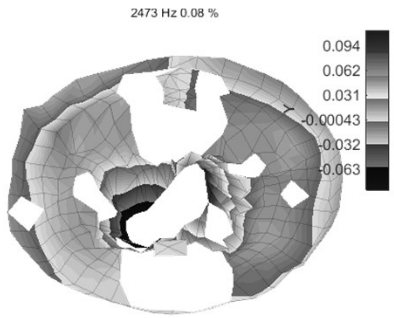
Correlation

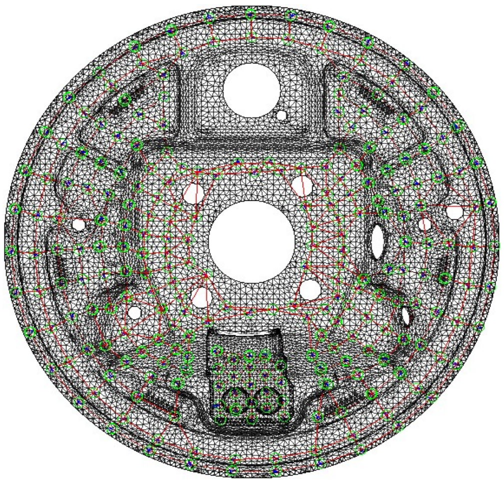
- Observation (poly section 9.1 topology correlation)
- Sensor placement objectives and pathologies
- Correlation criteria (section 9.2 and 9.3)
- Expansion (section 9.5)

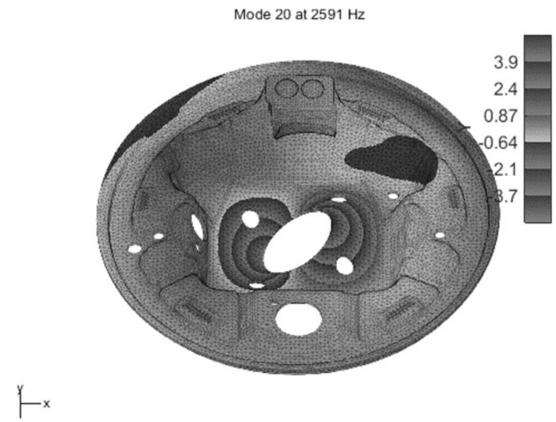
MS2SC http://savoir.ensam.eu/moodle/course/view.php?id=1874
PROVIR http://savoir.ensam.eu/moodle/course/view.php?id=490

Hybrid test & FEM



Identification error
- Noisy measurements
- Identification bias
- NL, time varying, …





FEM error
- Geometry
- Material parameters
…

Topology correlation= observe FEM @ sensors $\{y\} = [c]\{q\}$

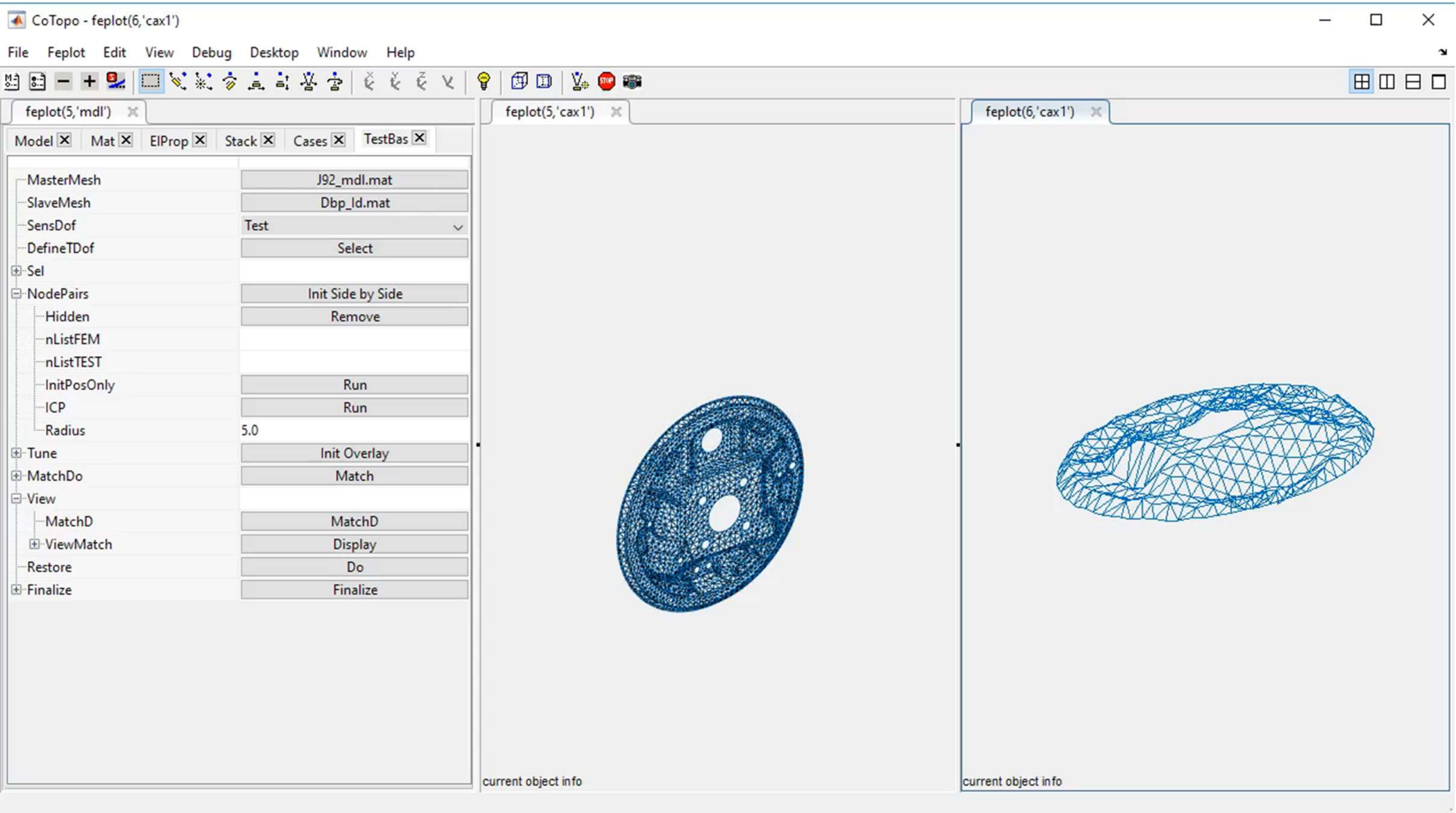
Correlation = distance between test & FEM $J = \|\{y\}, [c]\{q\}\|$

Updating = estimate parameters using correlation objective

Expansion = estimate all DOF knowing test and model

State estimation, hybrid twin

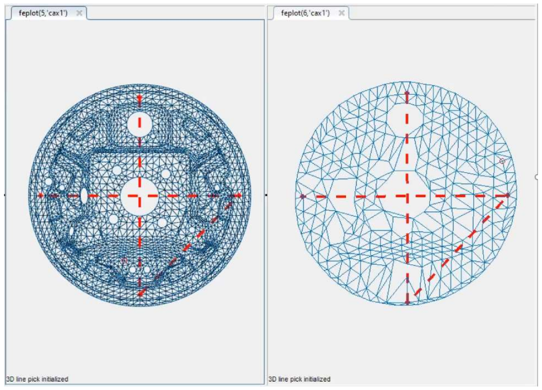
Basic topology correlation process

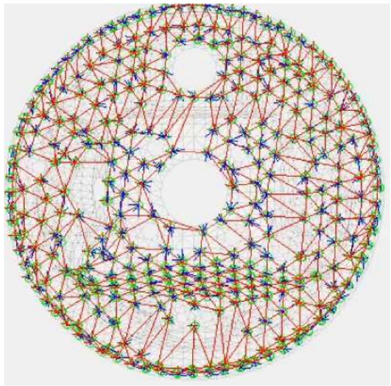


https://www.youtube.com/channel/UCy5PHLw70NqfDDvg24tXJWg

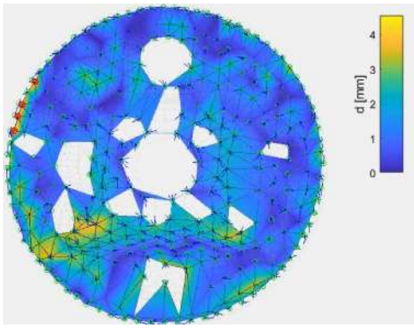
Topology correlation process

- Initial orient (coarse)
- Give reference points & reposition





- Optimize orientation to minimize distance
- Analyze errors



Observation strategy

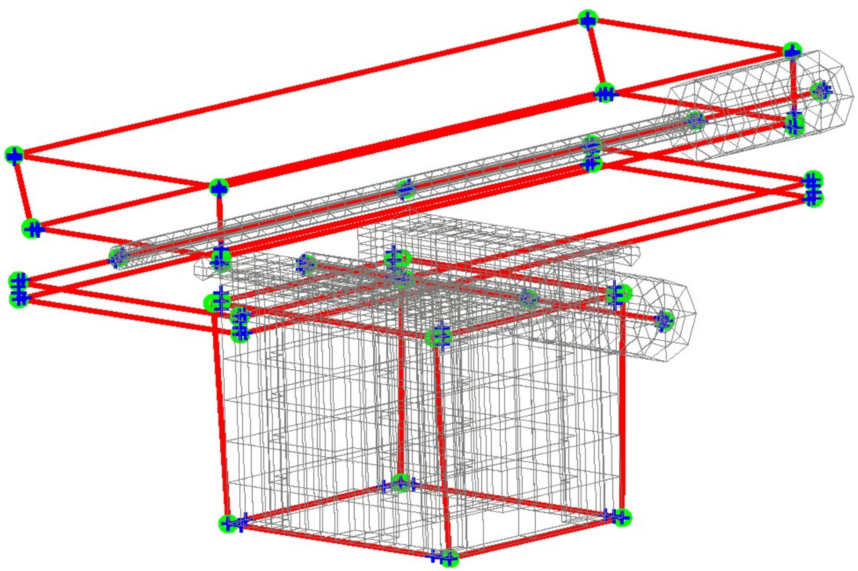
Linear observation equation

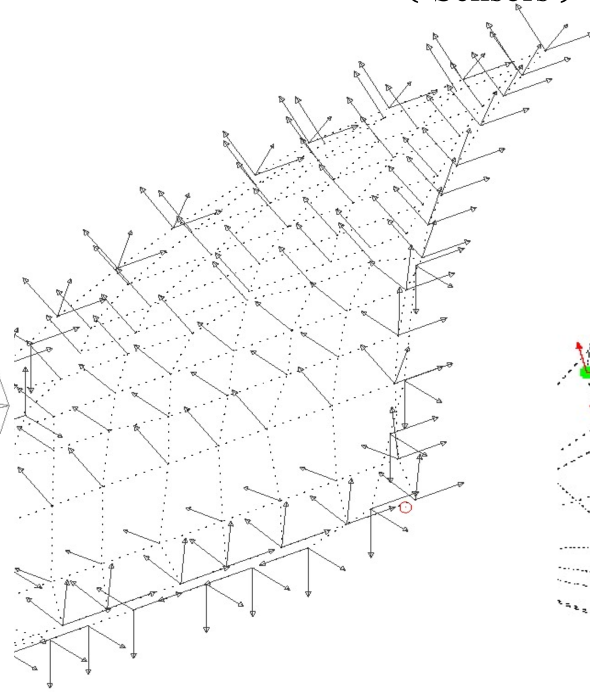
$$\{y\}_{NS\times 1} = [c]_{NS\times N}\{q\}_{N\times 1}$$

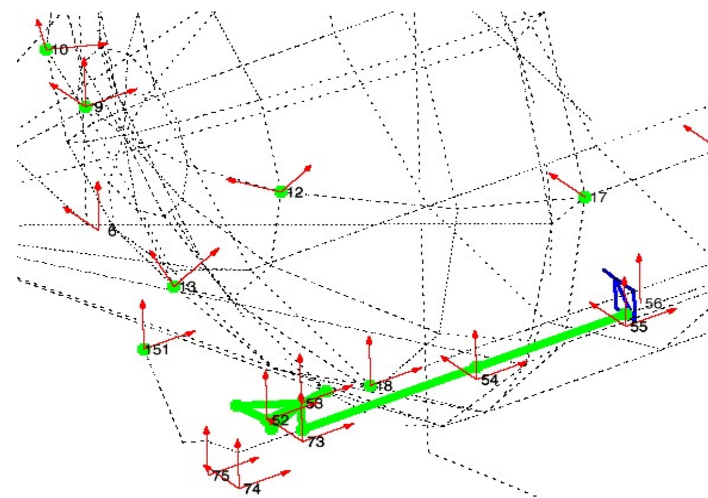
- translations, rotations, deformations
- non-coincident test/FEM
- arbitrary orientations

