/uncertainty

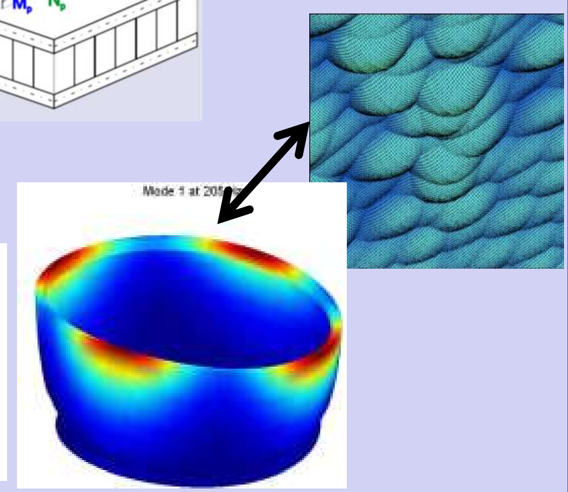






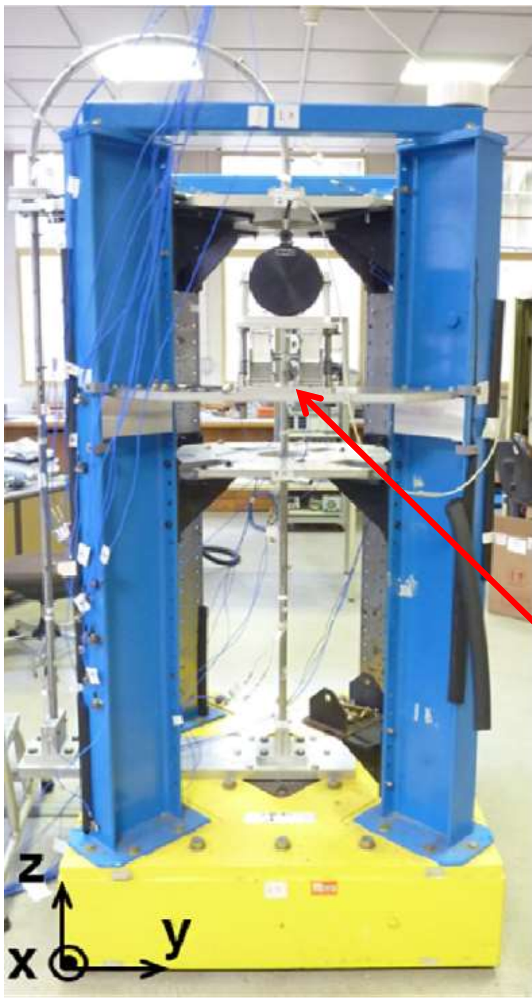


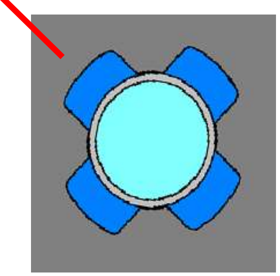


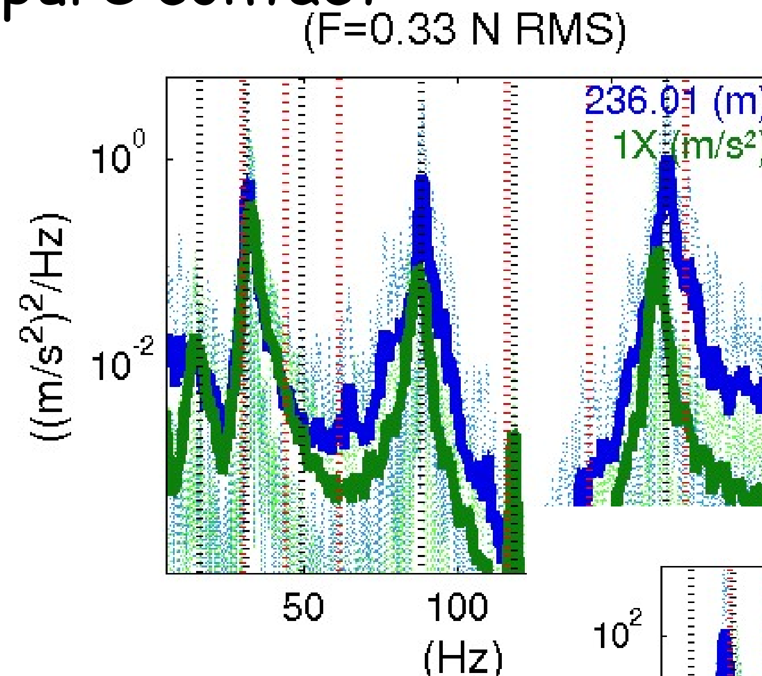


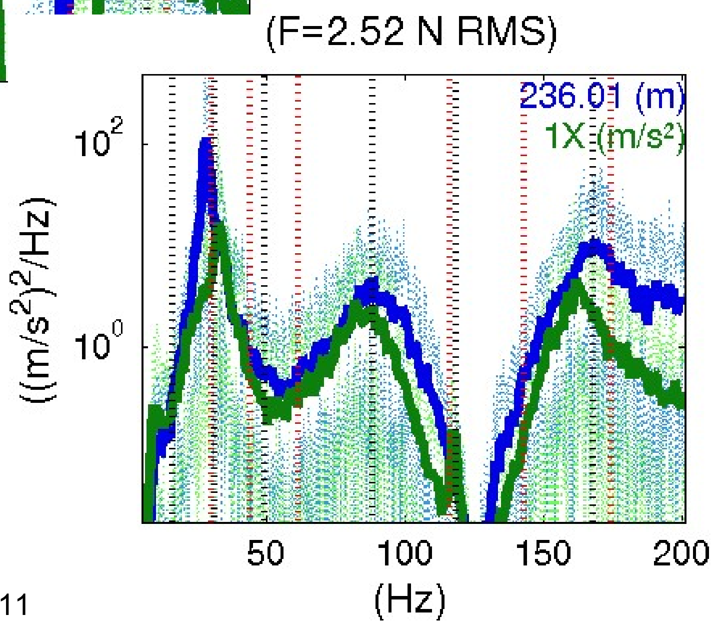
Energy coupling & NL system

Bending beam with gap & contact
Apparent damping with pure contact



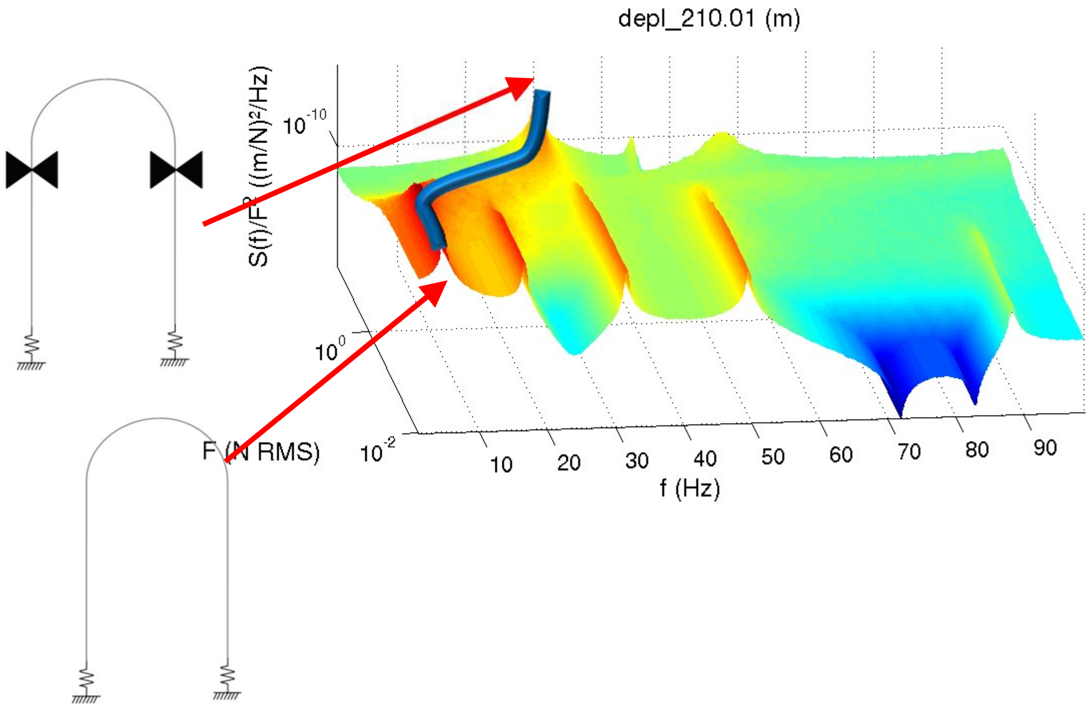


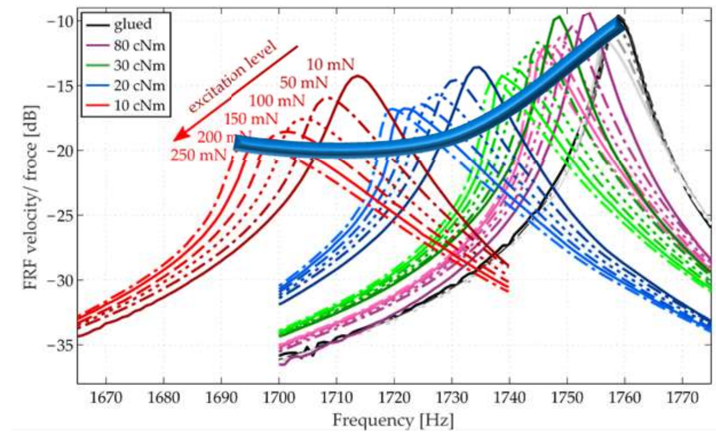




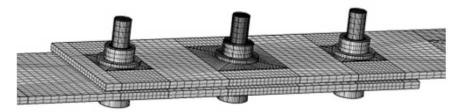
PhD. Thénint 2011

Damping occurs in the transition



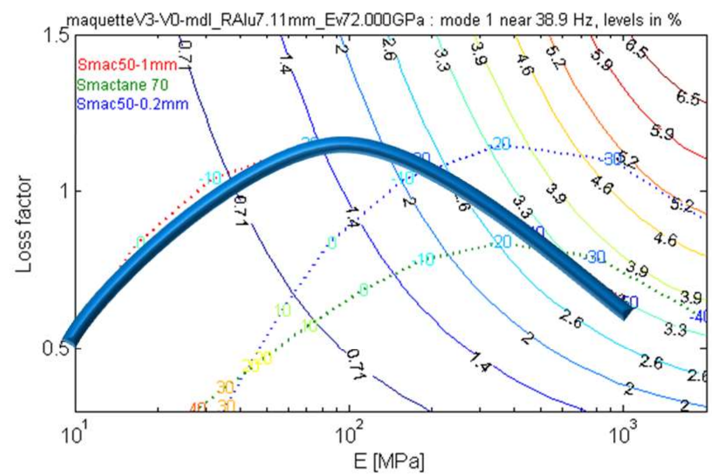


FEMTo-ST Orion Testbed



 Transition

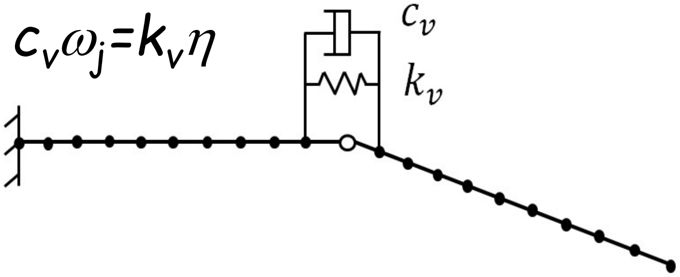
- Contact with broadband excitation level
- Friction : amplitude/normal force dependence
- Viscoelasticity : temperature/frequency dependence

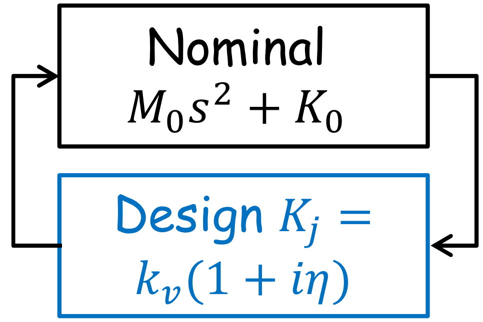


ENSAM/PhD. Hammami 2014
Hammami/Balmes/Guskov, MSSP 2015

A mechanical root locus (coupling)