$$\{t_{\text{Test nodes}}\} = [c_{NA}] \quad \{q_{\text{All DOFs}}\}$$

$$\{t_{\text{Sensors}}\} = [c_{TN}] \quad \{t_{\text{Noeuds Capteurs}}\}$$

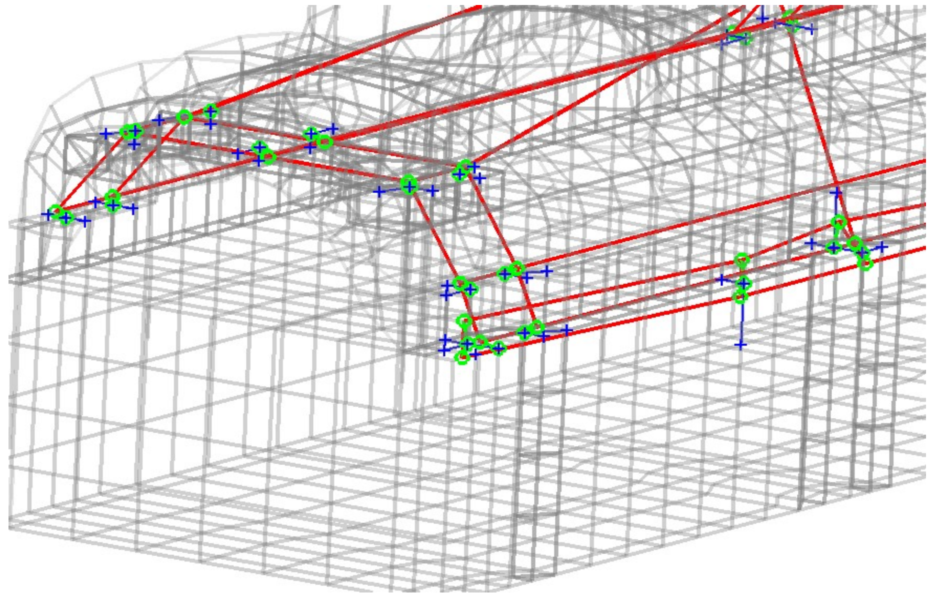


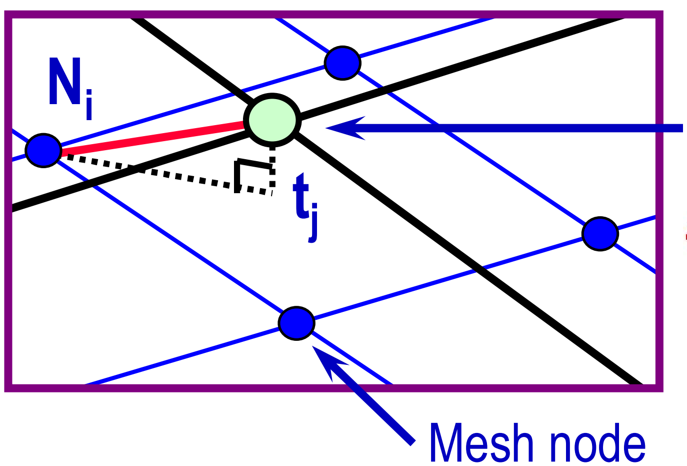


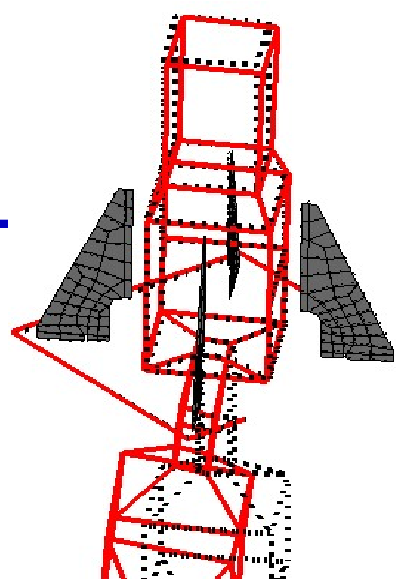


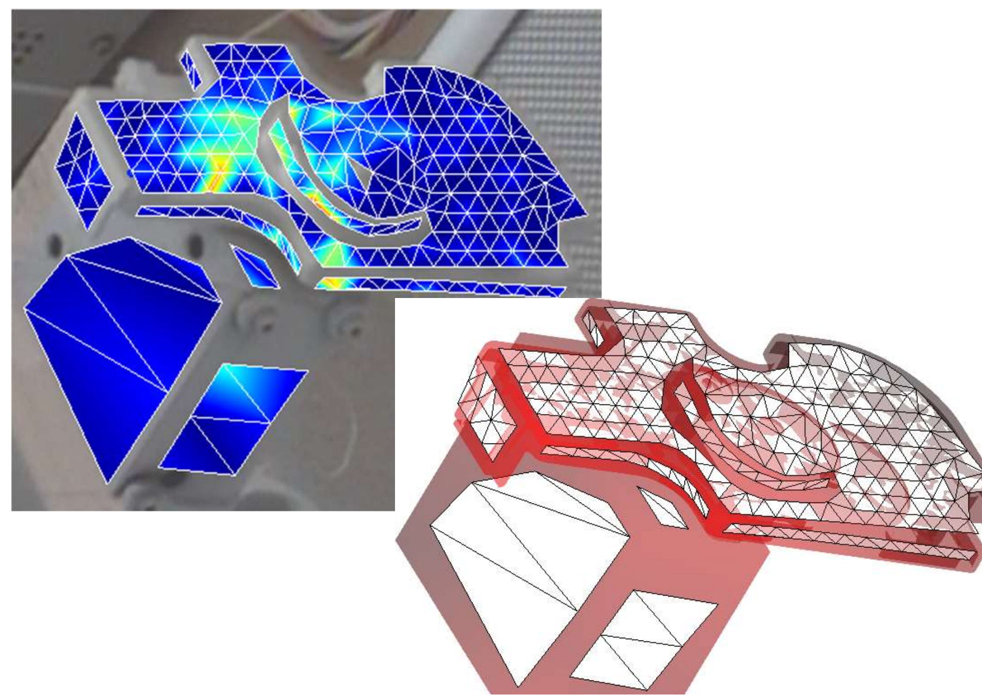
Topology Correlation problems

- Determine matching surface
- Geometry mismatch
- Handle offsets for shells
- Exact location in presence of camera distortion
- …



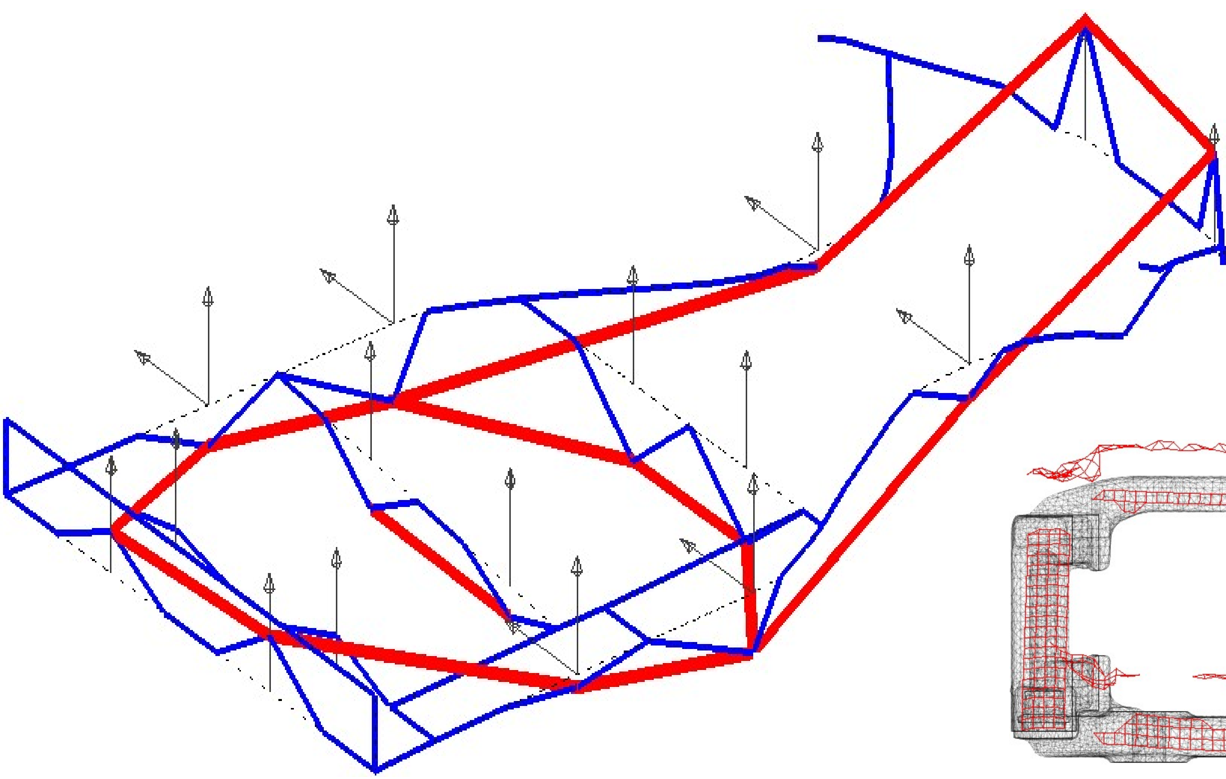


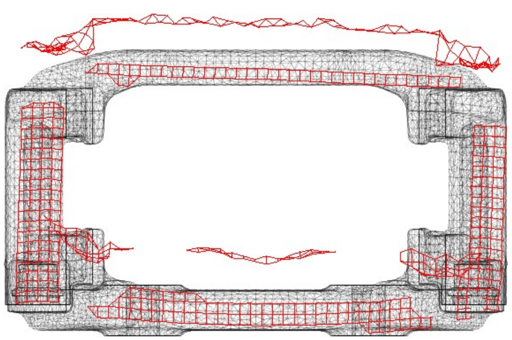


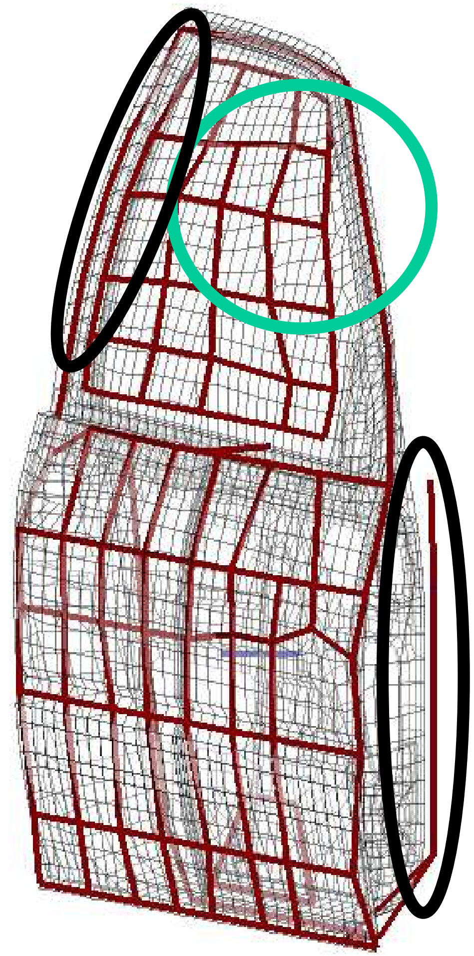


View & correlate

- Connect to allow viewing
- Use regularity to avoid distortion
- Use exact topology



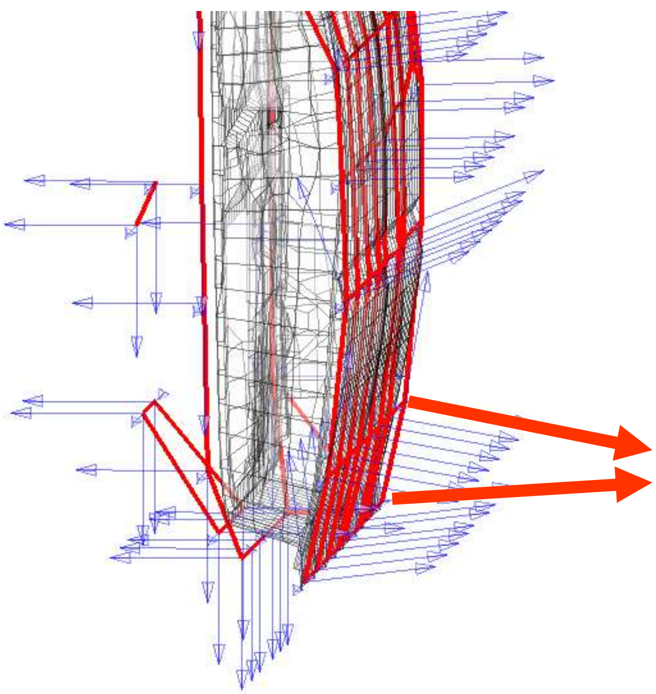


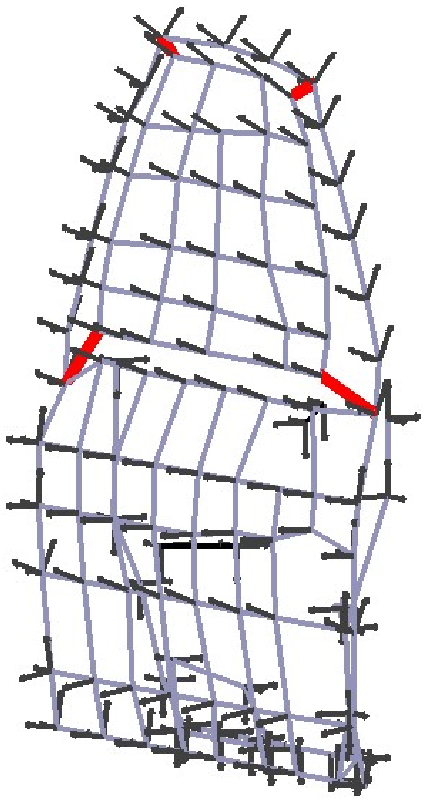


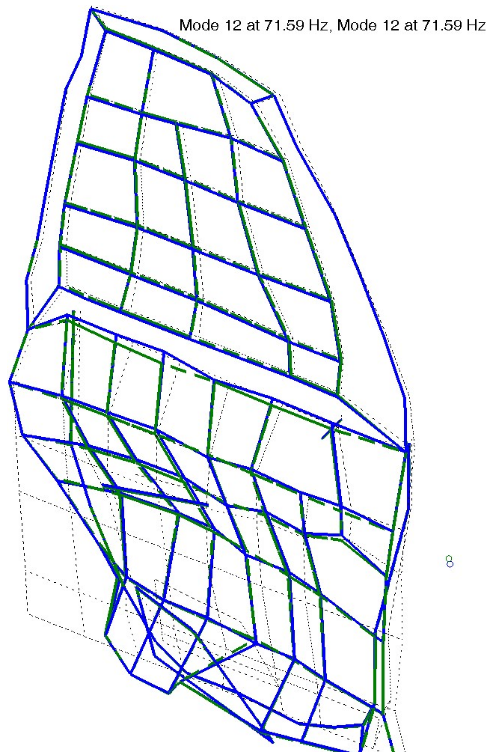
Avoid distortions : triax or not

- Non triaxial measurements can induce perceived deformation
- Wire-frame as truss and expand

Expansion ⇔ efficient use of channel count







Correlation criteria

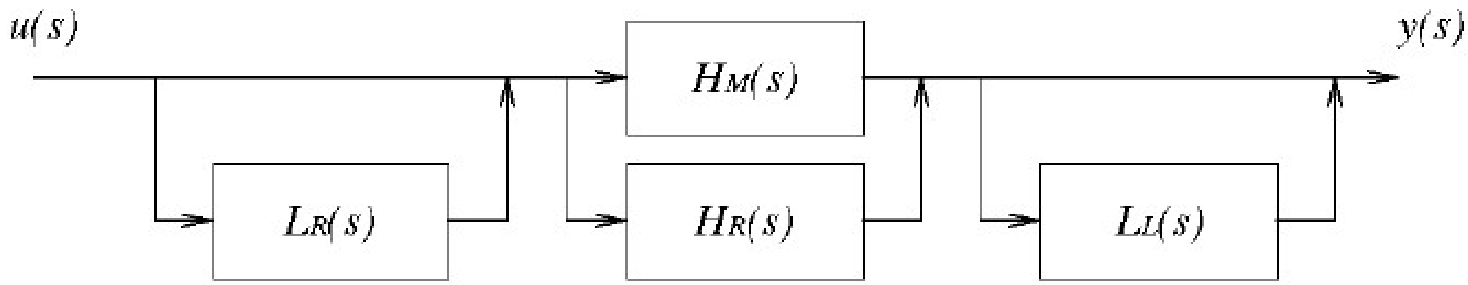
- FRF comparison (section 9.6)
- Sensor based : frequencies, MAC, relative error, MACCo (9.2)
- Model based : orthogonality, dynamic residual (9.3)

Needed tools

- Topology correlation / observation
- Modeshape expansion / estimation

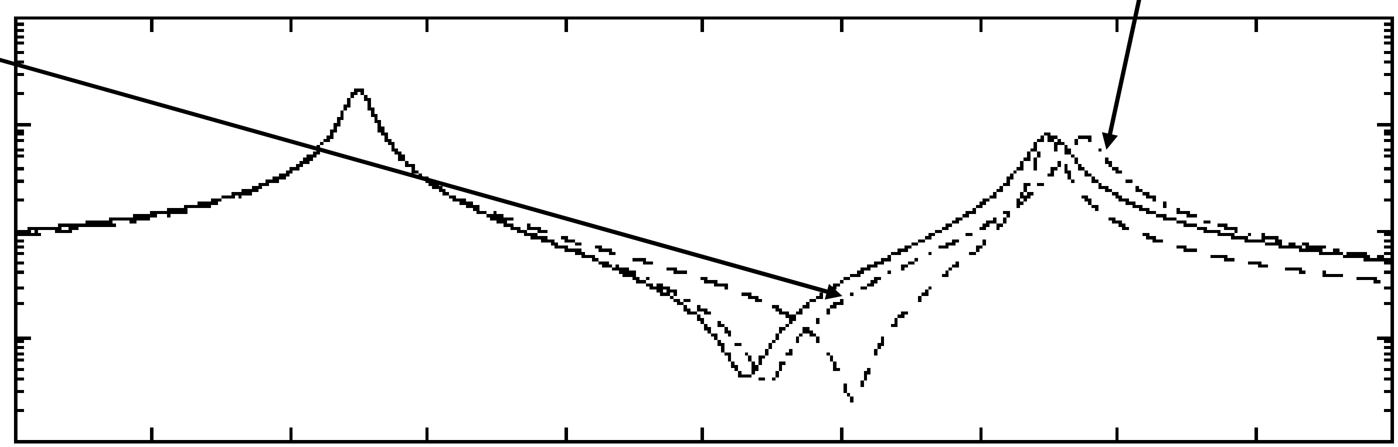
FRF comparison

$$[H(s)] = [I + L_L(s)]\,[H_M(s) + H_R(s)]\,[I + L_R(s)]$$

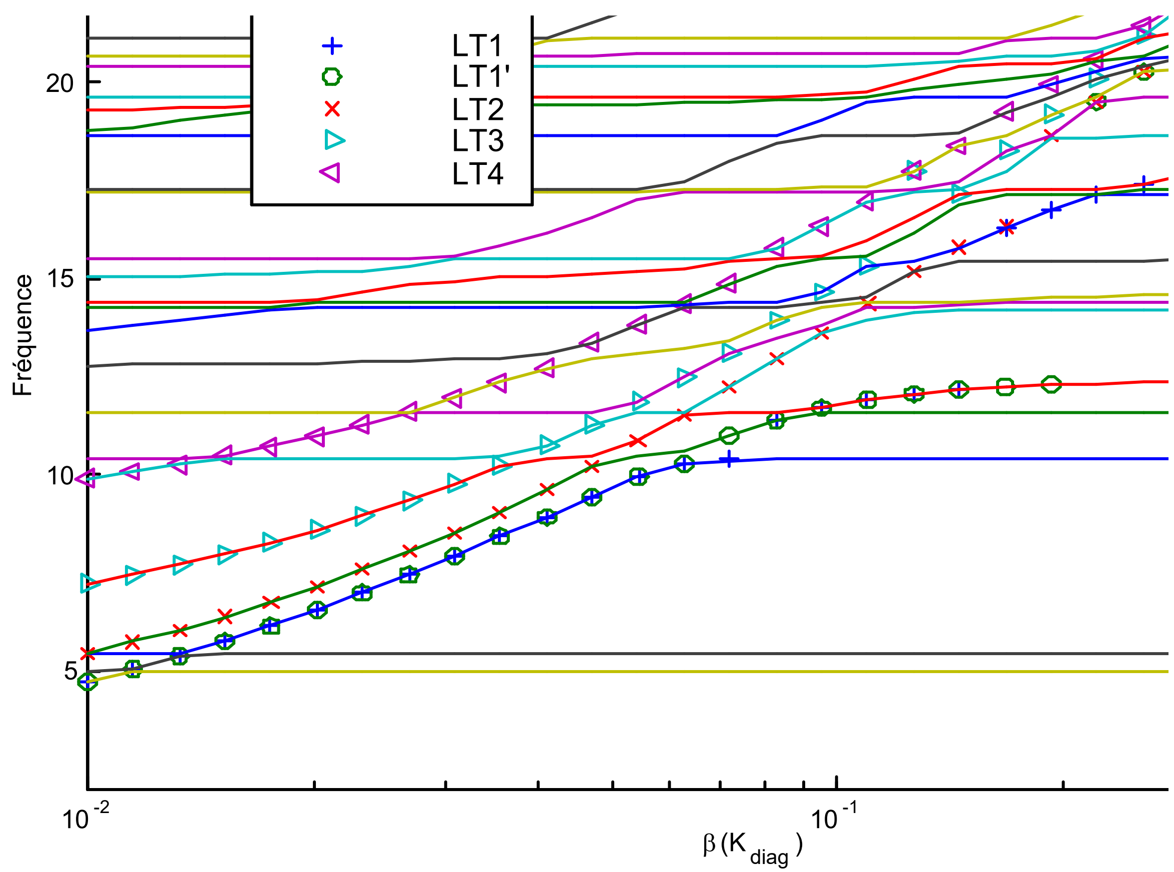


Typical FEM : closer with multiplicative

Typical ID : closer with additive



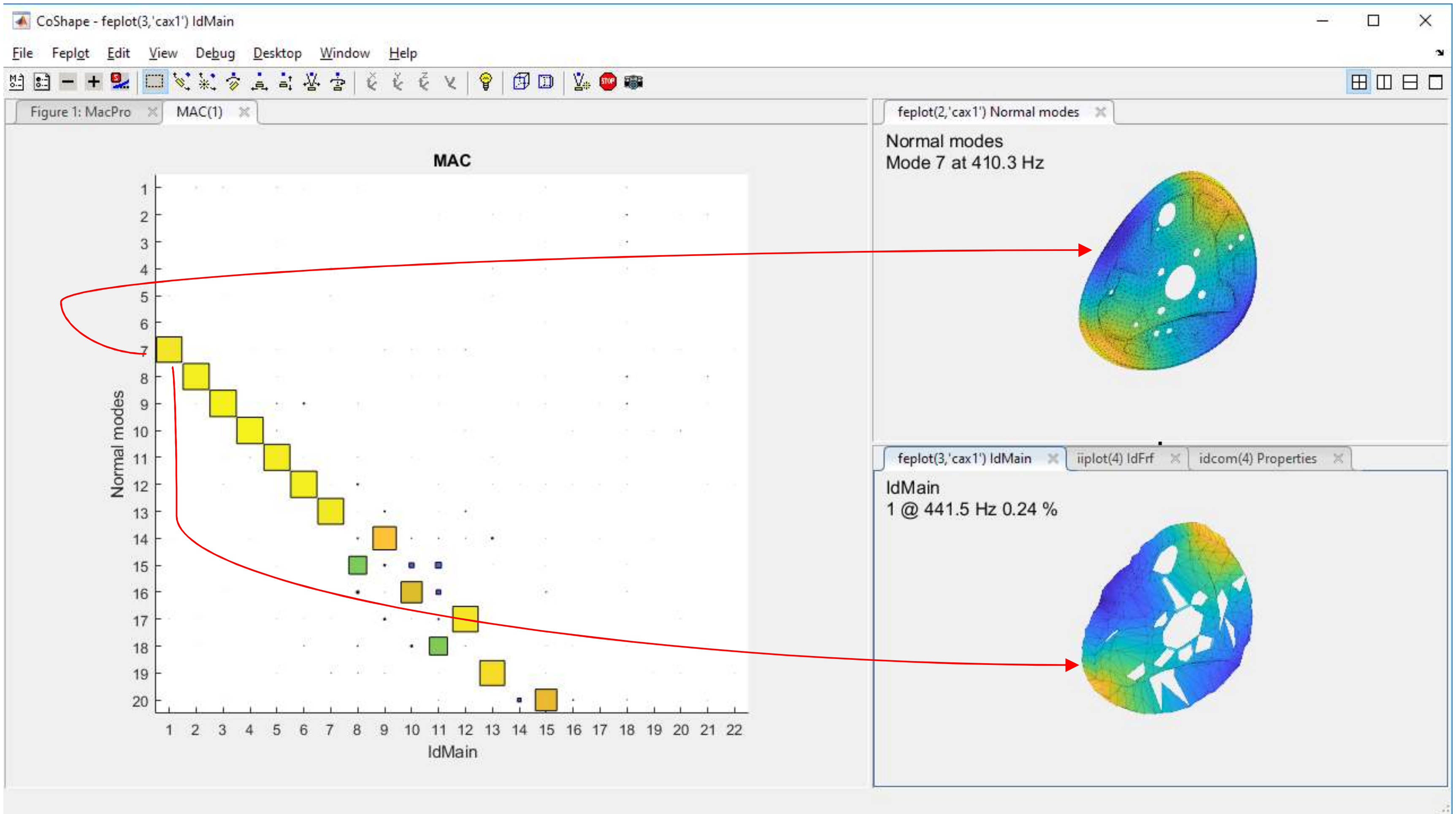
Frequency comparison : needs pairing



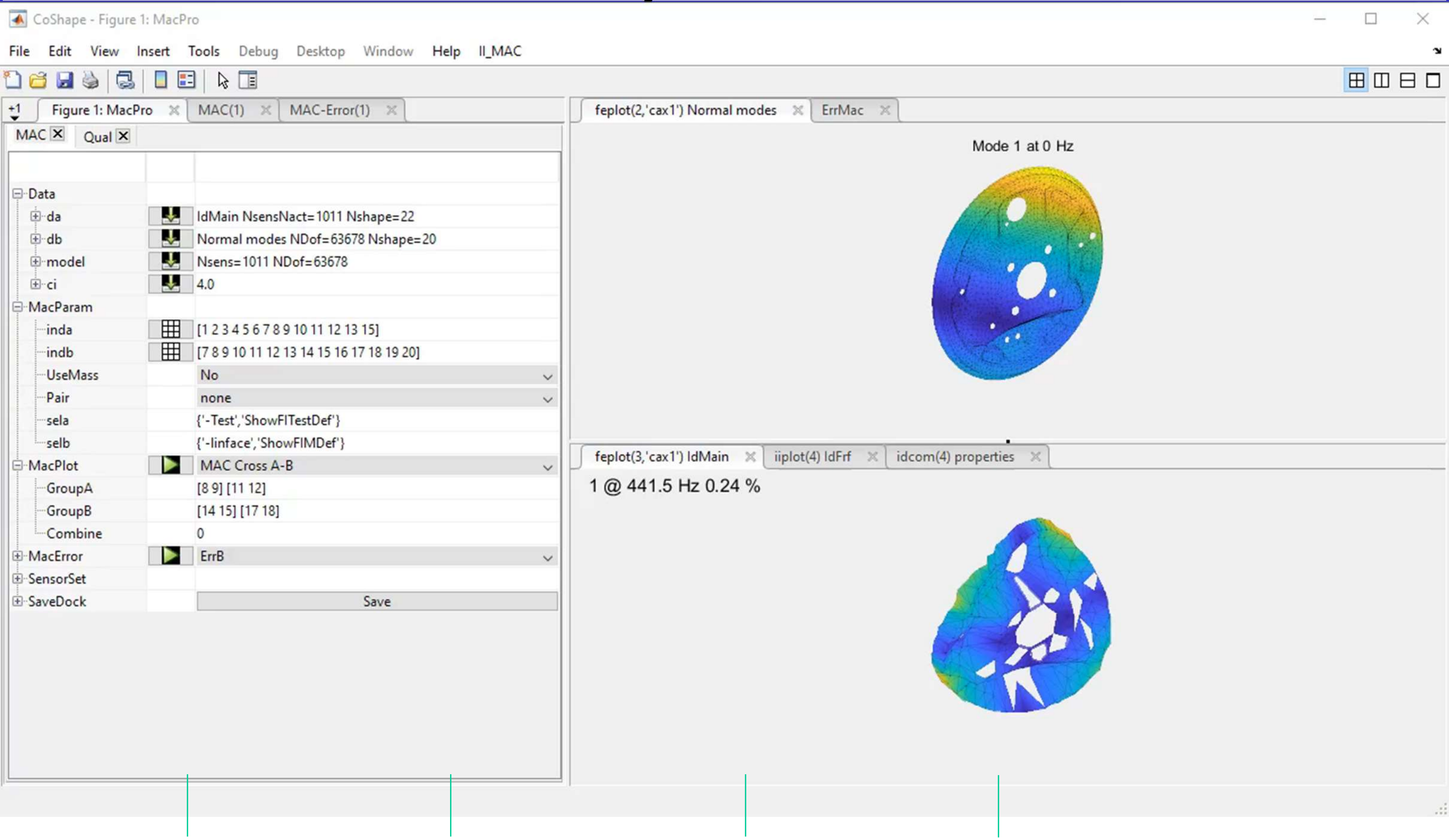
MAC and others

- Discuss MAC on blackboard (section 9.2.2)
- Show full and reduced versions (this requires talking about reduction)
- Relation with orthogonality conditions
- Relative error with or without scaling

Modal Assurance Criterion



Basic shape correlation

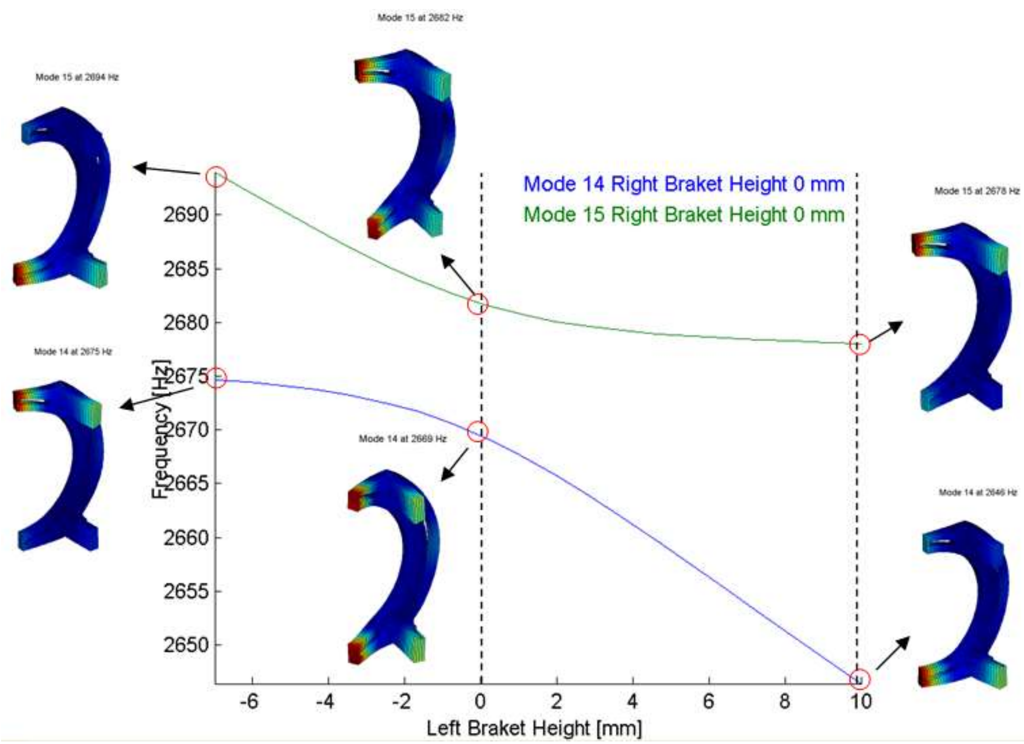


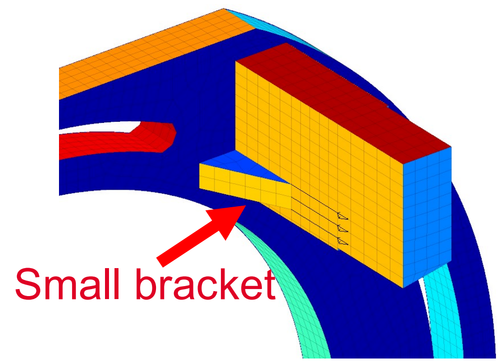
https://youtu.be/tVV-R2r-3M8

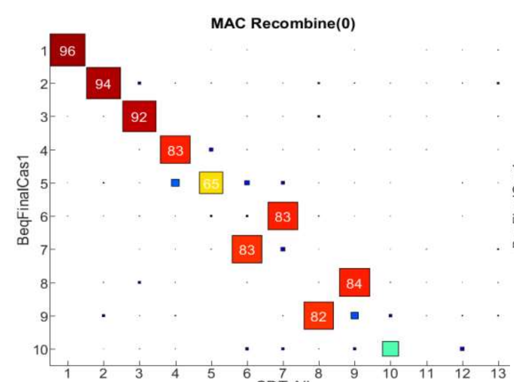
Correlation : mode crossing & MAC combine