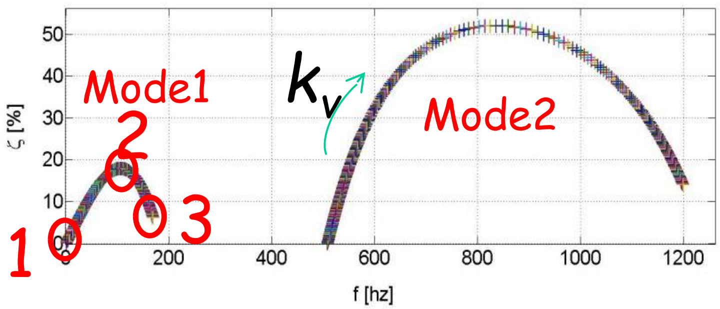
$c_v\omega_j=k_v\eta$

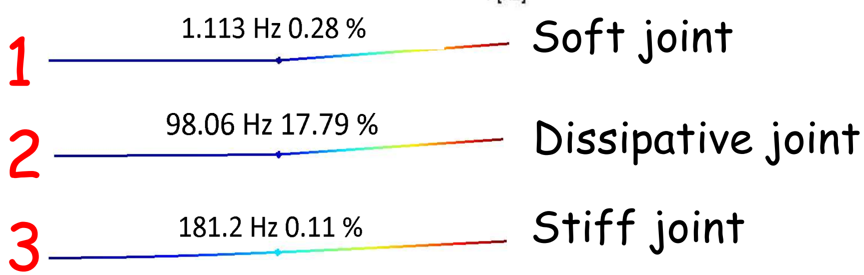




k_v variable, $\eta=1$ constant loss

Output : $f(k_v,\eta)$ $\zeta(k_v,\eta)$





Damping can only exist if joint is « working »

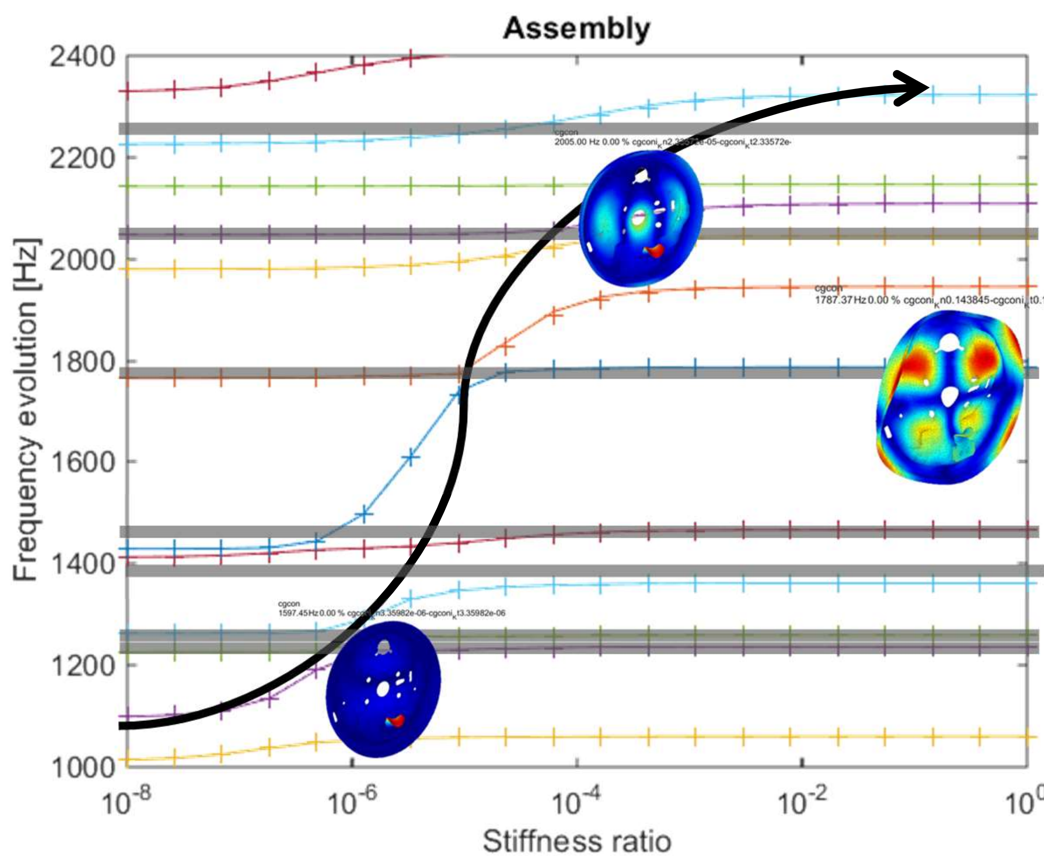
⬇

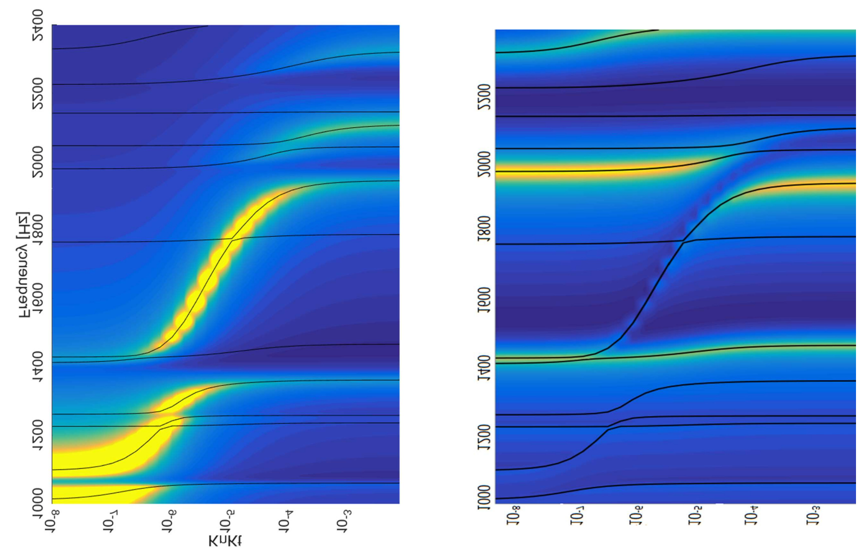
Energy fraction in joint must be « sufficient » (depends on k_v)

Related ideas : electro-mechanical coupling coefficient, pole/zero distance, modal strain energy

Coupling & mode crossing

- Variable contact surface, contact, sliding can generate coupling
- Horizontal lines : global modes
- S curve : local cable guide mode





Sensors can help
But can we track modes better ?

Lab work 3 : tracking & reanalysis

Q3 : Damping mechanisms

Damping : mechanism by which the energy in the considered system is diminished

- Non elastic behavior
  - Material : viscoelastic, plastic, …
  - Joint friction
  - Inter-frequency coupling through non-linearities
- Coupling with other systems
  - Fluid/structure, particle, soil, thermal, … interactions
  - Active control systems (piezo, electromagnetic, …)
  - Rotating parts

Linear, Non-linear

Viscoelasticity = LTI material

- Viscoelasticity (Bolzmannian material), stress f(strain history) linear in amplitude
- Viscous relaxation : convolution with displacement or velocity (non-parametric)

$$F=\int_{-\infty}^{+\infty} K(t-r)x(r)\mathrm{d}r=\int_{-\infty}^{+\infty} R(t-r)\dot{x}(r)\mathrm{d}r$$

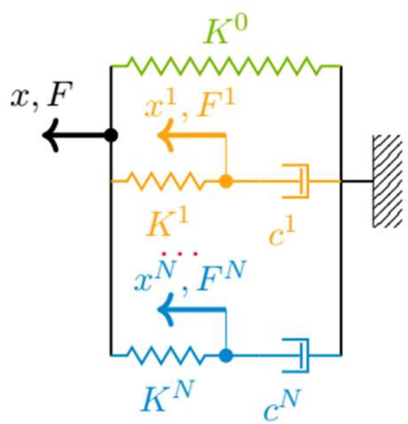
- Parametric model representation (Maxwell, rational fraction)

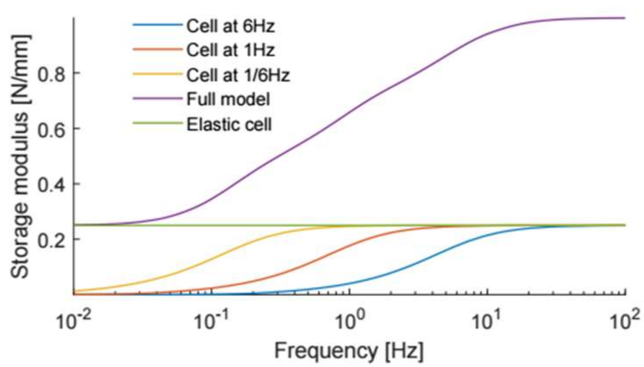
$$K(s)=K^0\prod_{i=1}^{N}\frac{1-{s}/{z^i}}{1-{s}/{p^i}}=K^\infty\left(1+\sum_{i=1}^{N}-\frac{g^i}{{s}/{\omega^i}+1}\right)=K^\infty\left(g^0+\sum_{i=1}^{N}-\frac{g^i s}{s+\omega^i}\right)$$

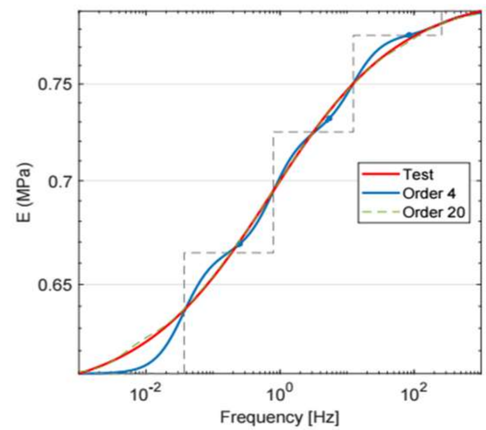
- Differential equation formulation (stress or stress rate relaxation)

$\frac{\dot{F}^i}{\omega^i}+F^i=-g^iF^\infty(x)$, or $\dot{F}^i+\omega^iF^i=g^i\dot{F}^\infty(x)$

- Order selection: place poles and zeros in frequency domain (> 1/decade)



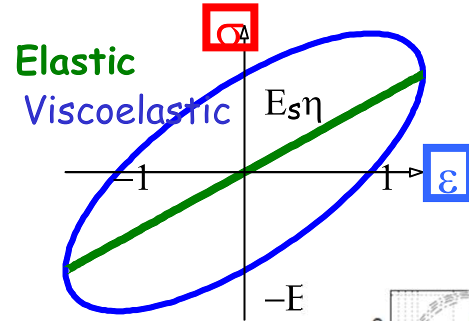




F. Renaud & al. : A new identification method of viscoelastic behavior DOI 10.1016/j.ymssp.2010.09.002
Rafael Penas, PhD ENSAM 2021, MSSP 2022

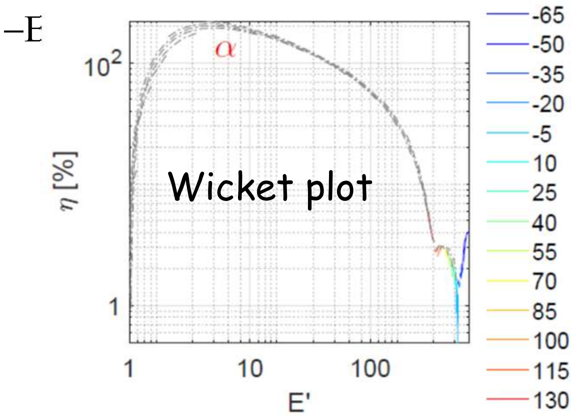
Viscoelasticity 2 : frequency domain

- Time convolution $\sigma(t) = \int_{-\infty}^{t} C(t-\tau)\epsilon(\tau)$
- Frequency complex modulus $\sigma = E(\omega, T, \epsilon_S)\epsilon$



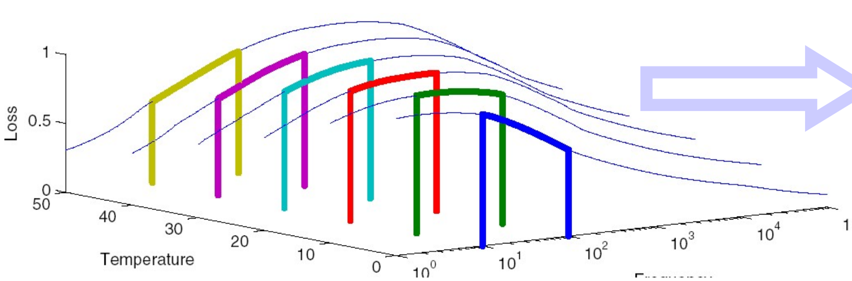
$$E = E'(1 + i\eta)$$

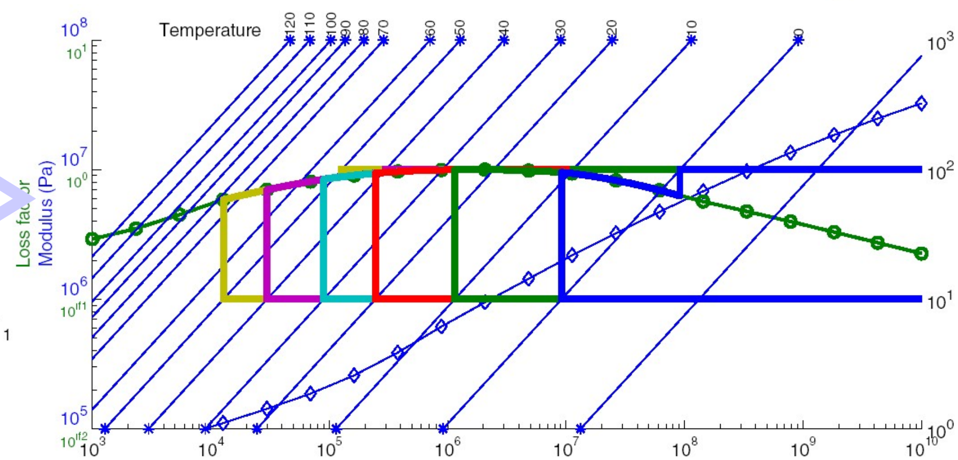
E' storage modulus, $\eta = \tan(\delta)$ loss factor