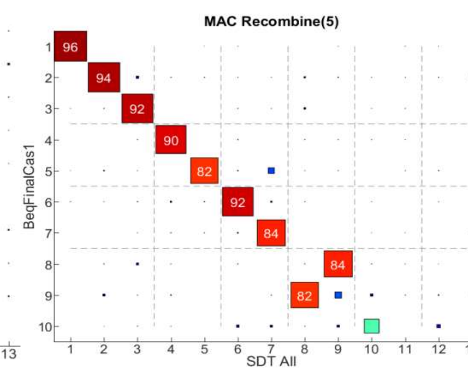
- High sensitivity for close modes associated with mode crossing
- MAC can drop to 0.5
- MAC combine linear combination of close modes = stays at 1.
- Orthonormal combination of modes in a group

$$A = argmin\left\|[\phi_{test}]_{gA} - [c\phi_{FEM}]_{gB}[A]\right\|$$







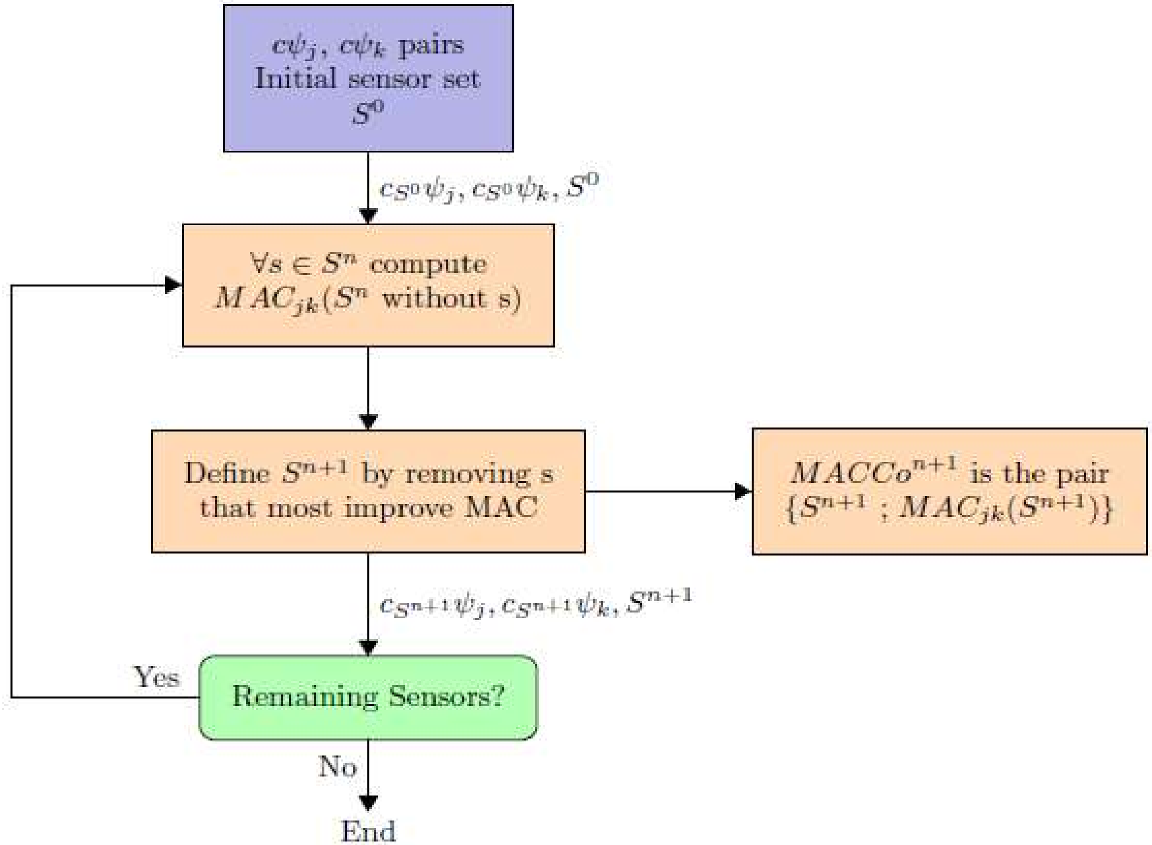


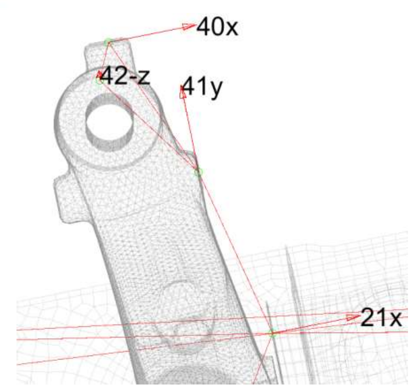
MACCo algorithm

What if analysis sorting sensors
by contribution to poor correlation

Common errors

- Poor identification
- Poor sensor position
- Sensor orientation error

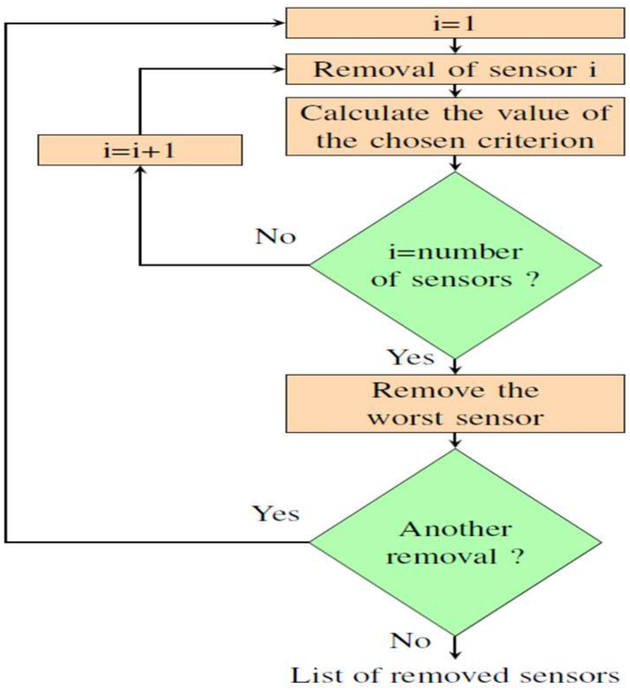




#	1	#	2	#	3	#	4	#	6	#	8	#	9
	88		82		84		44		48		46		69
35z	92	32z	84	17z	85	**36z**	55	28z	54	**36z**	56	4z	74
36z	94	40z	86	14z	86	**35z**	63	7z	59	7z	62	32z	79
28z	95	**36z**	88	**36z**	87	6z	68	10z	63	6z	65	24z	81
26z	96	28z	89	22z	87	39z	71	34z	67	**35z**	68	21z	83
31z	97	6z	90	26z	88	22z	74	9z	69	8z	72	3z	85

MAC & sensors : historical methods

- MAC on sensor sets
  - ☺ Localize area of poor correlation
  - ☹ No automated set definition
- COMAC : is a specific sensor bad ?
  - correlation of multiple modes at one sensor
  - eCOMAC : allow incorrect modeshape scaling
  - Not per mode / sensor
- MACCo : does correlation improve with less sensors ?
  - ☺ Easy to implement
  - ☺ Sensor sets per mode or global
  - ☹ No understanding of why each sensor is removed.



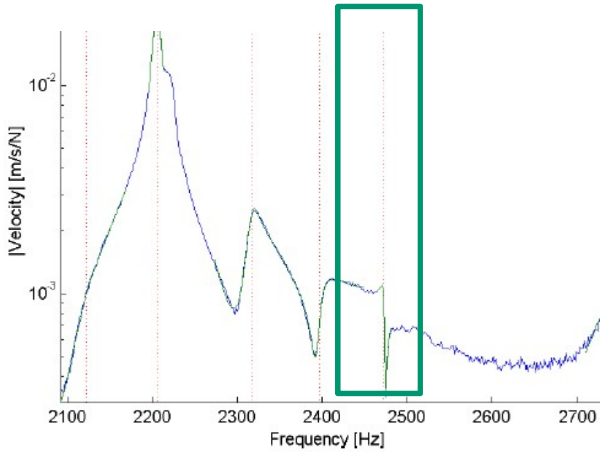
[1] N. A. J. Lieven, D. J. Ewins, « Spatial Correlation of Modeshapes, The Coordinate Modal Assurance Criterion (COMAC) », IMAC, 1988.
[2] Brughmans, Leuridan, Blauwkamp, « The application of FEM-EMA correlation and validation techniques on a body-in-white », IMAC, 1993
[3] Martin, Balmes, Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP, vol. 88, p. 62-80, may 2017

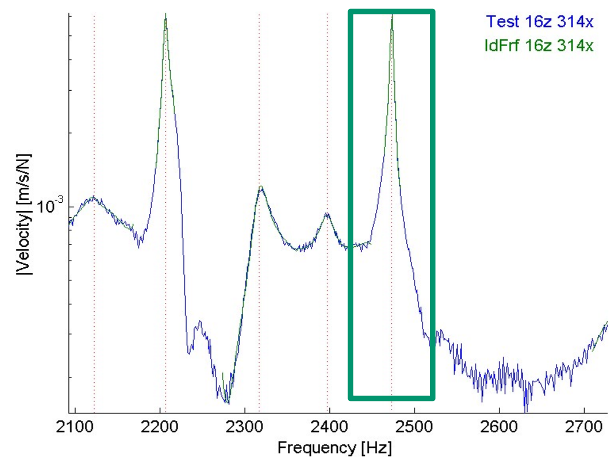
Test/identification error

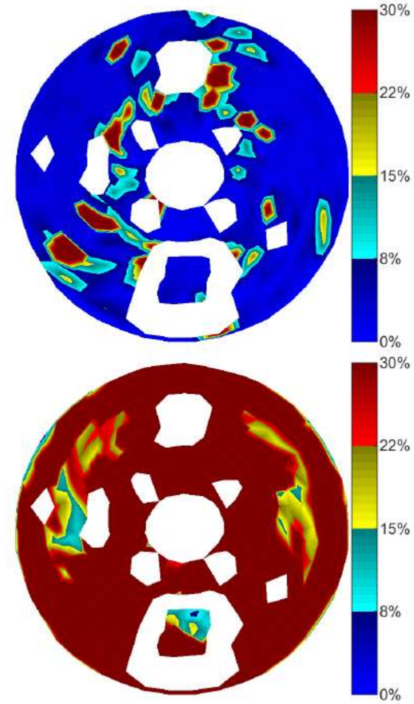
Contribution --

Contribution ++

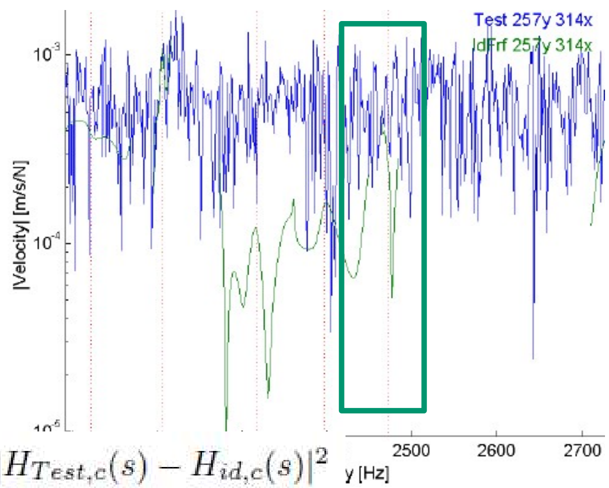
Error --

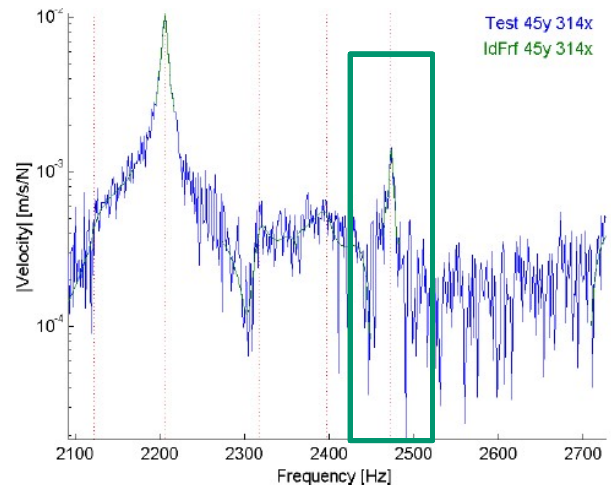






Error ++





$$e_{j,c} = \frac{\int_{\omega_j(1-\alpha\zeta_j)}^{\omega_j(1+\alpha\zeta_j)} |H_{Test,c}(s) - H_{id,c}(s)|^2}{\int_{\omega_j(1-\alpha\zeta_j)}^{\omega_j(1+\alpha\zeta_j)} |H_{id,c}(s)|^2}$$

Modes

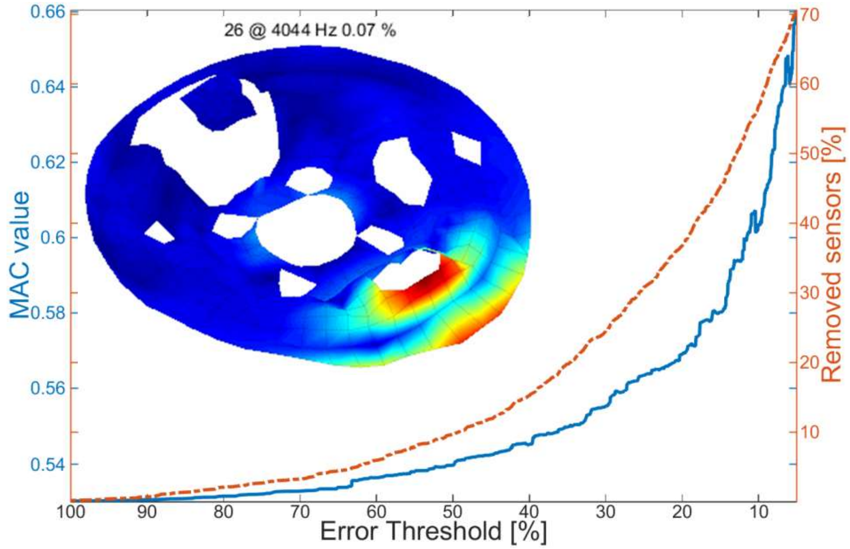
Mode ...	Freq[Hz]	Damp...	Error[%]	Contri...	MPC[%]	max(...
1	23	21,39	43	6	84	30
2	49	4,23	7	38	76	7
3	65	3,94	5	62	99	13
4	186	2,29	6	11	56	7
5	371	2,99	4	73	100	3
6	736	2,96	10	46	100	3
7	971	3,23	10	45	100	3
8	1424	3,50	4	11	99	3
9	1483	1,75	4	79	87	2

I/O Pairs

Mode	Out	In	Error	Level	Contrib	NOS
6	1285,03	,03	15,0%	1,5%	67,0%	0,2%
6	1411,03	,03	14,8%	4,5%	1,1%	0,7%
6	1174,03	,03	14,4%	5,7%	45,6%	0,8%
6	1490,03	,03	14,3%	4,1%	40,5%	0,6%
6	1403,03	,03	14,3%	4,8%	67,2%	0,7%
6	1244,03	,03	14,1%	6,9%	35,5%	1,0%
6	1283,03	,03	13,8%	2,1%	27,8%	0,3%
6	1153,03	,03	13,7%	4,1%	19,4%	0,6%
6	1395,03	,03	13,6%	2,4%	74,2%	0,3%

Per mode/sensor error/noise & contribution

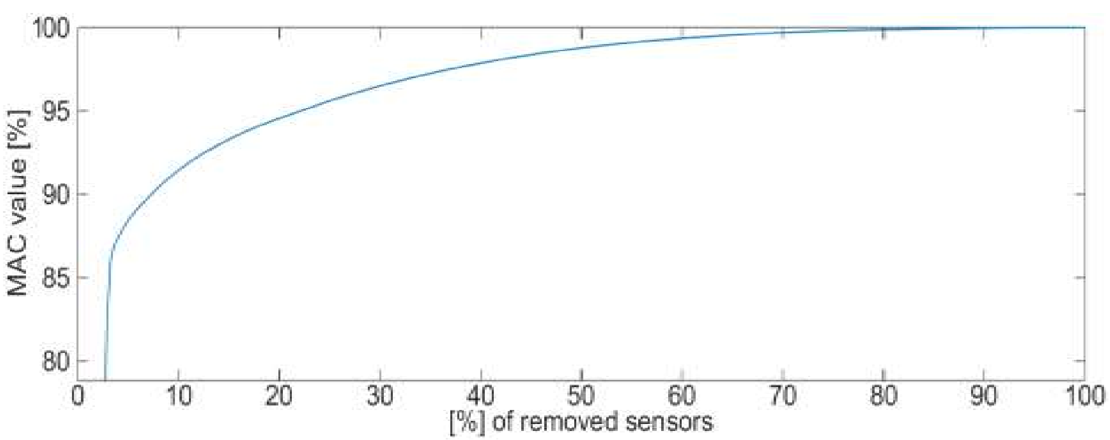
MAC & sensors : MACCo variants



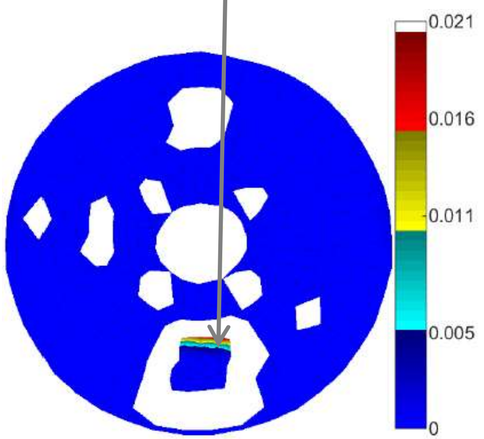
MACErr : Ignore sensors with bad identification

MACCo : sort sensors by impact on MAC
Removed sensors may indicate

- Bad measurements
- Model errors

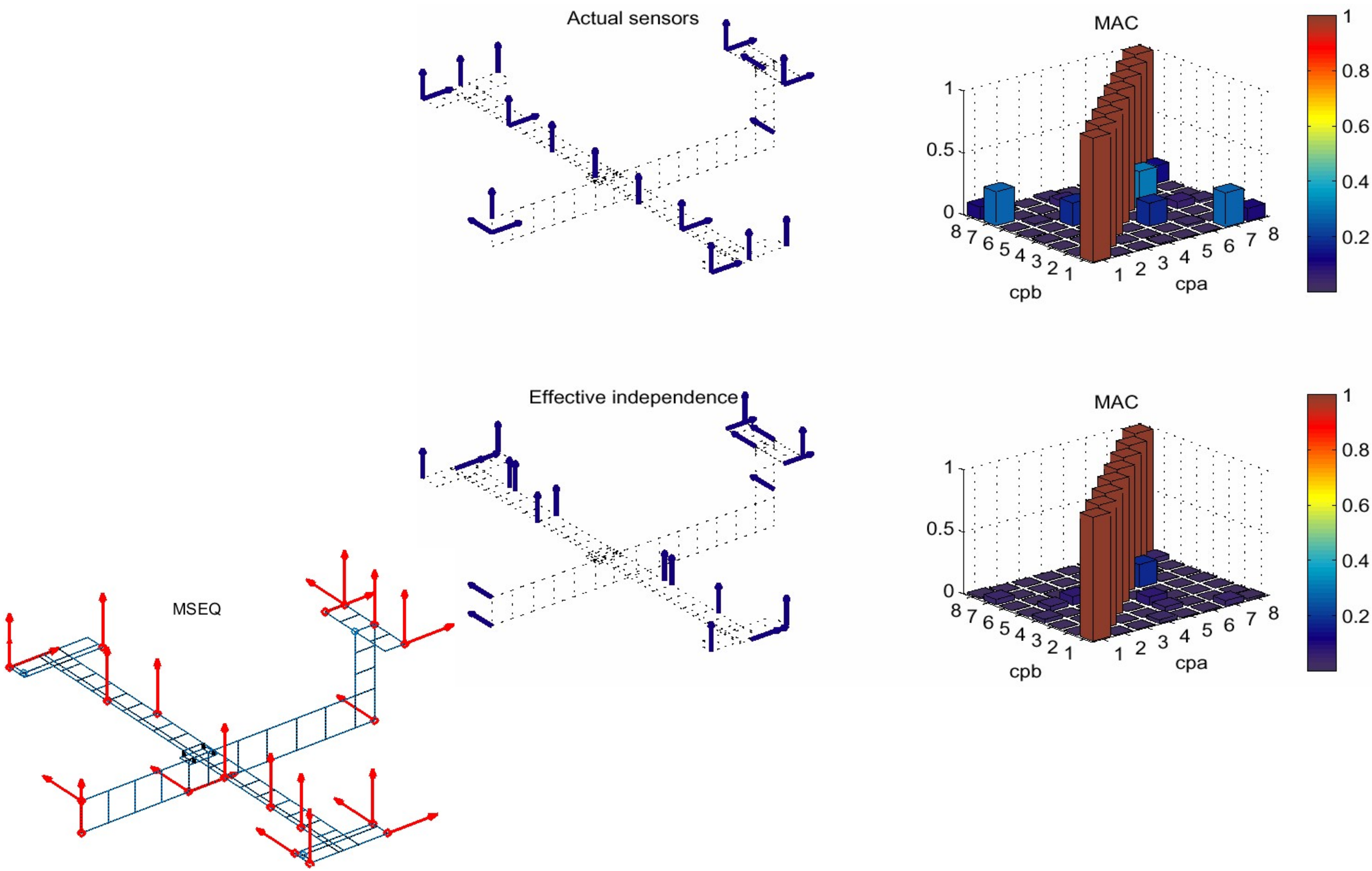


Each sensor removed improves MAC by 2%



White areas = sensors removed based on test

MAC : sensor placement



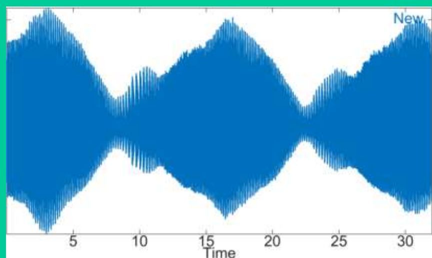
Expansion in structural dynamics : a perspective gained from success and errors in test/FEM twin building

Etienne BALMES (ENSAM PIMM & SDTools)
G. MARTIN, G. Vermot des Roches (SDTools)
T. CHANCELIER, S. THOUVIOT, Hitachi

CSMA, Giens, May 18, 2022

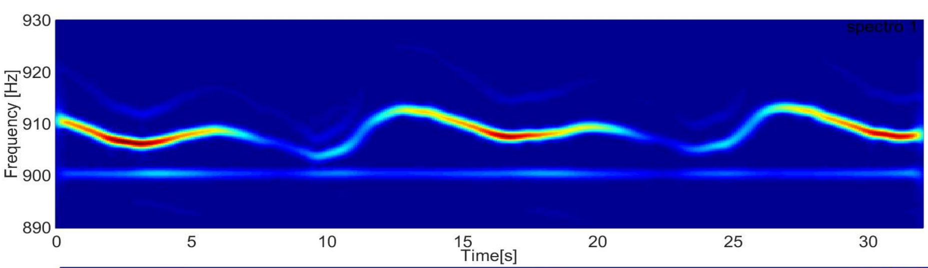
Learning experimental shapes in squeal event

Time measurement during squeal

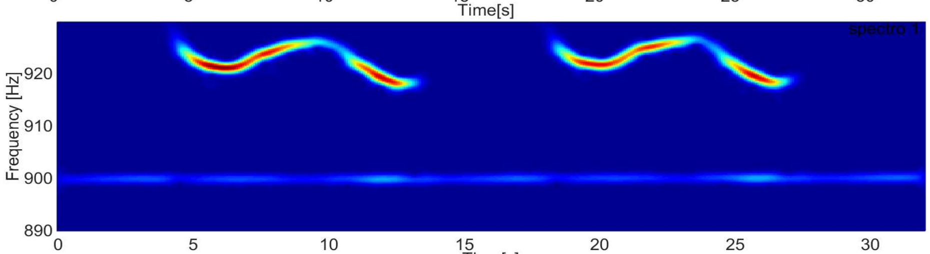


Variability
- influence of wheel angle

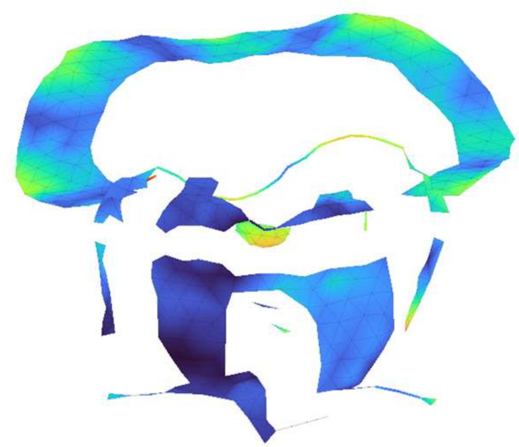
Reproductibility
- Multiple events



Braking event 1

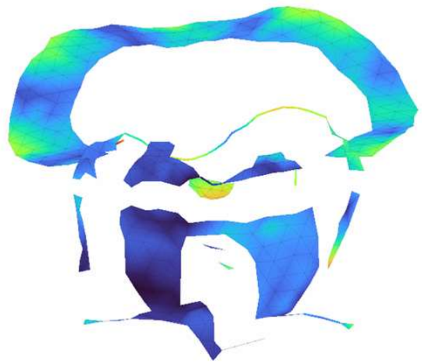


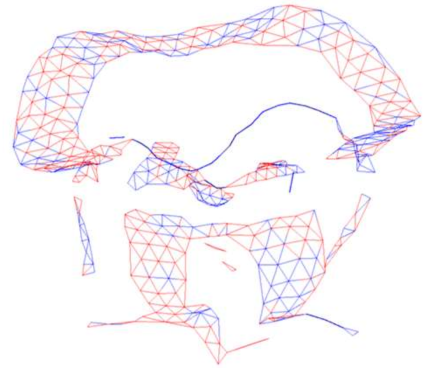
Braking event 2

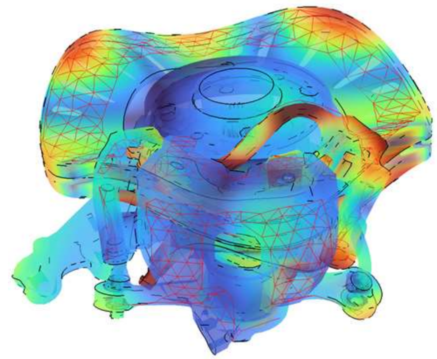


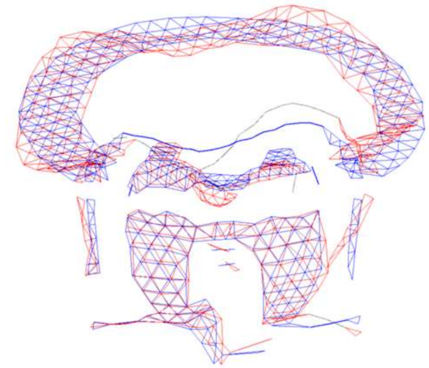
Complex ODS from 3D-SLDV
- 431 points
- f = 4050 Hz

Observation / test error

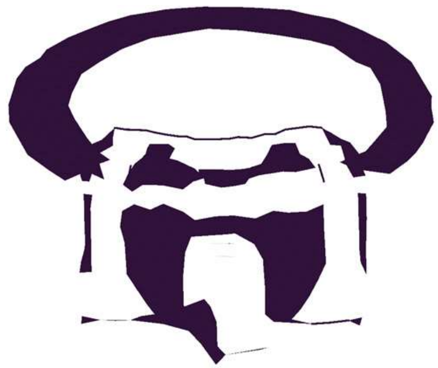


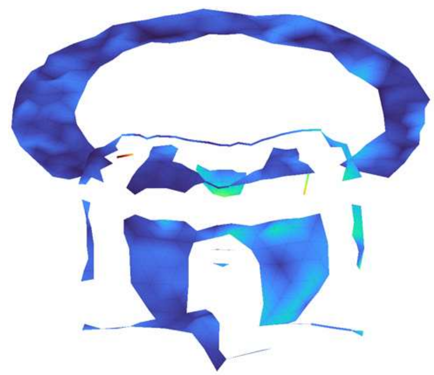


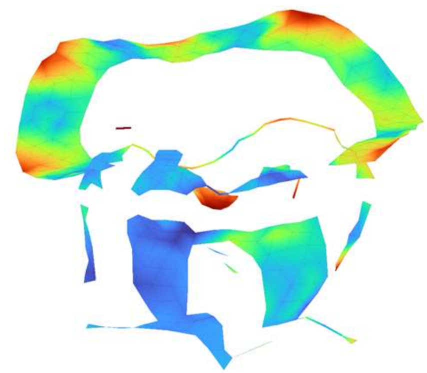




$\epsilon_{Test} =$
$\left\| \{y_{Test}\} - [c]\{q_{Exp}\} \right\|_Q^2$







$$\epsilon_{Test}^{R} = \frac{\epsilon_{Test}}{\|\{y_{Test}\}\|^2}$$

$\epsilon_{Test}^{R} = 0\%$

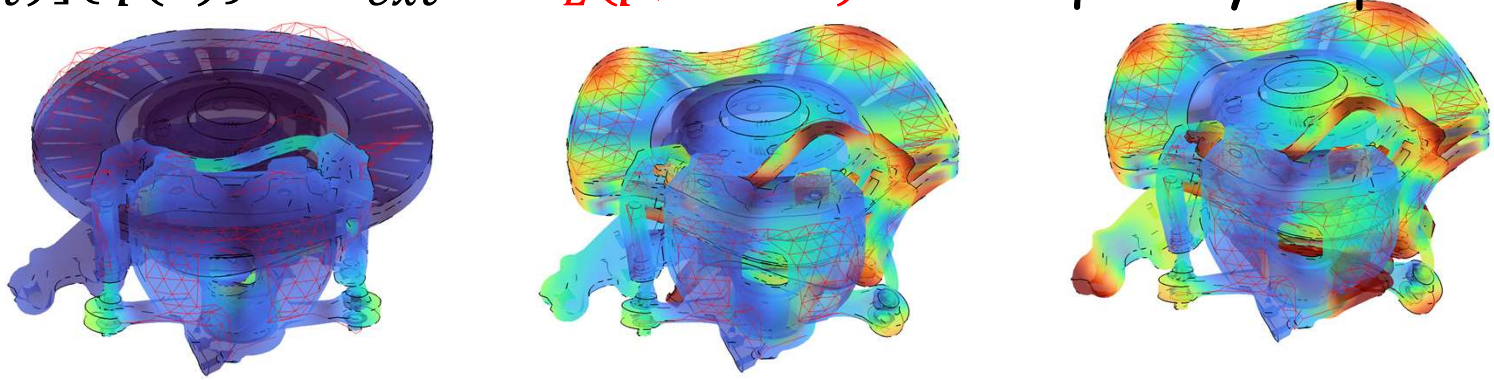
$\epsilon_{Test}^{R} = 14\%$

$\epsilon_{Test}^{R} = 95\%$

Equilibrium / model error / ignorance

$[M]\{\ddot{q}\} + [C]\{q\} + [K(p_{Est})]\{q\} = F_{ext} + R_L(p, noise)$ in time = Kalman

$[Ms^2 + Cs + K(p_{Est})]\{q(s)\} = F_{ext} + R_L(p, noise)$ in frequency expansio