- Temperature/frequency equivalence

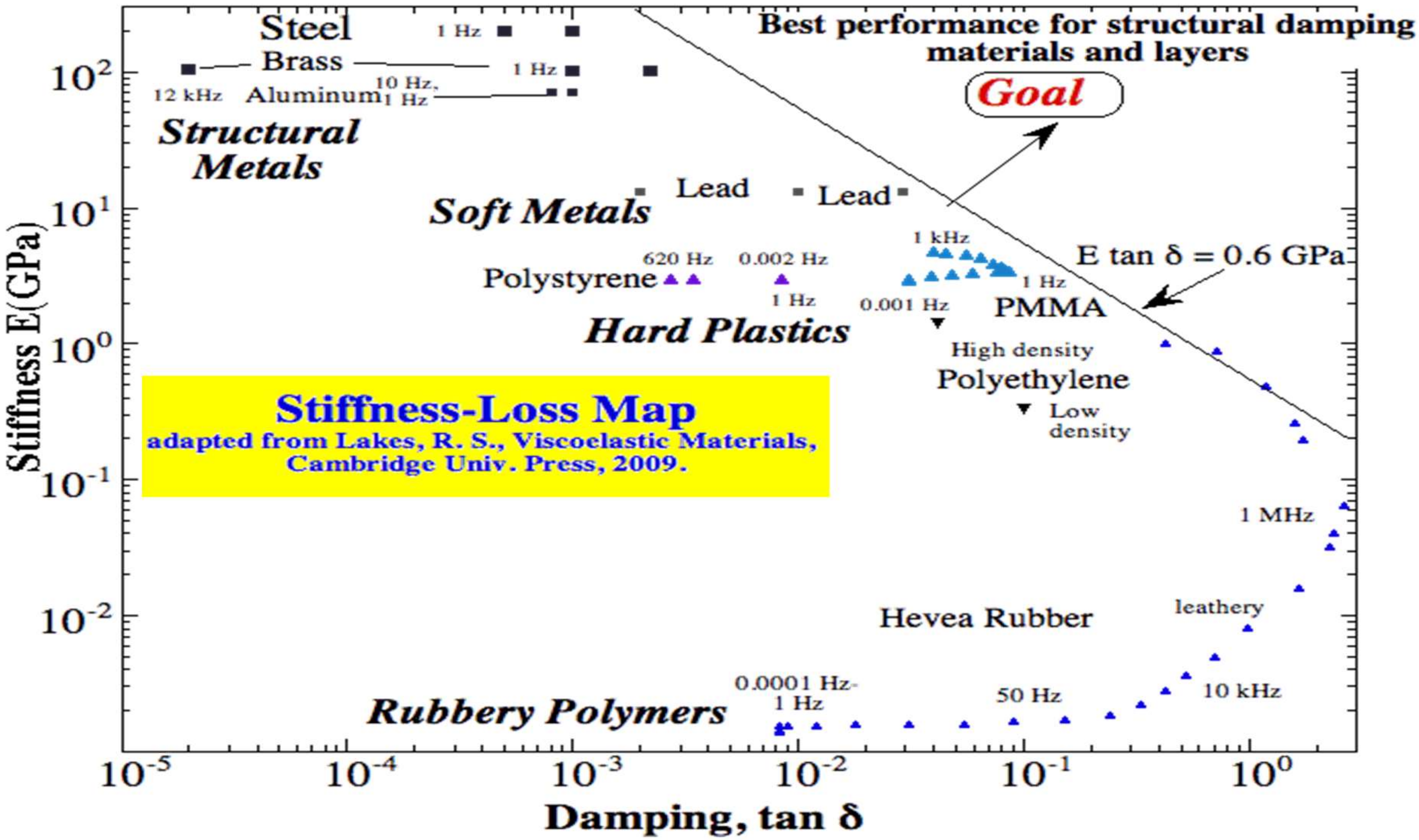
$E(\omega, T, \epsilon_S) = E(\omega\alpha(T, \epsilon_s))$ generalized coordinate in Wicket plot





[1] A. D. Nashif, D. I. G. Jones, and J. P. Henderson, *Vibration Damping*. John Wiley and Sons, 1985.
[2] J. Salençon, *Viscoélasticité*. Presse des Ponts et Chaussés, Paris, 1983.
[3] SDT/Visc www.sdtools.com/pdf/visc.pdf

Modulus / loss map of common materials



$$\sigma(\omega) = E_{complex}\,\epsilon(\omega) = E_{real}\big(1 + i\eta(\omega)\big)\epsilon = E_{real}\big(1 + i\,tan(\delta)(\omega)\big)\epsilon$$

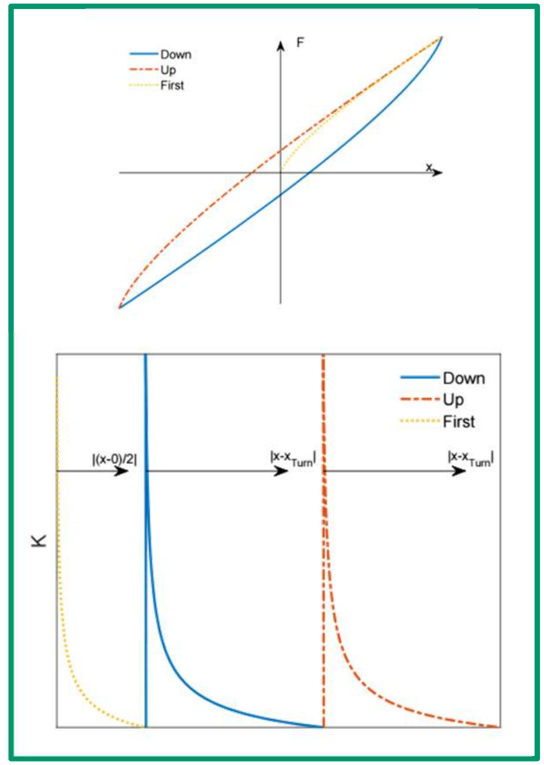
Hysteretic damping : keywords

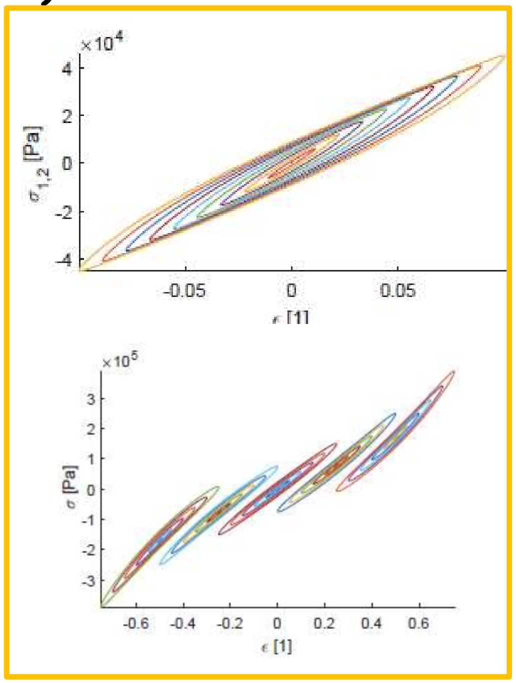
- Stress = fct (strain history)
  but independent on rate

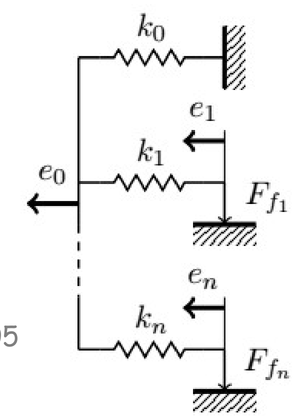
- Non parametric : triangular testing
  - Masing rule (homotethy + offset)
  - Madelung assumption (loop closure)

  - Payne effect (E ↓ amplitude ↑)
  - Mullins (first loading adaptation)

- Parametric models :
  Iwan, Dahl, Berg







[1] D. J. Segalman, "A Four-Parameter Iwan Model for Lap-Type Joints," *Journal of Applied Mechanics*, vol. 72, no. 5, pp. 752–760, Sep. 2005
[2] B. Bourgeteau, "Modélisation numérique des articulations en caoutchouc de la liaison au sol automobile en simulation multi-corps transitoire," Ecole Centrale Paris, 2009
[3] R. Penas, " Models for dissipative bushings in multibody dynamics ", ENSAM, 2021

Rate independent hysteresis

- Hysteretic relaxation (non-parametric)

$$F(x) = F(x_{turn}) + \int_0^{|x-x_{turn}|} K_f(r)\mathrm{d}r$$

- Selected order parametric representation Jenkins cells, in an Iwan series arrangement
- Force formulation

$F^i = \frac{g^i}{g^0} F^0$, if $F^i \mathrm{sign}(\dot{x}) < F_f^i$, and $F^i = \mathrm{sign}(\dot{x}) F_f^i$, if $\frac{g^i}{g^0}|F^0| \geq F_f^i$

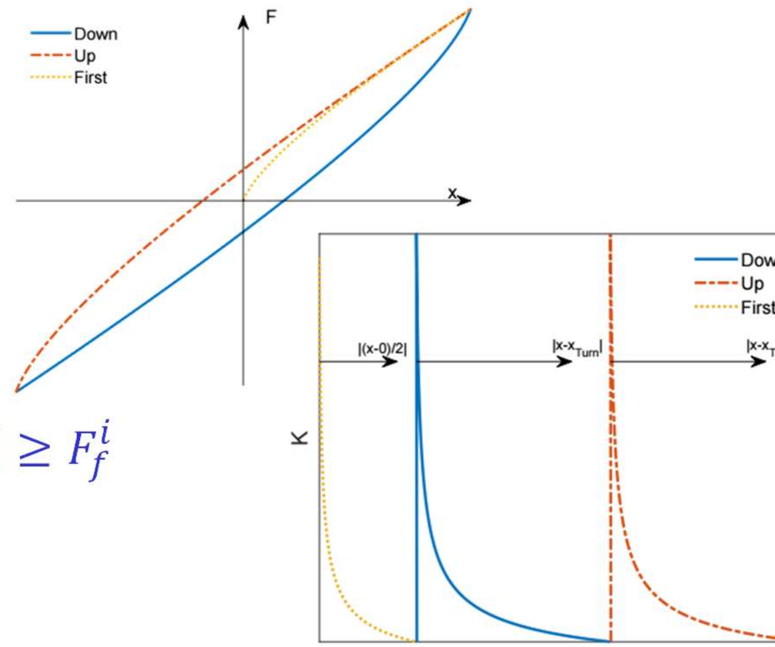
- Displacement formulation

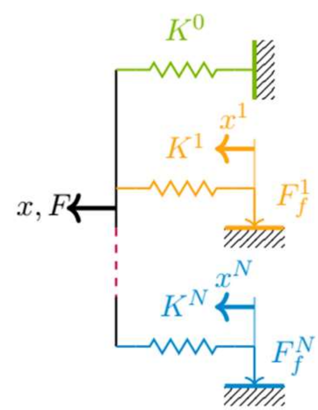
$\dot{x}^i = 0$, if $|x - x^i| < \frac{F_f^i}{K^i}$, and $\dot{x}^i = \dot{x}$, if $|x - x^i| < \frac{F_f^i}{K^i}$

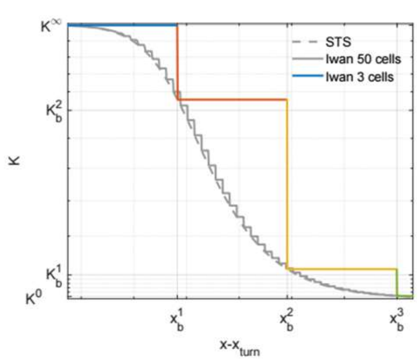
- Definition of break points and stiffness

$K_b^k = \sum_{i<k} K^i$, and $x_b^k = x_{turn} + \frac{F_f^k}{K^k}$

- Order selection: place x_b and K_b in the hysteretic relaxation curve

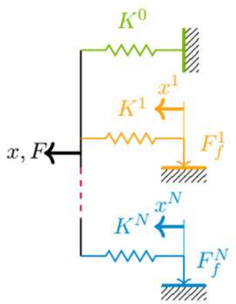


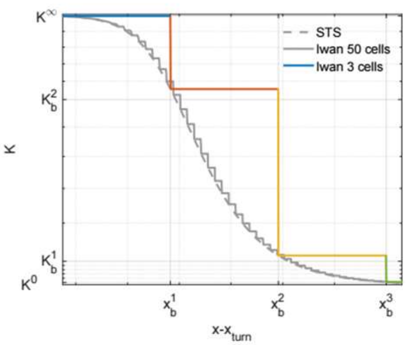


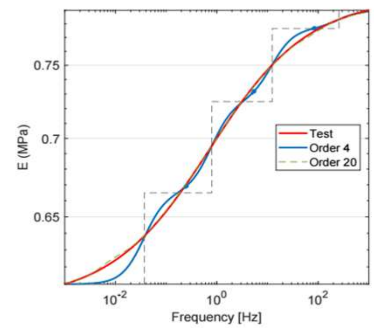


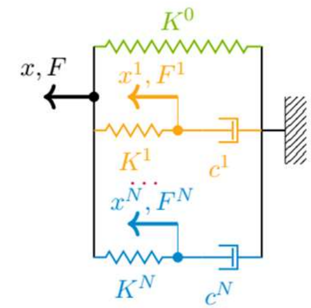
Parallel between viscoelasticity & hysteresis

- Fit non-parametric curves : discretize with selected order



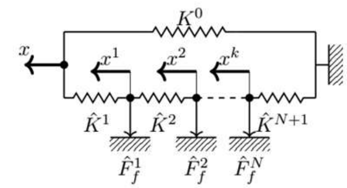




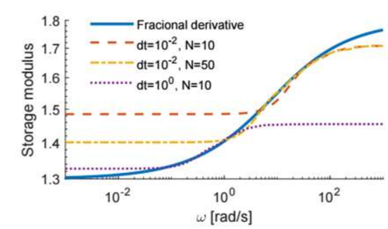


- Identify order independent parameters

  - STS $$F - F_{turn} = \left(\sqrt{NC|x - x_{turn}| + \frac{NC}{2\hat{K}^0}} - \frac{NC}{2\hat{K}^1}\right)\mathrm{sign}(x - x_{turn})$$



  - Segalman (4 parameter friction)
  - Fractional derivative $$F(s) + \frac{1}{\omega_c}\frac{\mathrm{d}^\alpha F}{\mathrm{d}t^\alpha} = K^0 x + \frac{1}{\omega_c^\alpha} K^\infty \frac{\mathrm{d}^\alpha x}{\mathrm{d}t^\alpha}$$

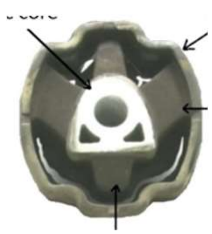


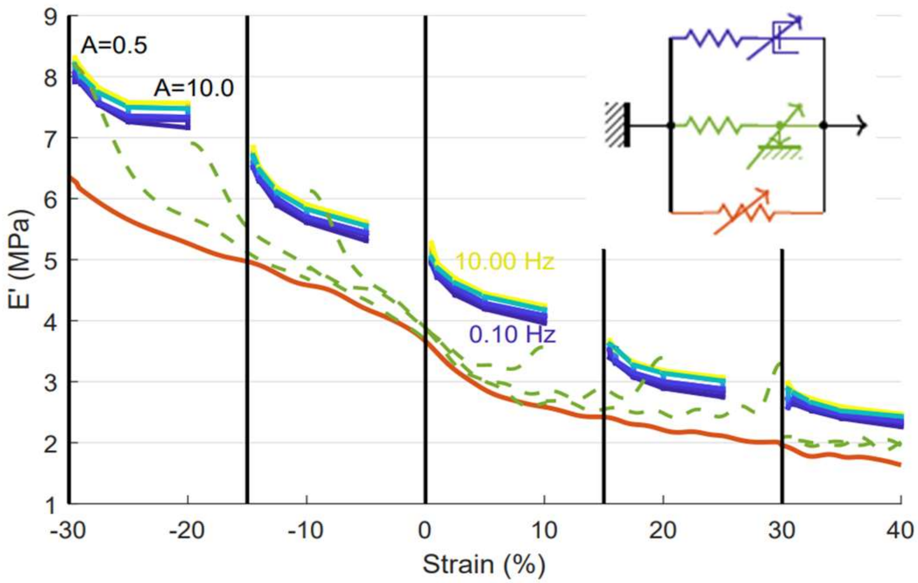
  - Implementation requires discretization = order selection

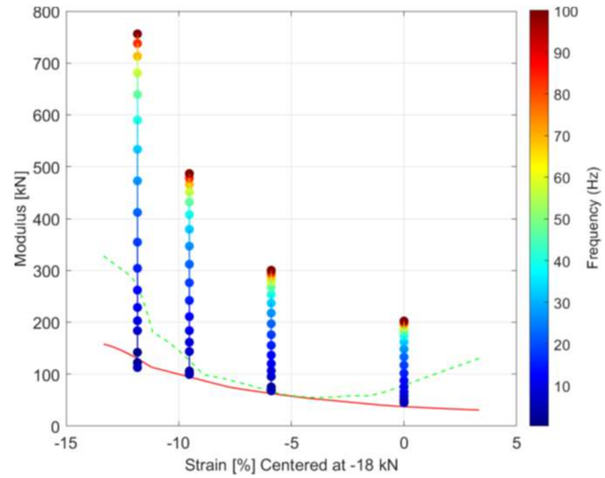
Adding hyperelasticity & combination

Material/mount model split in
static, path and dynamic effects

- Non-parametric models for: hyperelasticity, hysteresis and viscoelasticity $E(\omega, A, \epsilon_0)$





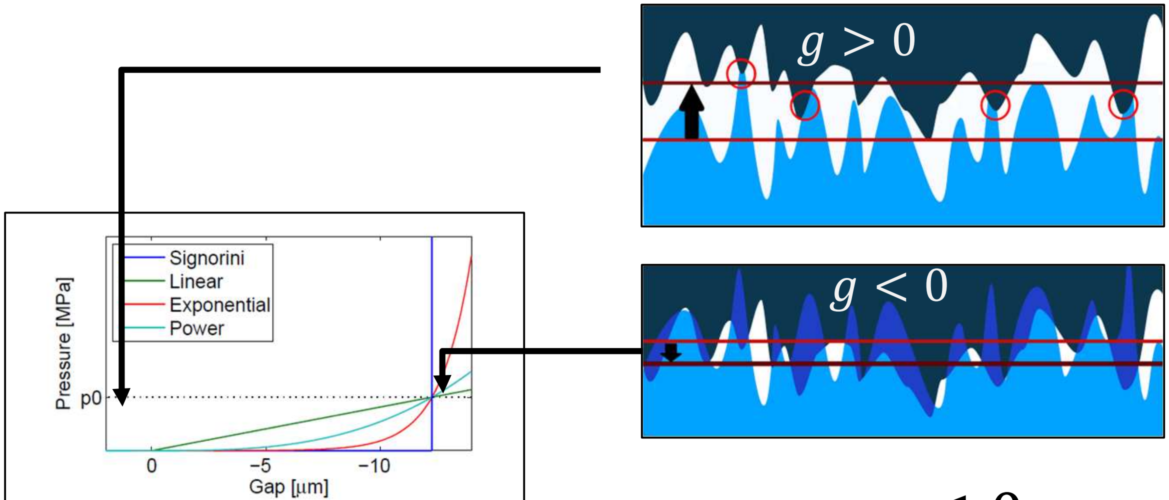


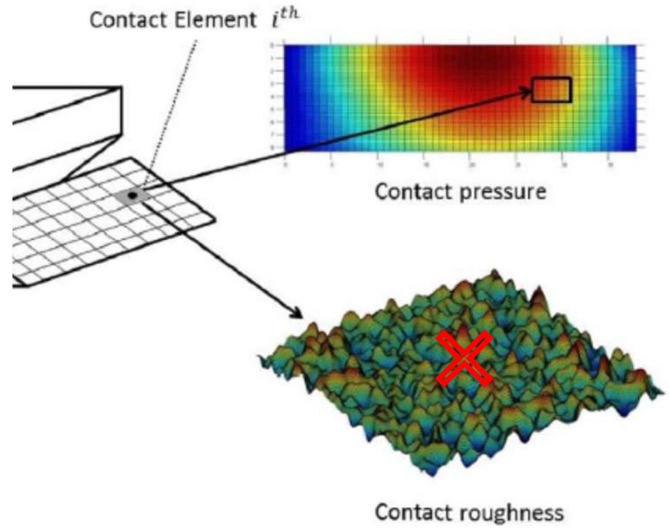
$$\dot{F}^i + \left(\omega^i + \frac{|\dot{x}|}{x_f^i}\right) F^i = \frac{g^i}{g_0} \dot{F}^0(x)$$

- Among all cell models that combine HE, HYST and VE
  - stress rate relaxation $\dot{F}^0(x)$ is needed
  - NL relaxation times $\omega^i + \frac{|\dot{x}|}{x_f^i}$ explain Paine effect

Contact constitutive law + friction

- Macro-scale surface not flat (1 gauss)
- Macro load function of gauss strain

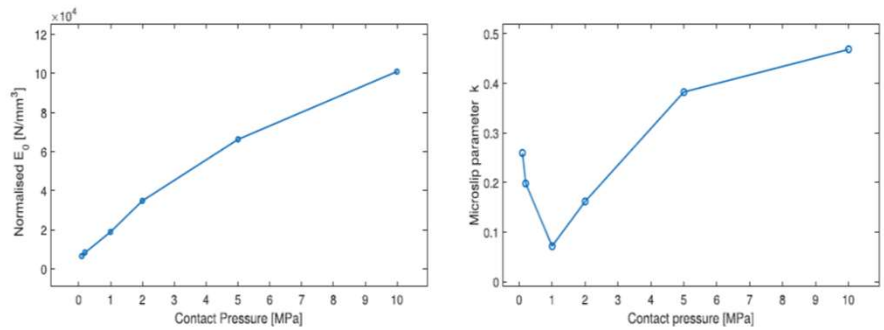




From : L. Pesaresi. JSV 2018

- Idealization Signorini
- Reality $p(\mathrm{g})$

$$g \leq 0$$
$$p > 0$$
$$pg = 0$$



- Friction : coulomb $\sigma_t = -\mu p \dfrac{v_t}{\|v_t\|}$

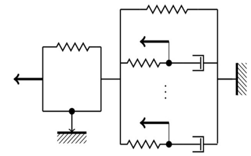
Q4 : from material to structure model

Test of visco mount

$F_x = f(x, \dot{x})$
- Measure disp/load
- Macro : results mix material and geometry

Identification **(p)**

0D / macro / meta-model



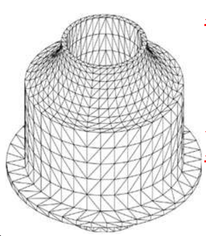
Behavior
- material & structure
- $F_x = f(x, \dot{x}, x_i, \boldsymbol{p})$

Identification **(p)**

Reduction [1] &
Hyper-reduction [2,3]

Transient simulation

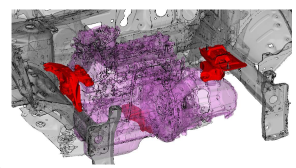
3D physical / micro



Knowledge
- FEM structure
- Implicit/explicit time

Behavior
- $\{\sigma\} = f(\{\varepsilon\}, \{\dot{\varepsilon}\}, \{\varepsilon_i\}, p)$
- Hyper-visco-hyster. mat

Multi-body / system simulation



Use cases
- Engine suspension
- Drive line
- Comfort, endurance

[1] Hammami, Balmes, Guskov, « Numerical design and test on an assembled structure of a bolted joint with viscoelastic damping », *MSSP*, v70, p.714–724, 2015

[2] Farhat, Avery, Chapman, Cortial, « Dimensional reduction of nonlinear finite element dynamic models with finite rotations and energy-based mesh sampling and weighting for computational efficiency», *IJNME*, vol. 98, nº 9, p. 625-662, 2014.

[3] F. Casenave, N. Akkari, F. Bordeu, C. Rey, and D. Ryckelynck, "A nonintrusive distributed reduced-order modeling framework for nonlinear structural mechanics—Application to elastoviscoplastic computations," IJNME, vol. 121, no. 1, pp. 32–53, Jan. 2020, doi: 10.1002/nme.6187.