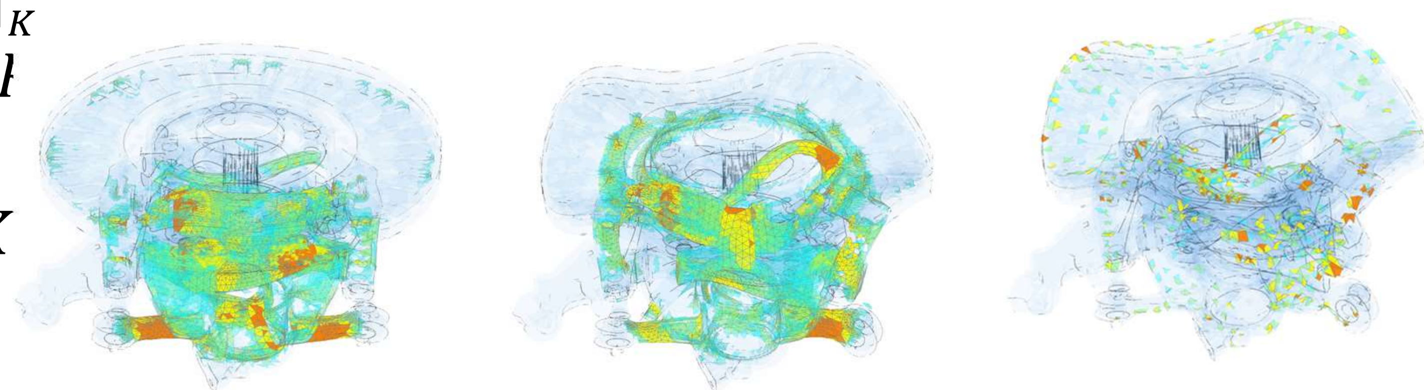
Measure residuals $\|\{R_L\}\|_K$

1. Work $\{R_D\} = [K]^{-1}\{I$
2. Energy

$\epsilon_{Mod} = \|\{R_L\}\|_K^2 = R_D^T K$

3. Relative error

$$\epsilon_{Mod}^R = \frac{\epsilon_{Mod}}{\|\{q_{Exp}\}\|_K^2}$$

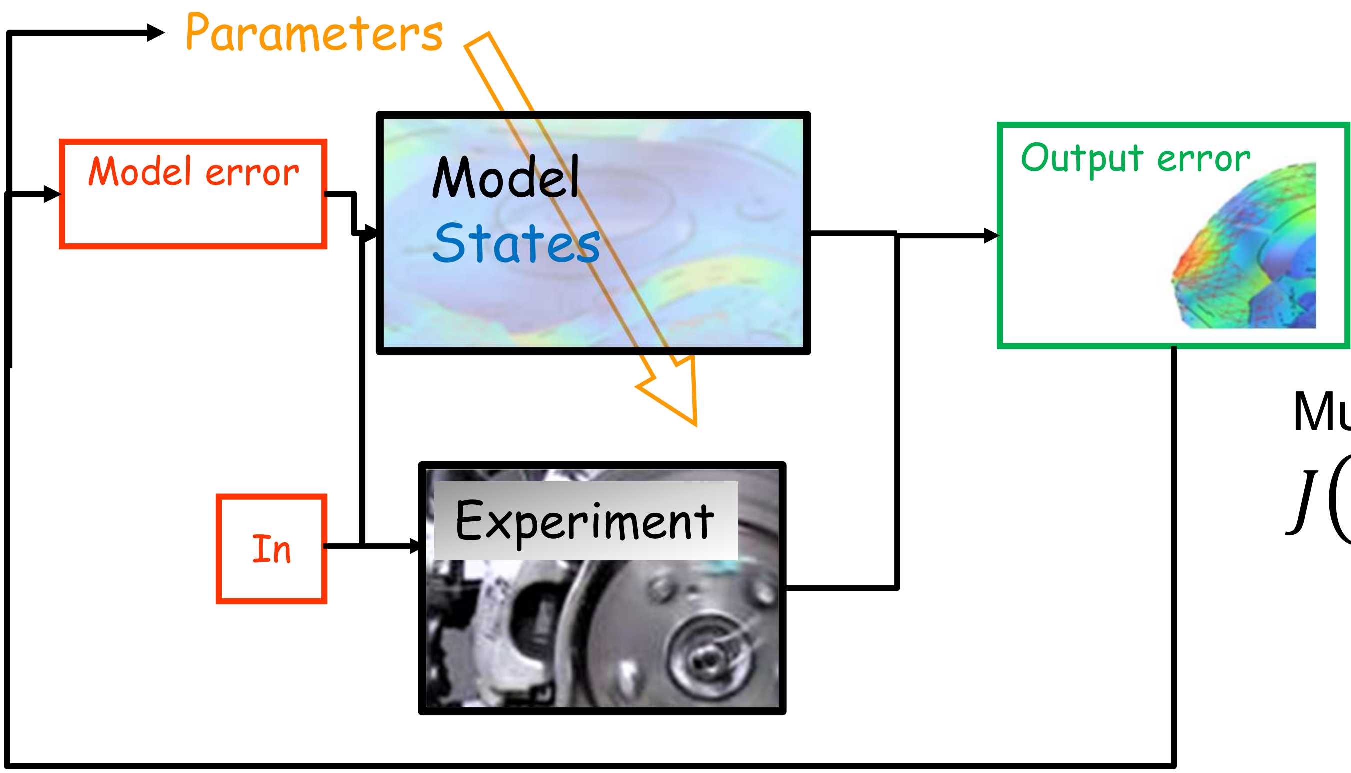


$\epsilon_{Mod}^R = 0\%$ — Complex mode

$\epsilon_{Mod}^R = 3\%$ — Our target

$\epsilon_{Mod}^R = 82\%$ — Dynamic expansion

Model based estimation / expansion / hybrid twin



Multi-objective optimization

$$J\left(q_{Exp}, \gamma, p\right) = \|R_L\|_K + \boldsymbol{\gamma} \, \|cq - y_{Test}\|_Q$$

Outline

1. Coefficient γ to balance between the two errors
2. States, reduction & numerical cost
3. Ouput-error norm Q
4. Analyze model error K-norm

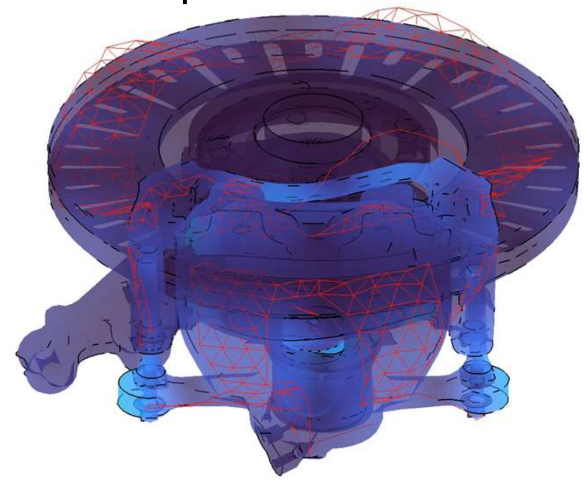
1. Test / model tradeoff

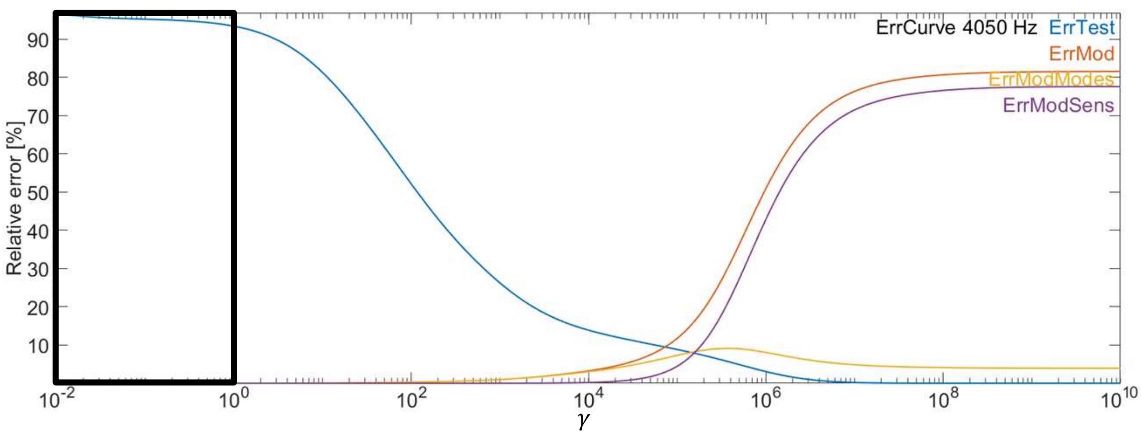
Modeling and test error minimization : multi-objective cost function

$$J(q_{Exp}, \gamma) = \epsilon_{Mod} + \boldsymbol{\gamma} \epsilon_{Test}$$

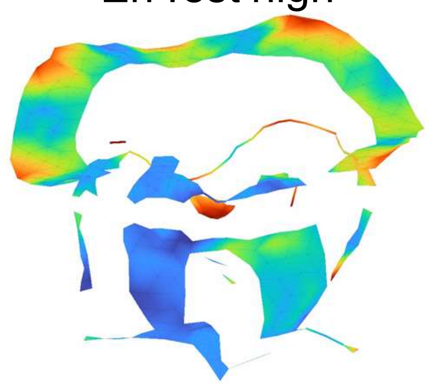
γ too low : no motion (no external input)

Expansion + Test

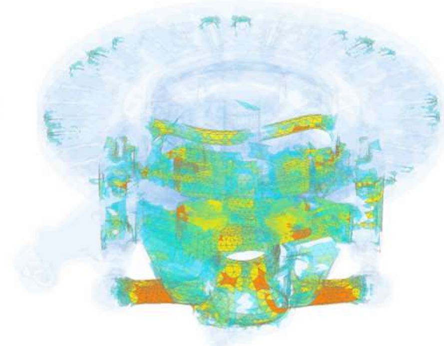




ErrTest high



ErrMod low



1. Test / model tradeoff

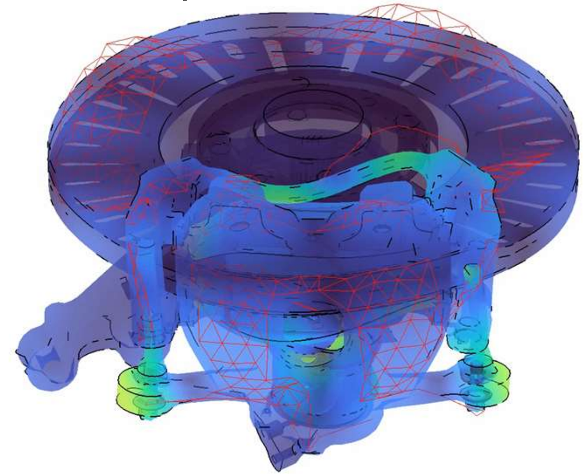
Modeling and test error minimization : multi-objective cost function

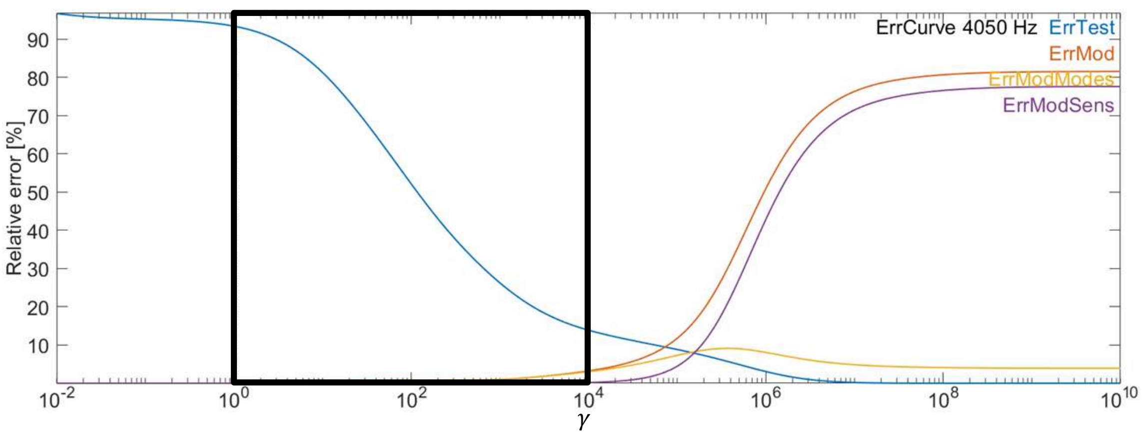
$$J(q_{Exp},\gamma) = \epsilon_{Mod} + \boldsymbol{\gamma}\epsilon_{Test}$$

γ too low : no motion

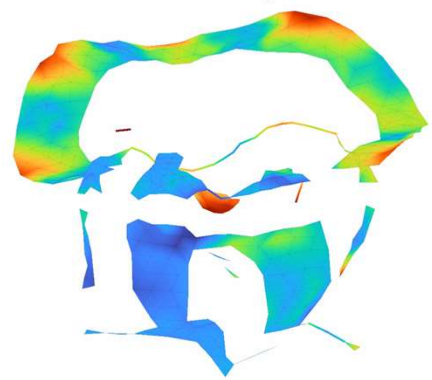
γ increase : expanded shape gets closer to the measurement

Expansion + Test

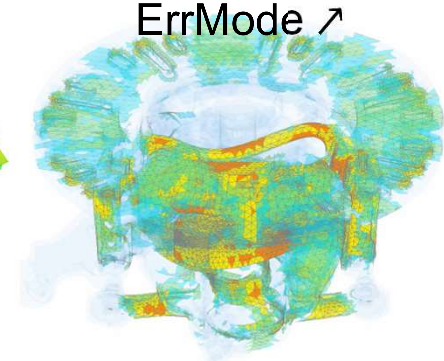




ErrTest ↘



ErrMode ↗



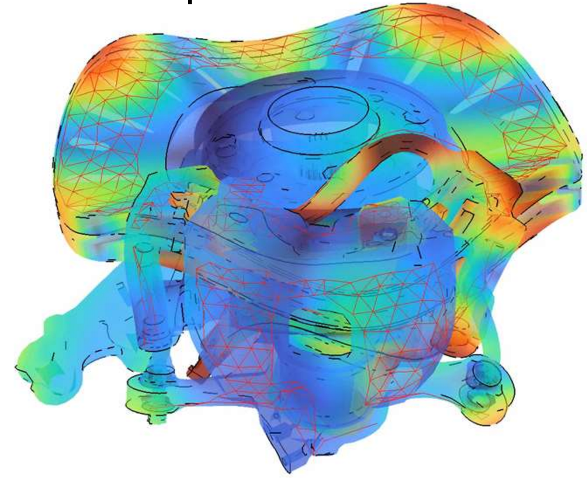
1. Test / model tradeoff

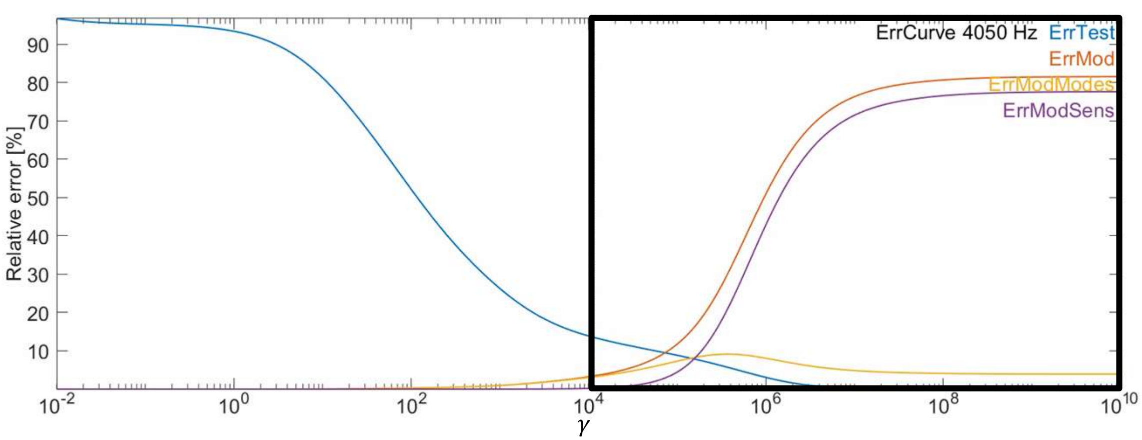
Modeling and test error minimization : multi-objective cost function

$$J(q_{Exp}, \gamma) = \epsilon_{Mod} + \boldsymbol{\gamma} \epsilon_{Test}$$

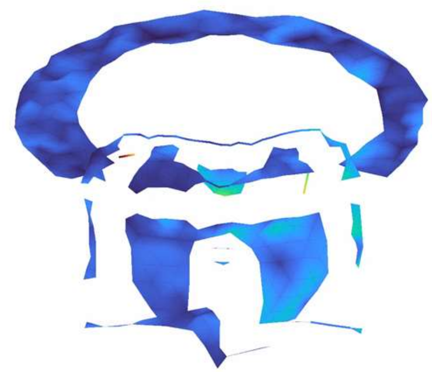
γ too low : no motion
γ increase : expanded shape gets closer to the measurement
γ too high : model error concentration at sensors

Expansion + Test





ErrTest ≈ noise

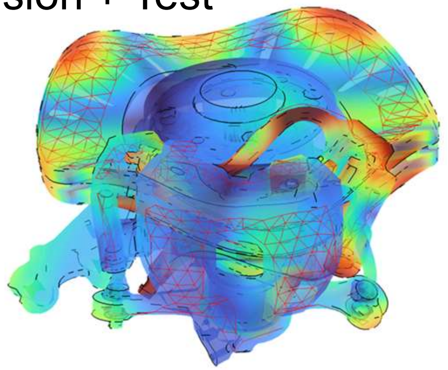


ErrMod at sensors

1. Test / model tradeoff : conclusion

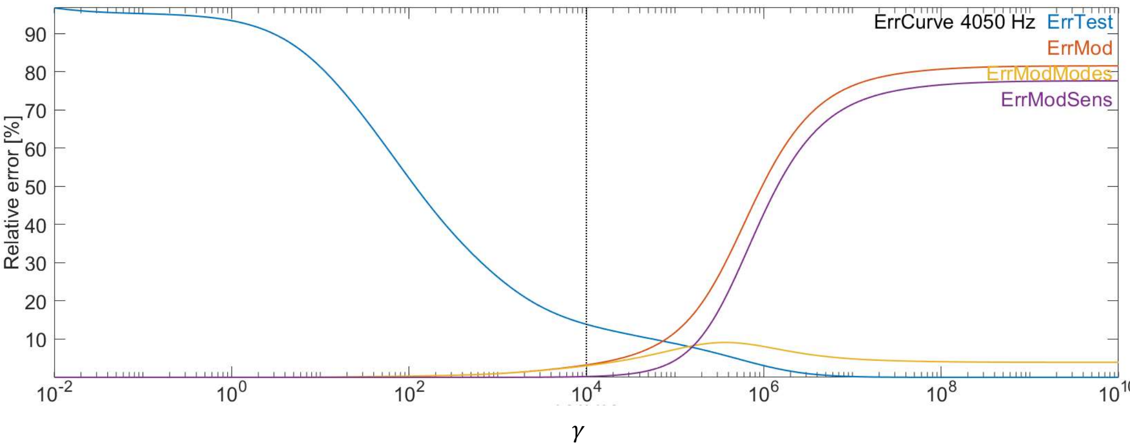
$$J(q_{Exp},\gamma) = \epsilon_{Mod} + \boldsymbol{\gamma}\epsilon_{Test}$$