Historical method : V_0 = reduction modes (poor)

MSE (modal strain energy) is a kinematic reduction using elastic modes and general damping

$$\boxed{\{q\} = [T]\{q_r\} = [\phi_1 ... \phi_{NM}]\{q_r\}}$$

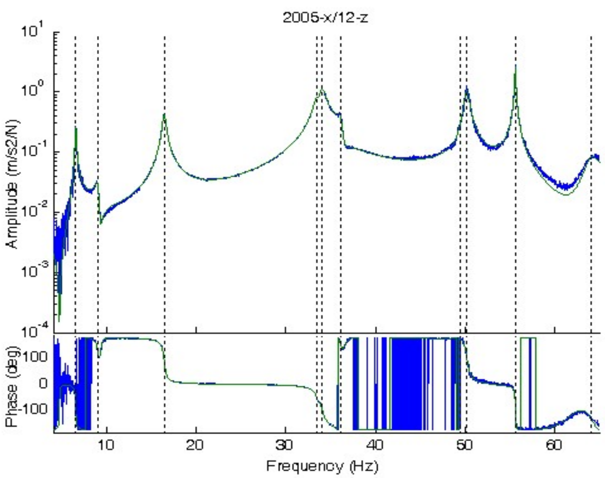
$$\left[s^2[I] + \boxed{[\phi]^T [Im(Z(s))][\phi]} + \left[\setminus \omega_j^2 \setminus\right]\right]\{p\}(s) = [\phi]^T \{F(s)\}$$

NASTRAN

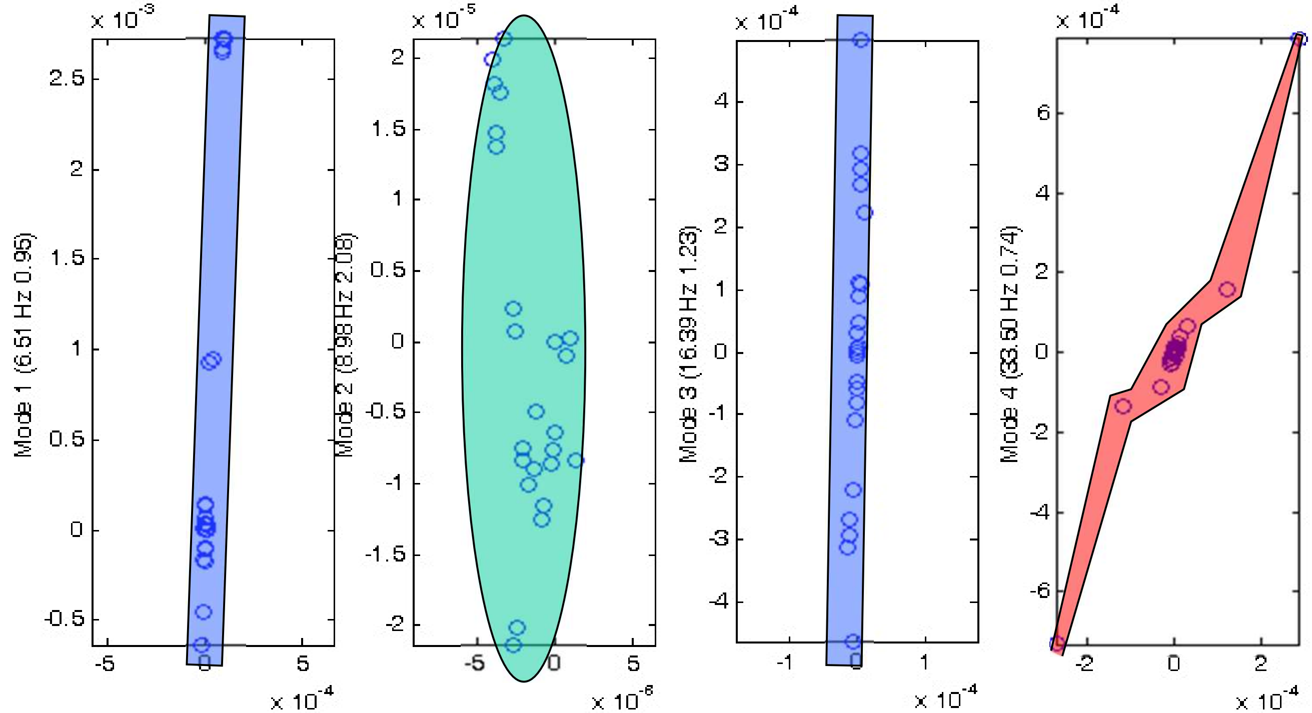
- SOL110 modal complex modes
- SOL111 modal frequency response
- SOL112 modal transient

When does MSE work ?

Experimental modes are often “real”



Garteur SM-AG19 test



“Real modes”	Residues for I/O pairs line up
“Complex modes”	Residues have a phase spread
Poor modes	Have complex residues

When are complex modes nearly real ?

- Uncoupling criterion (Hasselman) ⇔

$$\boxed{\min(\zeta_1\omega_1, \zeta_2\omega_2)/|\omega_1 - \omega_2| \ll 1}$$

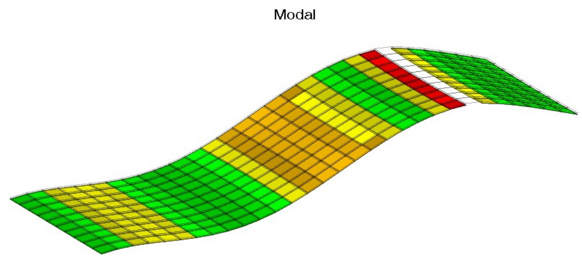
corresponds to non overlap of peaks : using real modes is then OK

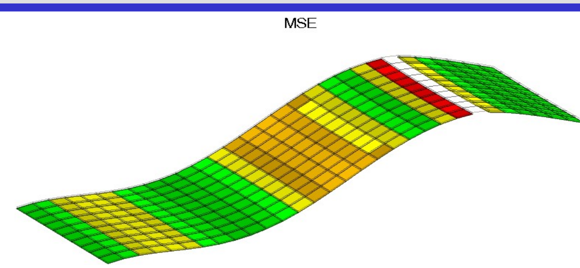
- Proof based on damping matrix positiveness

$$\gamma_{12}\gamma_{21}/(\gamma_{11}\gamma_{22}) < 1$$

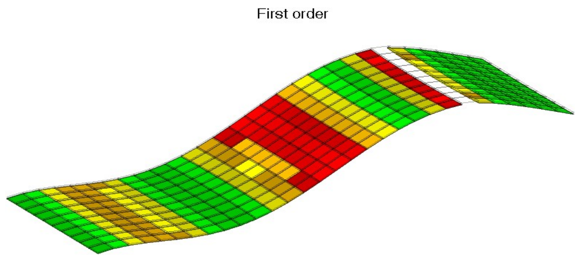
- When damping is enhanced by design, modes are often complex

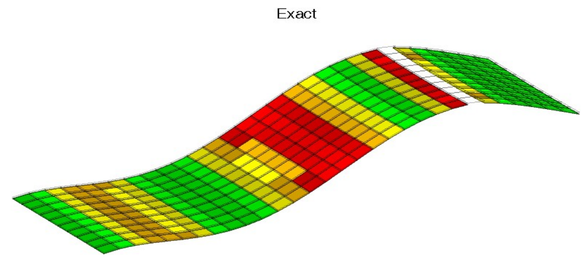
Historical V_1 = reduction modes + enrichment (good)

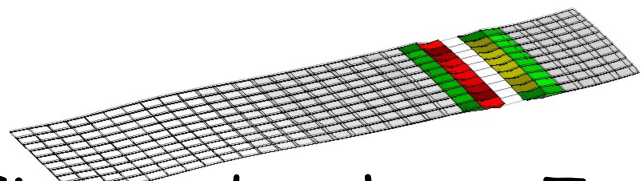




- Forced response around a resonance
- Objective correct strain energy







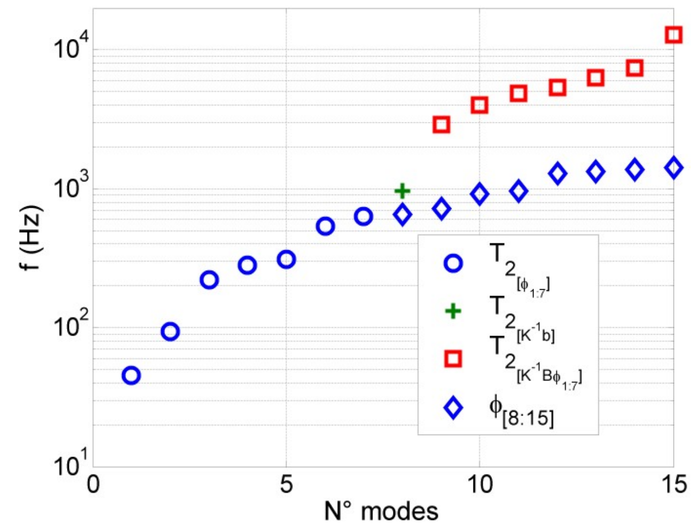
First order shape : $T = [K_o^{-1} \, [\mathrm{Im}(Z-Zo)] \, \phi_4 \,]_{orth}$

- Modes (MSE) $\quad T=[\Phi(p_0)]$
- Multi-model $\quad T=[\Phi(p_1) \; \Phi(p_2)]$
- Modes + static $\quad T=[\Phi(p_0) \; K_o^{-1} \, [\mathrm{Im}(Z-Zo)] \, \Phi(p_0)]$

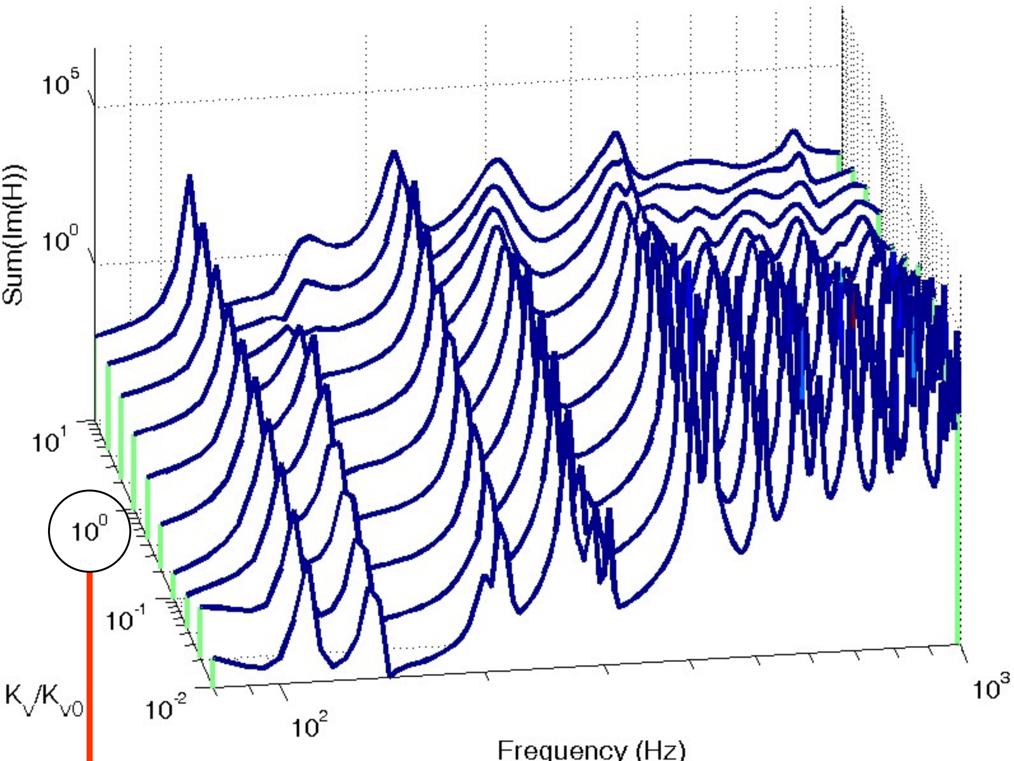
Reduction Bases/Starting Point

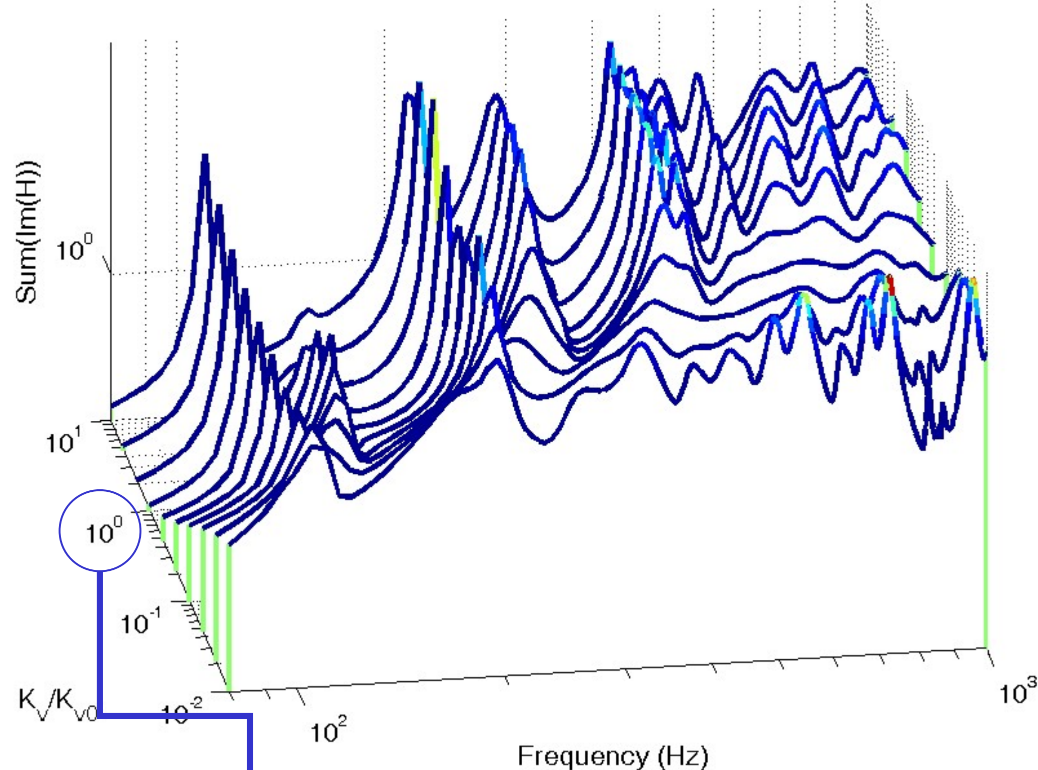
- Modal $T=[\Phi(p_0)]$
- Modal + static Responses to input $T=[\Phi(p_0)\ K^{-1}b]$
- Modal + static responses to representative loads $T=[\Phi(p_0)\ K^{-1}[b\ R_j]\ \Phi(p_k)]$