Expansion + Test

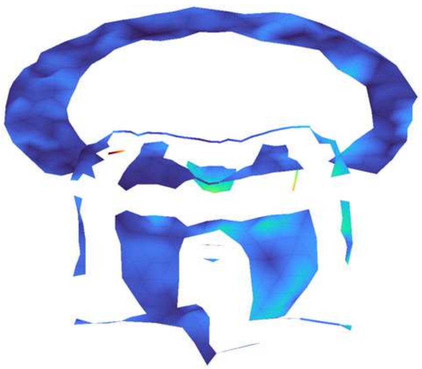


γ too low : no motion
γ too high : model error concentration at sensors

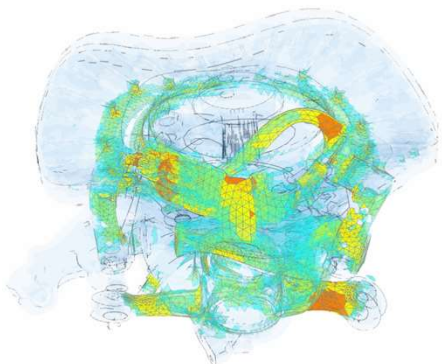
Good value : ϵ_{Test} low enough, ϵ_{Mod} smooth



ErrTest low « enough »



« Smooth » ErrMod



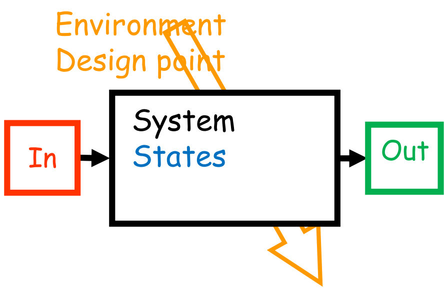
Necessities
- at least 20 values 4 orders of magnitude
- Interactive γ navigation
- Error viewing interfaces
- Relative error give local minimima & optima $\neq$ weighted error

2. States, reduction, numerical cost

Multi-objective problem $J(q_{Exp},\gamma) = \epsilon_{Mod} + \gamma\epsilon_{Test}$

Rewritten as extended saddle point

$$\begin{bmatrix} \hat{K} & -Z(\omega,p) \\ -Z(\omega,p) & \gamma[c]^T Q[c] \end{bmatrix} \begin{Bmatrix} R_D \\ q_{Ex} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \gamma[c]^T Q y_{test} \end{Bmatrix}$$

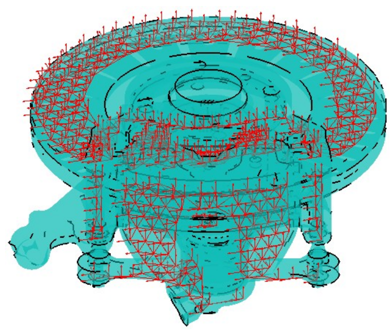


1.7e6 DOF
1239 sensor

Two levels of variable separation

- FEM $u(x,t) = N(x)q_i(t)$
- Modal analysis (Ritz) $\{q_i(t,p)\} = [T]\{q_R(t,p)\}$

$[T]$ chosen for bandwith, loads (external, parametric)



Needs for T learning

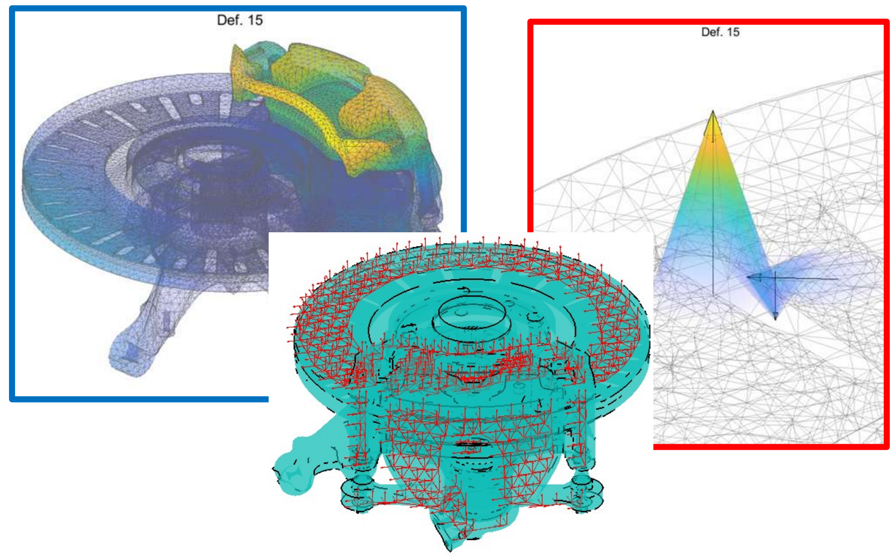
- Modes $\{\phi_j(p_0)\}$
- Loads at sensors $[c]^T$
- Parametric loads $[\partial Z/\partial p]\{\phi_j(p)\}$

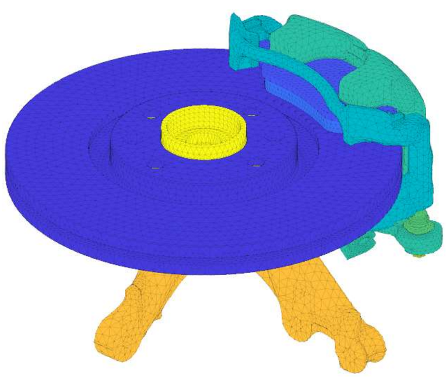
100 shapes
5*100 shapes
1239 shapes

2. States, reduction, numerical cost

Ritz reduction

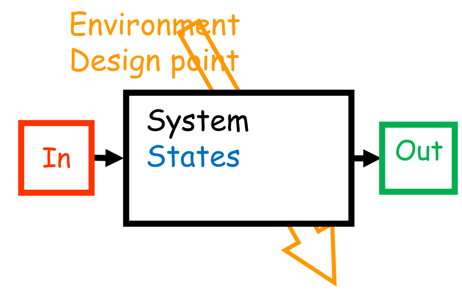
- Learning $\{\phi_j(p_1)\}$ $\{\phi_j(p_2)\}$ $\quad [\mathrm{K}]^{-1}\left[[\partial Z/\partial p]\{\phi_j(p)\} \quad [c]^T\right]$
- Basis building :
  - Full 1.7e6 DOF x (100 + 500 + 1239) vectors = 23 GB
  - Full global modes (SVD/Eig) 1.3 GB
  - Sparse sensor shape $[\mathrm{K}]^{-1}[c]^T$ (LU) 7 MB
  - Other basis building at SDTools
    - Full parametric shapes (could be sparse)
    - Component modes block-wise & two level reduction (PhD Vermot, 2011)





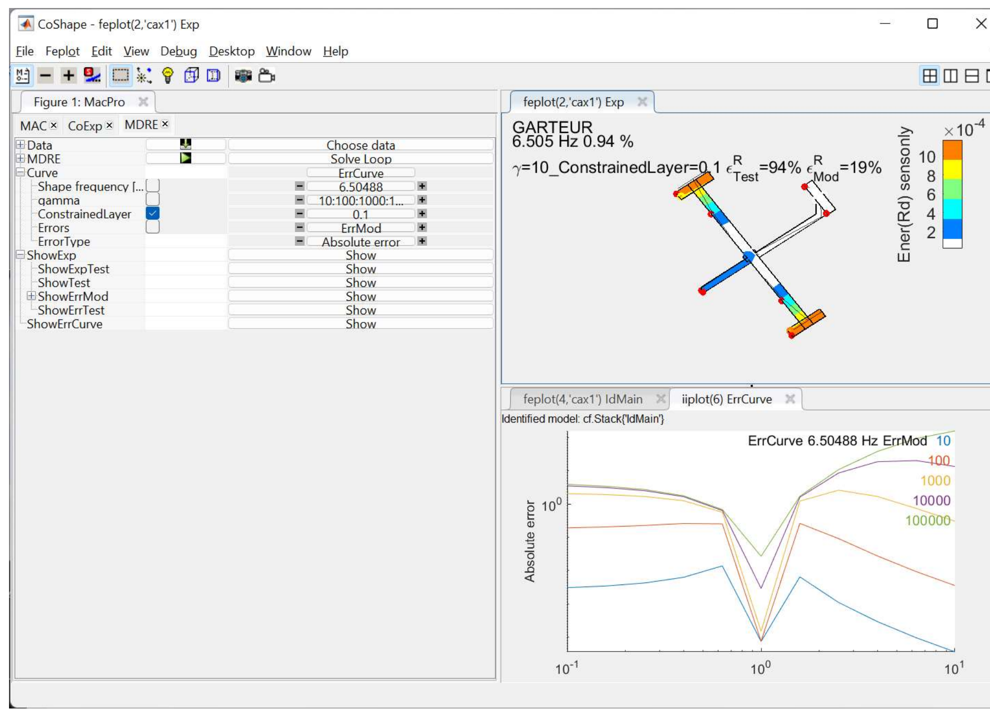
2. States, reduction, numerical cost

Ritz reduction (offline ∝1 h, once)



Expansion $\begin{bmatrix} \hat{K} & -Z(\omega,p) \\ -Z(\omega,p) & \gamma[c]^T Q[c] \end{bmatrix} \begin{Bmatrix} R_D \\ q_{Ex} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \gamma[c]^T Q y_{test} \end{Bmatrix}$

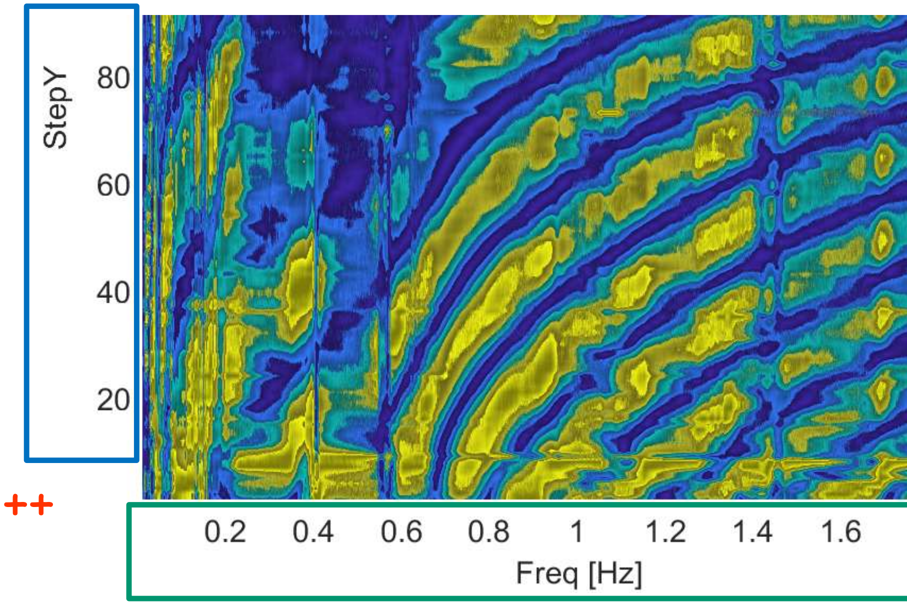
- Full NEVER
- Reduced ∝1s, 1e3 times
  - Analytic gradient if updating
- Restitution : online / interactive
  - Display ∝1ms, energy per element ∝1s
  - Online saves memory
    1000 shapes : full 25 GB, reduced 28 MB



3. Test error norm

Test error is frequency/sensor dependent

$$\left\| \{y_{Test}\} - [c]\{q_{Exp}\} \right\|^2_{Q(\omega)}$$

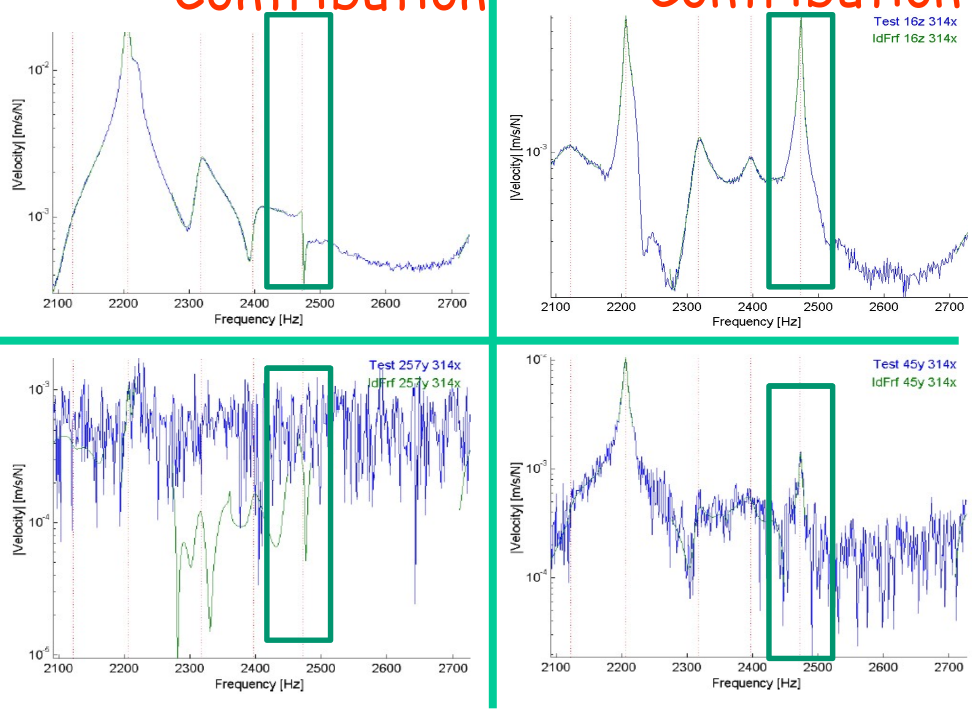


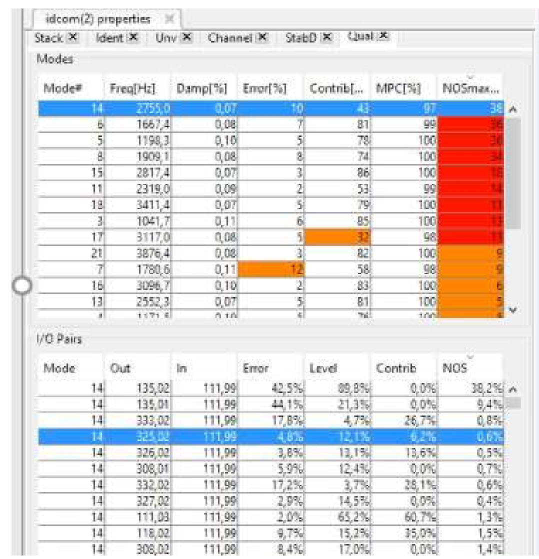
Contribution --

Contribution ++

Error --

Error +-




idcom(2) properties

Stack | Ident | Unv | Channel | StabD | Qual

Modes

Mode#	Freq[Hz]	Damp[%]	Error[%]	Contrib[...	MPC[%]	NOSmax...
14	2755,0	0,07	10	43	97	38
6	1667,4	0,08	7	81	99	36
5	1198,3	0,10	5	78	100	36
8	1909,1	0,08	8	74	100	34
15	2817,4	0,07	3	86	100	15
11	2319,0	0,09	2	53	99	14
18	3411,4	0,07	5	79	100	13
3	1041,7	0,11	6	85	100	13
17	3117,0	0,08	5	32	98	11
21	3876,4	0,08	3	82	100	9
7	1780,6	0,11	12	58	98	9
16	3096,7	0,10	2	83	100	6
13	2552,3	0,07	5	81	100	5