- Frequencies of corrections □ differ from modes ♦ : the added information is VERY different



MSE validity FRF prediction



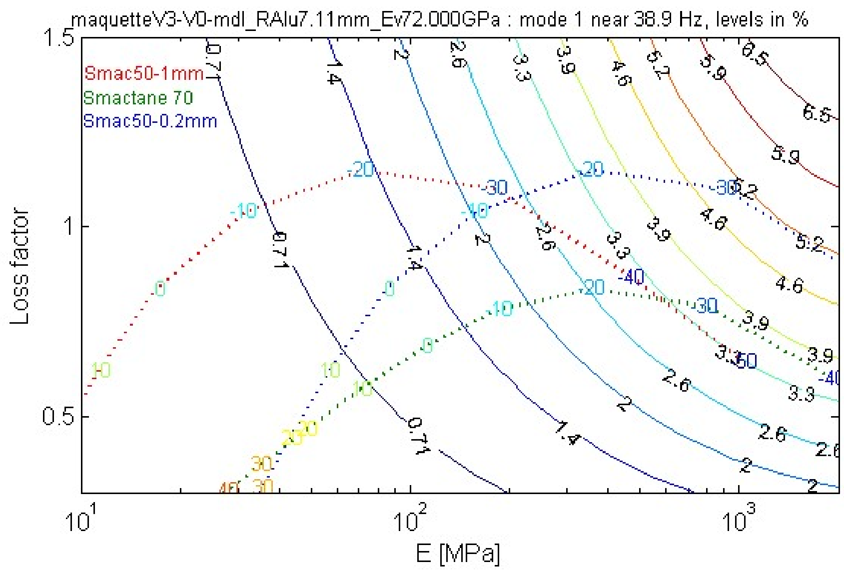


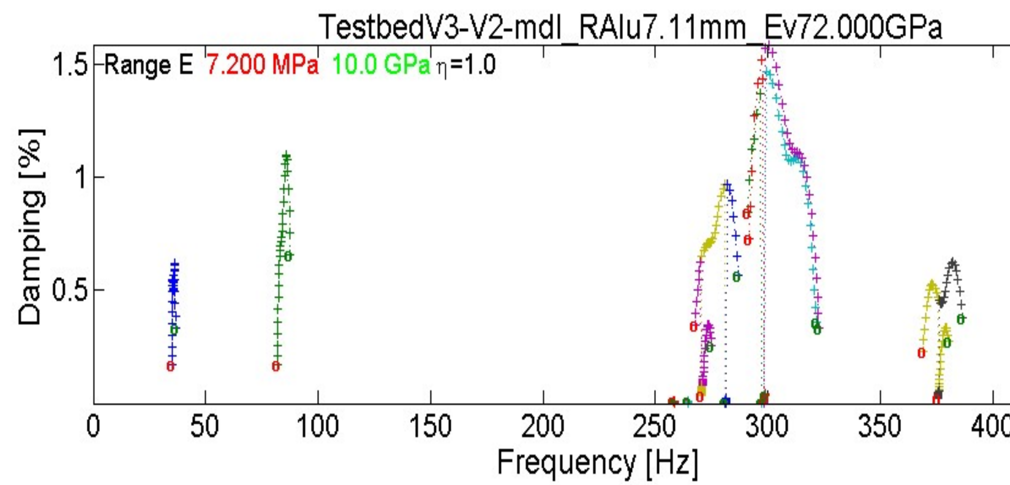
$$T = \boxed{\left[\boxed{\phi(k_v)_{1,NM}} \quad K_0^{-1} K_v \phi(k_v)_{1,NM}\right]}$$

Fixed nominal modes give a poor approximation

Eigenvalue extraction

- Fixed E sparse solvers are not robust and slow
  $\Rightarrow$ reduction T + full solver $\qquad [Z(E_i,\lambda_j)]\{\psi_j\} = \{0\}$
- Analytic E(s) not common + associated sparse eigenvalue not available $\Rightarrow$ only used for transient
- Need to track $\{\psi_j\}$, $\omega_j, \zeta_j = f(E(T,\omega_j), \eta(T,\omega_j))$
  $\Rightarrow$ identify from forced response
  $\Rightarrow$ interpolate from $f(E,\eta)$

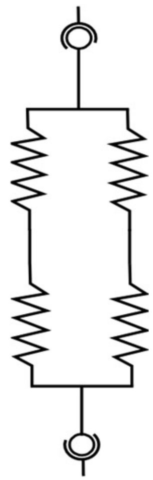


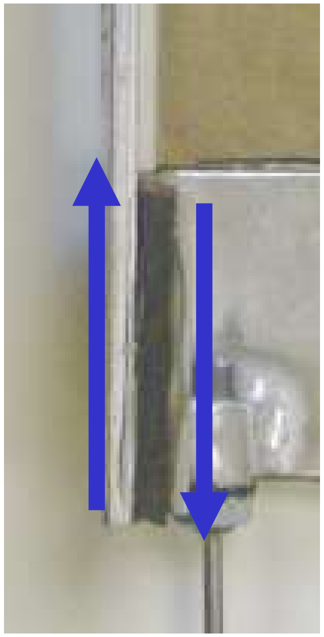


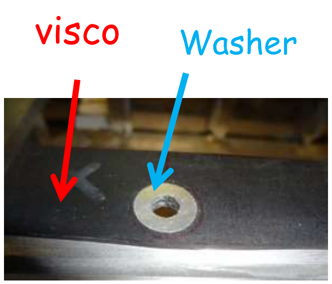
Structural mechanisms

- Compression & shear springs
  - Integration gives orders of magnitude choice in functional stiffness
  - Possibly structured junctions
- Free layer bending (compression)
  - Inefficient if damping material is too soft
- Constrained layer bending (shear)
- Trough thickness dynamics
  - Only efficient in resonant (high frequency behavior)

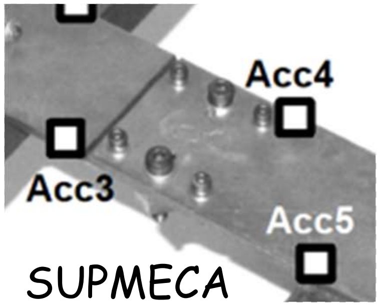
$$k_z(\omega) = G(\omega)\left(1 + i\eta(\omega)\right)\frac{S}{\tau}$$



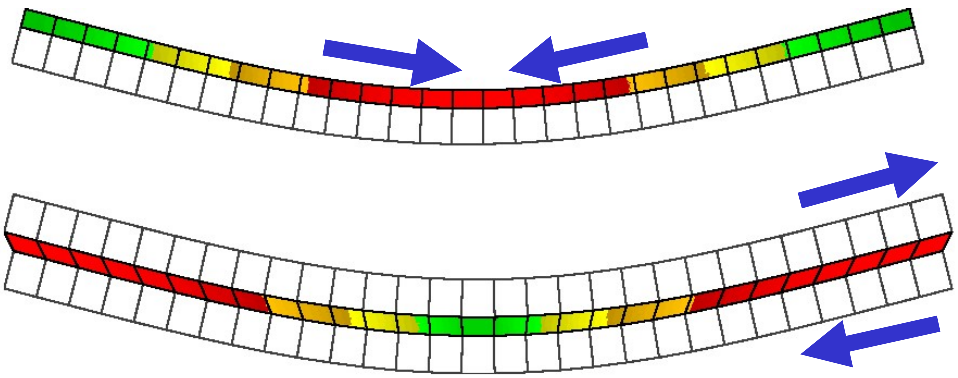




ENSAM



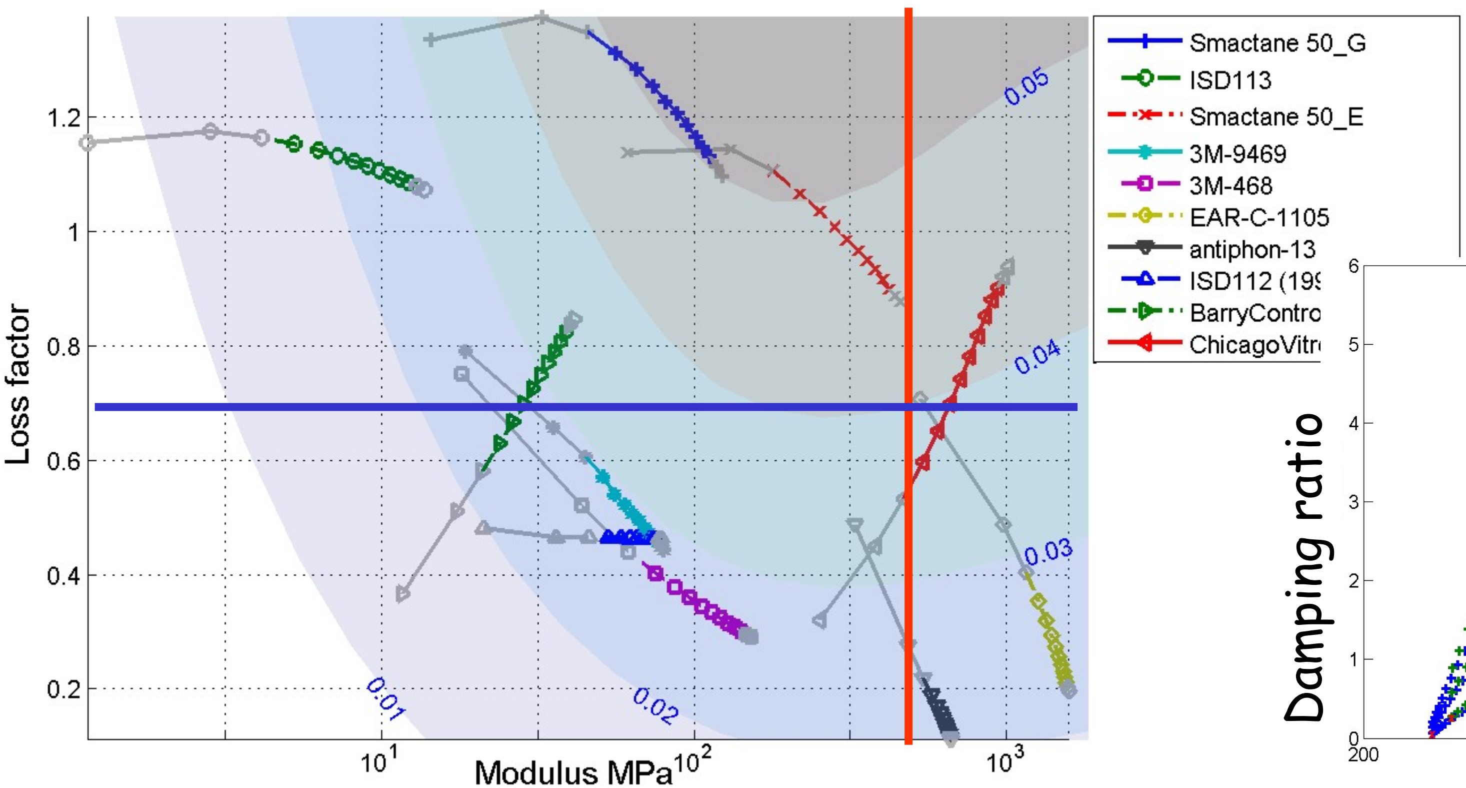
SUPMECA

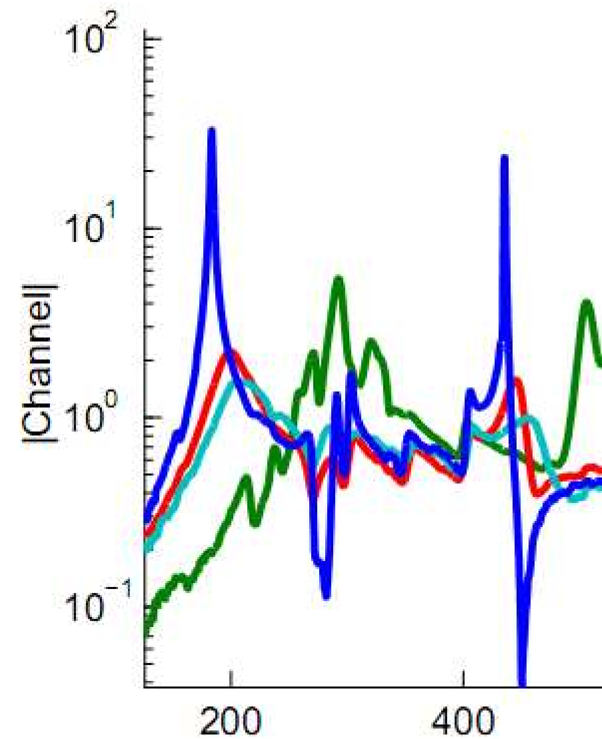


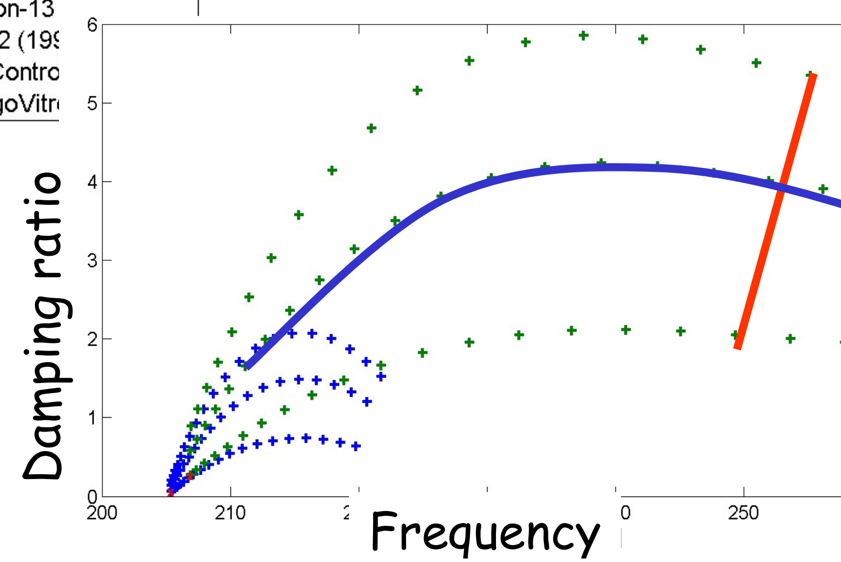
Optimization is necessary

- Design space : material, thickness, surface (GS/h or EA/h)
- Multiple orders of magnitude

Damping ratio target example

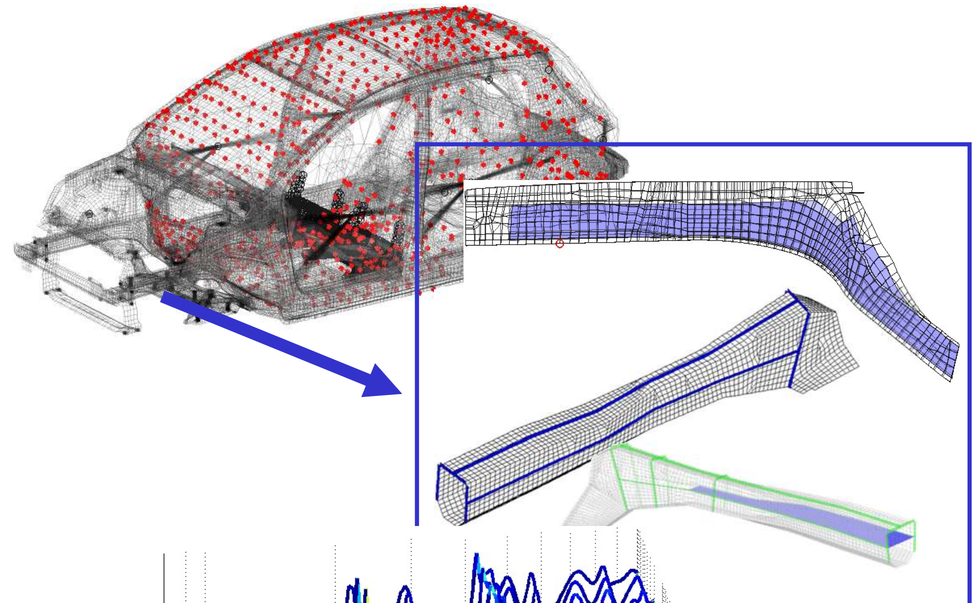


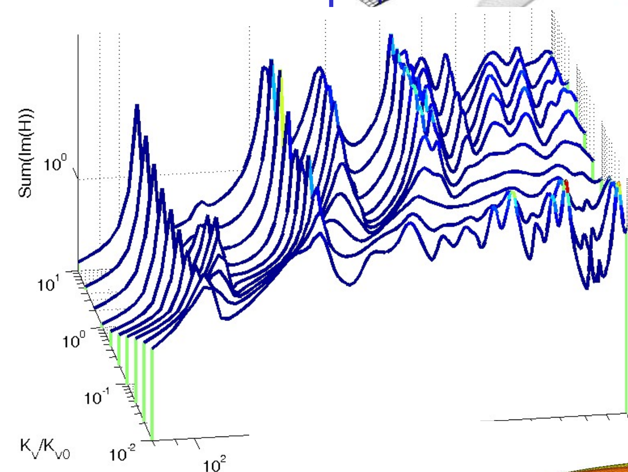


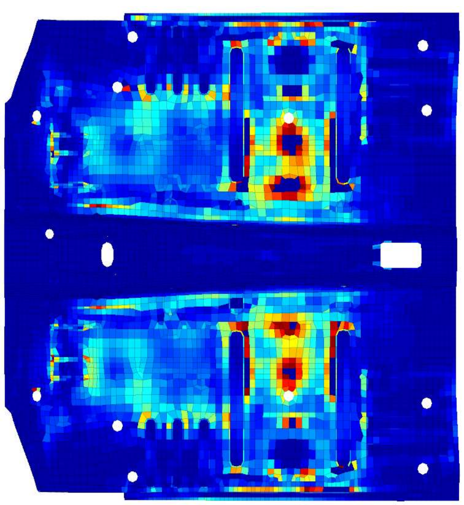


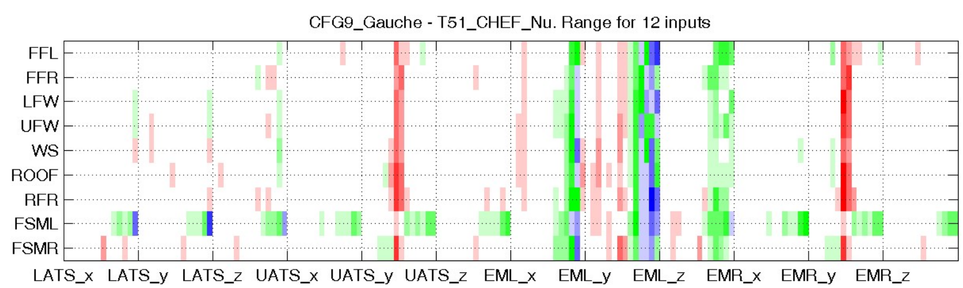
Why optimization is a challenge

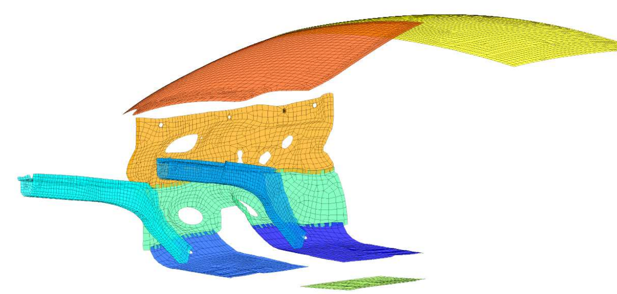
- Large models (x1e6 DOF)
- Many modes /frequencies to compute (x1e3)
- Multiple designs (x10)
- Large parametric variations (1e4)
- Complex objectives (x1e3 disp)





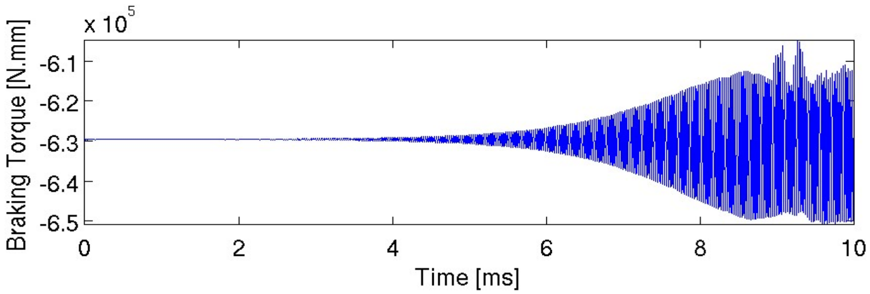


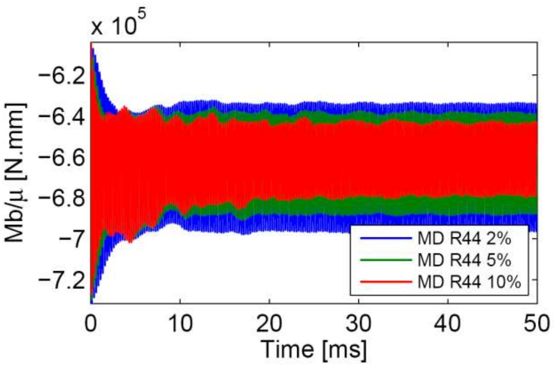


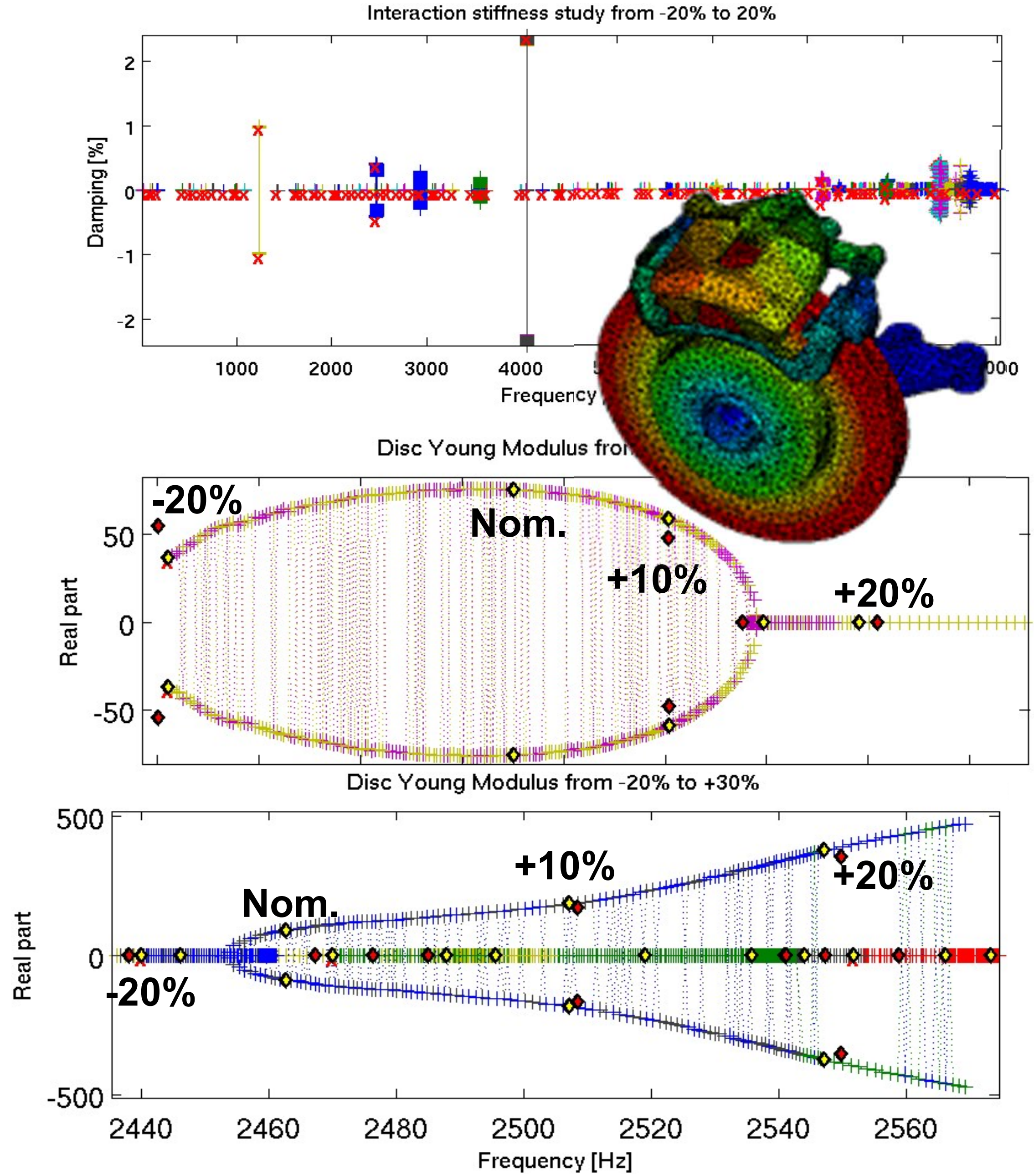


Historical V_2 = multi-model & CMS

Objectives:
- complex mode stability
- Transient simulation

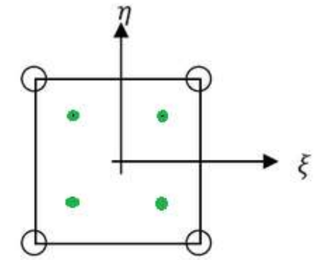






Historical V_3 = hyper reduction of transient NL

HR phase 1 : generic form of FEM problems



1. FEM kinematics $u(x,t) = [N(x)]\{q(t)\}$

$$\epsilon = [\mathbb{C}]_{(N\epsilon \times N_g)\times N}\{q\}$$

2. Material evolution : $S_g = f_{mat}\left(\epsilon_g, u_{int\ g}\right)$

3. Virtual work $M\ddot{q} + \mathcal{F}_{int}(S, u_{int}) = \mathcal{F}_{ext}$

- Integration at quadrature points g (disassembly)

$$\mathcal{F}_{int}(q, u_{int}) = \sum_{g} \mathbb{C}^T J_g w_g f_{mat}(\epsilon, u_{int})$$

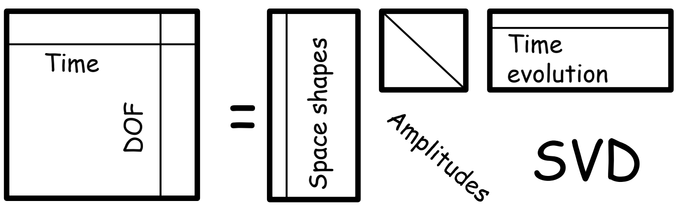
$$= [\mathbb{B}]_{N\times(N_g \times N_f)}\left\{f_{mat}\left([\mathbb{C}]_{(N\epsilon\times N_g)\times N}\{q\}, u_{int}\right)\right\}$$

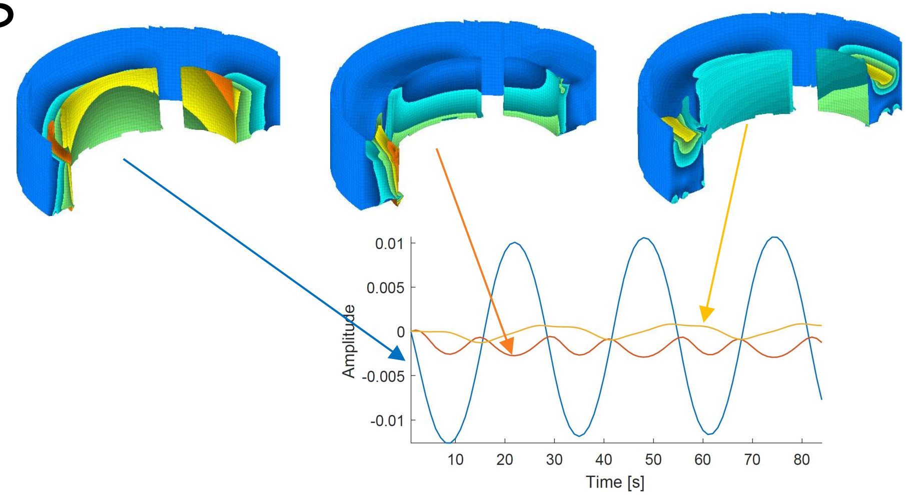
HR phase 2 : kinematic reduction

1/ Off-line learning (non-linear High Fidelity Simulation)
Snapshots or iterative method

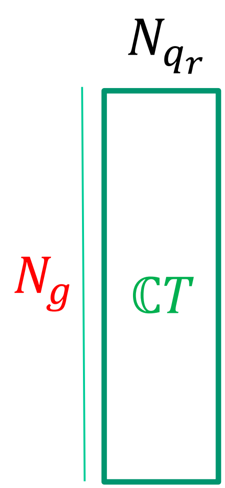
2/ Kinematic reduction using SVD, or CMS

$$q_{learn} = \sum_i \{U_i(x)\}\,(\sigma_i\, v_i(t))$$





Reduced equations of motion $T^T M T \ddot{q}_r + [T^T \mathbb{B}]_{Nqr \times Ng} f_{mat}(\mathbb{C}T q_r, u_{int}) = T^T F_{ext}$



- N_g (gauss points) remains large
- $f_{mat}(\mathbb{C}T q_r, u_{int})$ evaluation dominates

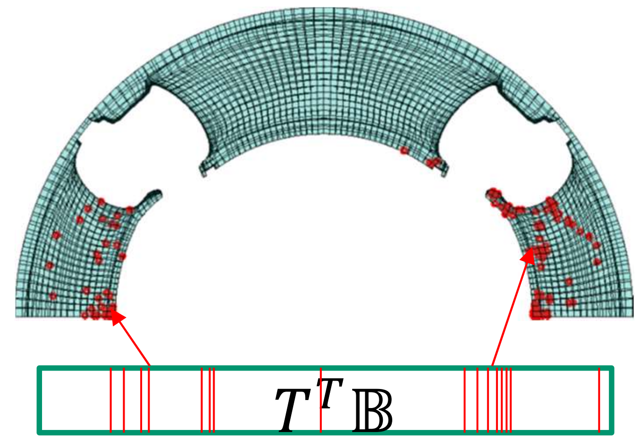
HR phase 3 : operator reduction

3/ Operator reduction (hyper-red.) = less points + new weights

$$\{F_{int}\} = \int_\Omega f_{mat}(x_g, t)\, \mathrm{d}V \approx \sum_{g} \mathbb{C}^T J_g w_g f_{mat\ g} = \mathbb{B}(x_g) f_{mat\ g}(q, t)$$

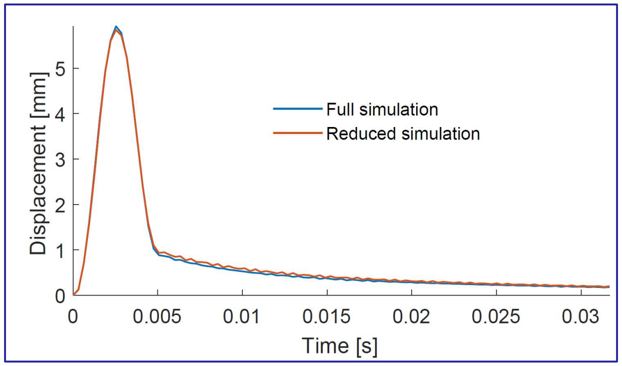
Choose NT learning points and minimize $\|w_g^*\|_0$ subject to[1]

$$\left\| \left[[T^T \mathbb{B}]\, S_l \right]_{(NT \times NR)} - \left[[T^T \mathbb{B}_g]\, S_{lg} \right]_{(NT \times NR) \times Ng} \{w_g^*\} \right\|_2 < \varepsilon_{tol} \text{ and } w_g^* > 0$$



4/ On-line usage (29s vs. 27h)

5/ Post-processing (estimate continuous fields)



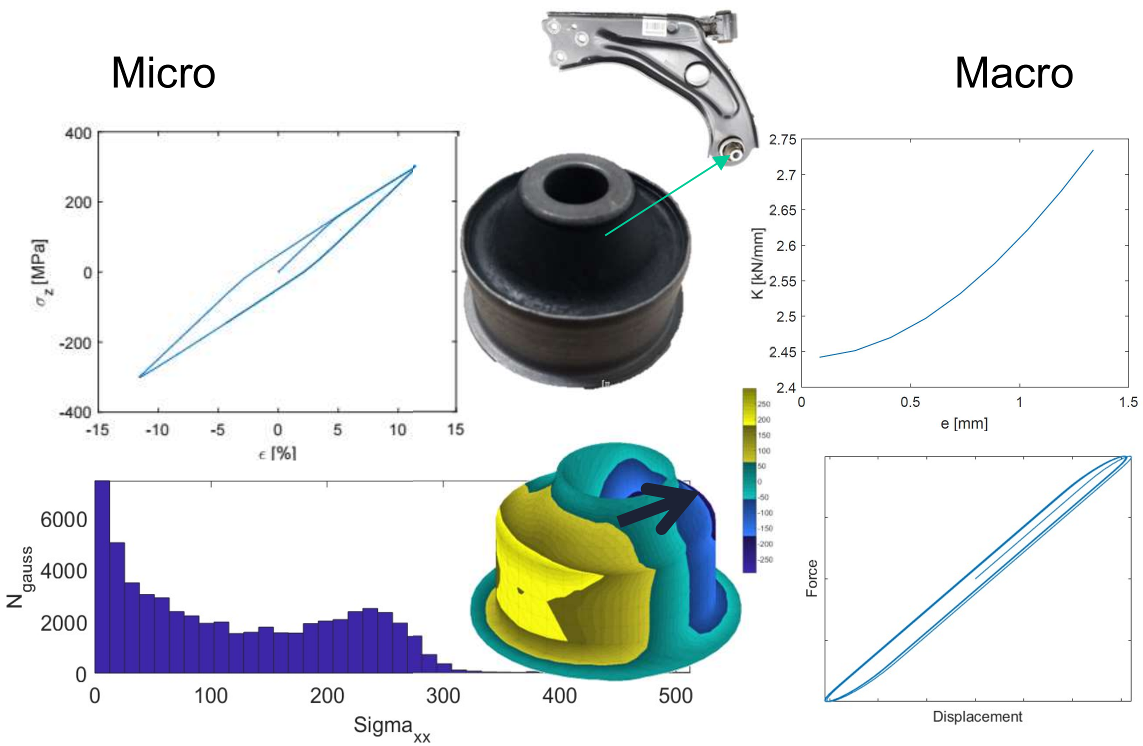
[1] Farhat, Avery, Chapman, Cortial, « Dimensional reduction of nonlinear finite element dynamic models with finite rotations and energy-based mesh sampling and weighting for computational efficiency», *IJNME*, vol. 98, nº 9, p. 625-662, 2014.

[2] F. Casenave, N. Akkari, F. Bordeu, C. Rey, and D. Ryckelynck, "A nonintrusive distributed reduced-order modeling framework for nonlinear structural mechanics—Application to elastoviscoplastic computations," IJNME, vol. 121, no. 1, pp. 32–53, Jan. 2020, doi: 10.1002/nme.6187

[3] Penas, Rafael, *Models of dissipative bushing in multibody dynamics*, PhD ENSAM 2021

System = macro / micro

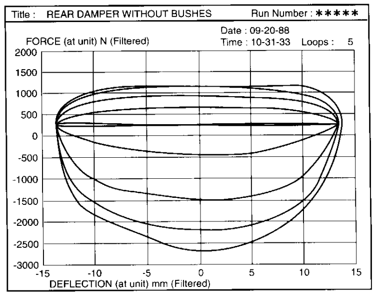
- Field distribution : macroscopic law not a simple function of material behavior



- Resultant (average in space)
- Loss factor (average in time & space)

$$\eta(X,T) = \frac{\frac{1}{2\pi}\int_0^T \sigma(X,t)\dot{\epsilon}(X,t)dt}{\frac{1}{2}\max_0^T\left(\sigma(X,t)\epsilon(X,t)\right)}$$

System/macro force-state map
F = f(q,t)
Meta-model



Internal strains or internal displacement

Vicoelastic stress field

$$E(s) = E_\infty - \left(\sum_{j=1}^{n} \frac{E_j}{s+\omega_j}\right)$$

1. FEM kinematics
2. Material evolution : $S = f_{mat}(\mathrm{u_{nl}}, u_{int})$

$$q_{vj} = -\frac{E_j}{(s+\omega_j)} q$$

3. Virtual work $M\ddot{q} + \mathcal{F}_{int}(q, S) = \mathcal{F}_{ext}$

$$\mathcal{F}_{int}(q, u_{int}) = [\mathbb{B}]_{N\times(N_g \times N_f)}\{f_{mat}(S)\}$$

Viscoelastic parametric matrix $\mathcal{F}_{int}(q, u_{int}) = \left(E^\infty + \sum_j \frac{E^j}{s+\omega_j}\right) K^u\{q\}$

Augmented state-space

$$E(s) = E_\infty - \left(\sum_{j=1}^{n} \frac{E_j}{s+\omega_j}\right) \Longrightarrow \quad q_{vj} = -\frac{E_j}{(s+\omega_j)} q$$

$$\left[\begin{bmatrix} M & 0 & 0 \\ 0 & M & 0 \\ 0 & 0 & M \end{bmatrix} s + \begin{bmatrix} 0 & -M & 0 \\ K_e + E_\infty K_v & 0 & K_v \\ E_j M & 0 & \omega_j M \end{bmatrix}\right] \begin{Bmatrix} q \\ sq \\ \boxed{q_v} \end{Bmatrix} = \begin{Bmatrix} 0 \\ F \\ 0 \end{Bmatrix}$$