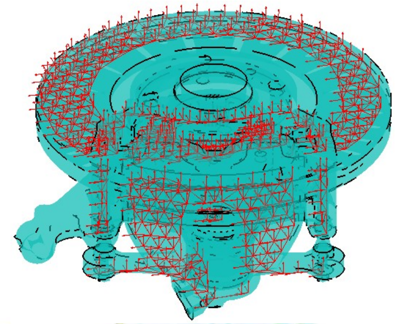
I/O Pairs

Mode	Out	In	Error	Level	Contrib	NOS
14	135,02	111,99	42,5%	89,8%	0,0%	38,2%
14	135,01	111,99	44,1%	21,3%	0,0%	9,4%
14	333,02	111,99	17,8%	4,7%	26,7%	0,8%
14	325,02	111,99	4,8%	12,1%	6,2%	0,6%
14	326,02	111,99	3,8%	13,1%	13,6%	0,5%
14	308,01	111,99	5,9%	12,4%	0,0%	0,7%
14	332,02	111,99	17,2%	3,7%	28,1%	0,6%
14	327,02	111,99	2,9%	14,5%	0,0%	0,4%
14	111,03	111,99	2,0%	65,2%	60,7%	1,3%
14	118,02	111,99	9,7%	15,2%	35,0%	1,5%
14	308,02	111,99	8,4%	17,0%	0,0%	1,4%

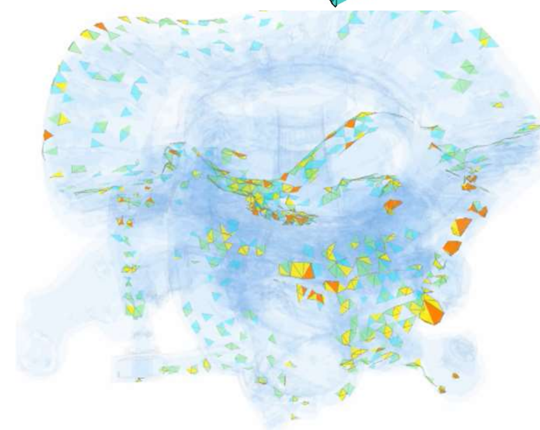
[1] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

4. Model error analysis

In $\begin{bmatrix} \hat{K} & -Z(\omega,p) \\ -Z(\omega,p) & \gamma[c]^T Q[c] \end{bmatrix} \begin{Bmatrix} R_D \\ q_{Ex} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \gamma[c]^T Q y_{test} \end{Bmatrix}$



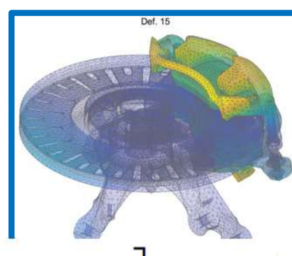
One has $[Z(\omega,p)]\{R_D\} = [c]^T \gamma(c q_{Ex} - y_{Test})$

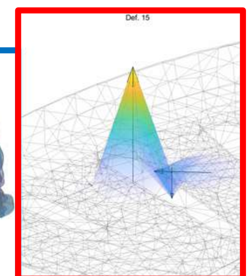


- The model error is approximated as dynamic response to point loads at sensors => stress concentrations

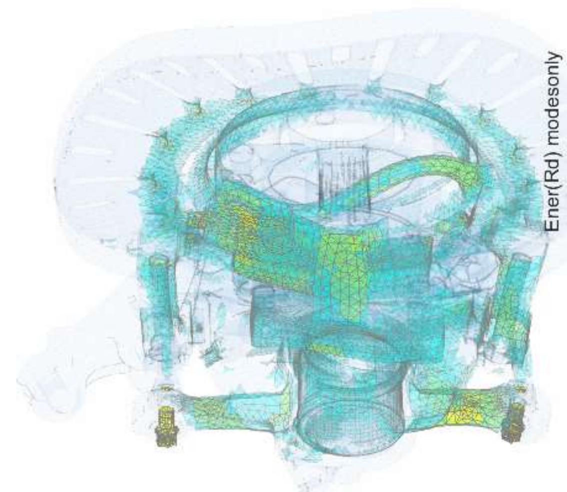
- But sensor enrichment can be split by regularity/frequency

$$[T] = [\phi_M \quad T_{Sens}^{\perp}]$$





$$\varepsilon_{Mod} = \{R_D\}|_M^T \begin{bmatrix} \ddots & & \\ & \omega_M^2 & \\ & & \ddots \end{bmatrix} \{R_D\}|_M + \{R_D\}|_\perp^T \begin{bmatrix} \ddots & & \\ & \omega_\perp^2 & \\ & & \ddots \end{bmatrix} \{R_D\}|_\perp$$



4. Model error analysis

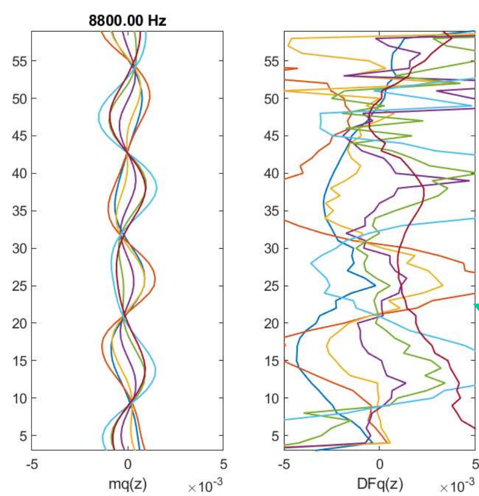
- Regularity may still not be sufficient
- Bottois (PhD 2019) used Bayesian inference

$$\begin{cases} \boldsymbol{\mu}_{\delta_1} = \tilde{E}_t \mathbf{K_t y} + \tilde{E}_f \mathbf{K_f y} \\ \boldsymbol{\Sigma}_{\delta_1} = (\tilde{E}_t \mathbf{K_t} + \tilde{E}_f \mathbf{K_f}) \sigma_n^2 \boldsymbol{\Sigma_n} (\tilde{E}_t \mathbf{K_t} + \tilde{E}_f \mathbf{K_f})^H \\ \boldsymbol{\mu}_{\delta_2} = \omega^2 \mathbf{M y} \\ \boldsymbol{\Sigma}_{\delta_2} = (\omega^2 \mathbf{M}) \sigma_n^2 \boldsymbol{\Sigma_n} (\omega^2 \mathbf{M})^H \end{cases}$$

$$\begin{cases} \boldsymbol{\Sigma}_{\delta} = \left({\boldsymbol{\Sigma}_{\delta_1}}^{-1} + {\boldsymbol{\Sigma}_{\delta_2}}^{-1} \right)^{-1} \\ \boldsymbol{\mu}_{\delta} = \boldsymbol{\Sigma}_{\delta} \left({\boldsymbol{\Sigma}_{\delta_1}}^{-1} \boldsymbol{\mu}_{\delta_1} + {\boldsymbol{\Sigma}_{\delta_2}}^{-1} \boldsymbol{\mu}_{\delta_2} \right) \end{cases}$$

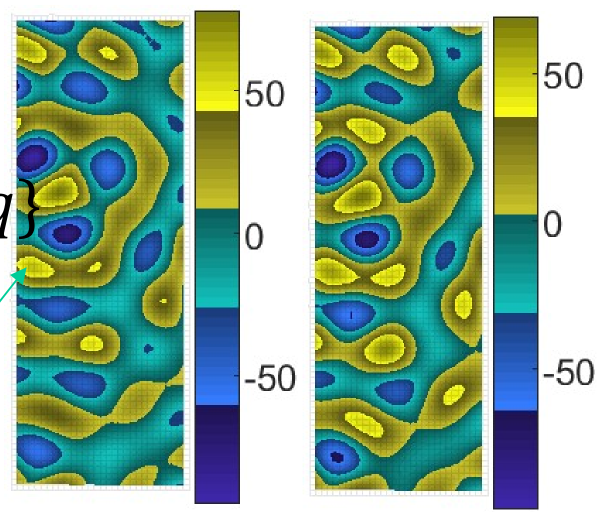
- MDRE expansion used to estimate shape q^{Exp}
- Good for displacement but not stress. Keep on edge where unknown force R_I occurs and to estimate acceleration field
- Estimate regularized interior displacement q_C^{Reg} (possible shift α)
- For true solution $q_C^{Exp} = q_C^{Reg}$

$$[M(s^2 - \alpha)] \begin{Bmatrix} q_I^{Exp} \\ q_C^{Exp} \end{Bmatrix} + \left(M\alpha + K(p) \right) \begin{Bmatrix} q_I^{Exp} \\ q_C^{Reg} \end{Bmatrix} = \begin{Bmatrix} R_I \\ \mathbf{0} \end{Bmatrix}$$



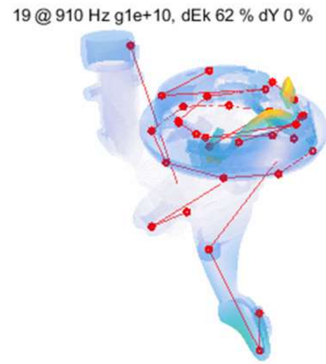
$$\omega^2 M\{q\} \approx [K]\{q\}$$

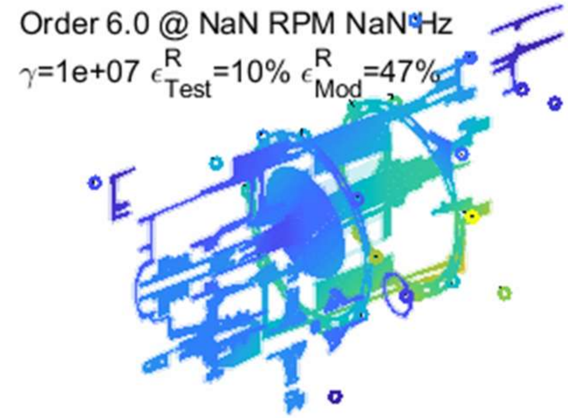
Bad / good

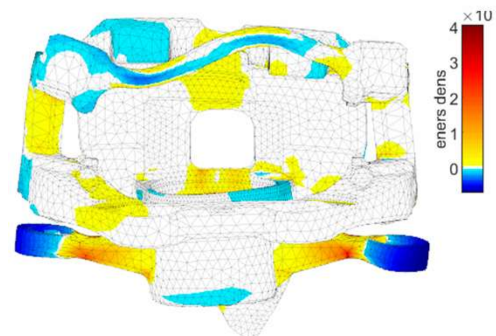


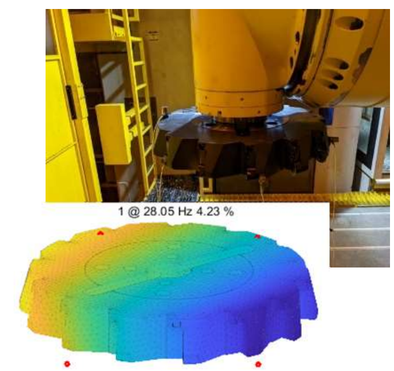
Conclusion

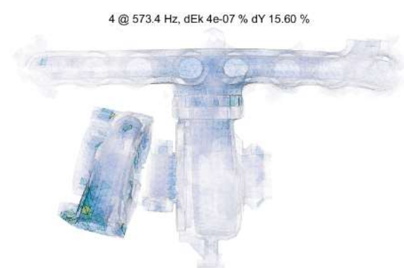
- Understanding multi-objective
  1e3 points needed
- Reduction is critical
  - Ritz reduction (offline ∝1h, once)
  - Learning $\{\phi_j(p_1)\}$ $\{\phi_j(p_2)\}$ $[\mathrm{K}]^{-1}[c]^T$
  - Basis building : SVD/EIG/LU
- Ongoing work
  - Parameter updating
  - Quantification of test errors
  - Non-linear transient
  - GUI / user friendliness

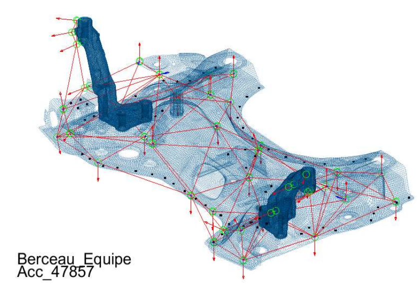








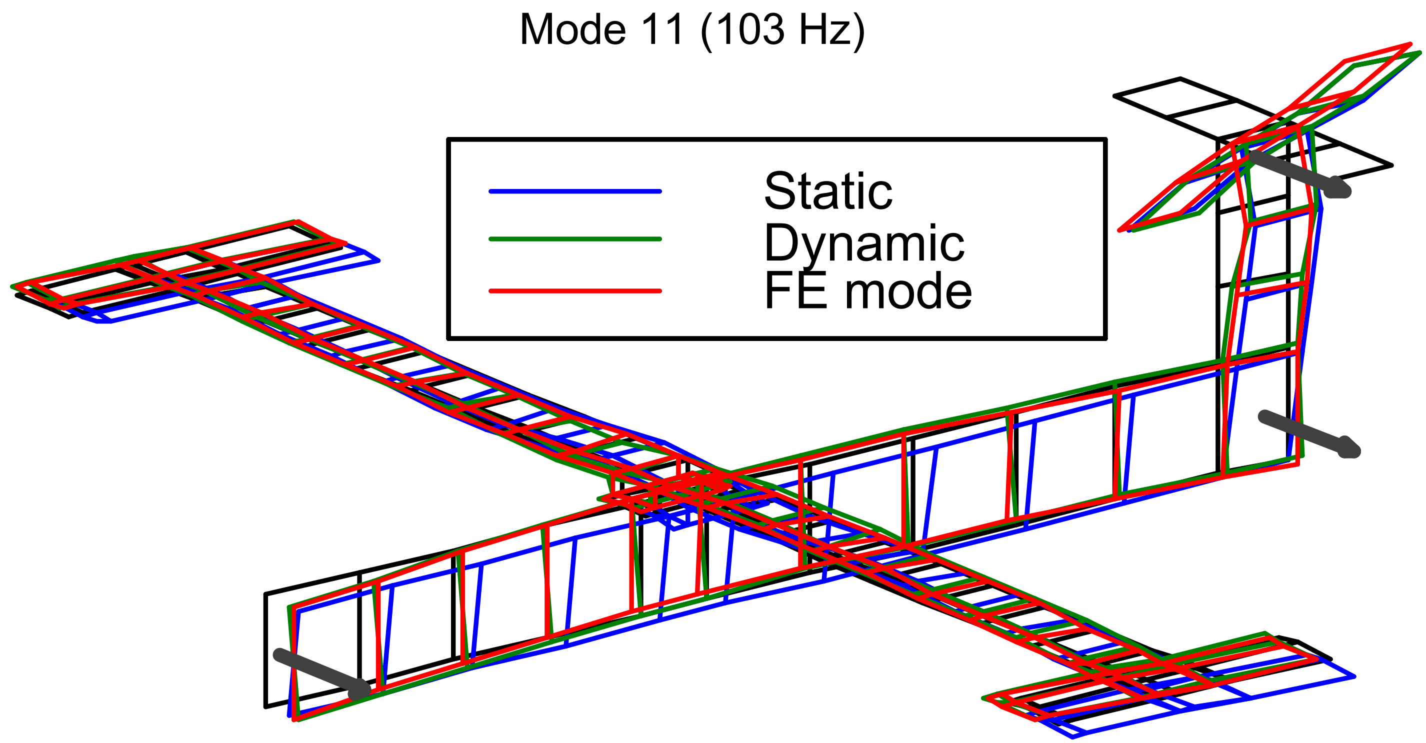




ARTS ET MÉTIERS ParisTech SDTools VIBRATION SOFTWARE & CONSULTING

Thanks to (alphabetical order) : Airbus, Arianespace, Bosch, …
Daimler, EDF, ESA, JE Electric, PSA/Stellantis, …

Static/dynamic expansion



- Static expansion has cut-off frequency
- Measurement errors are not accounted for

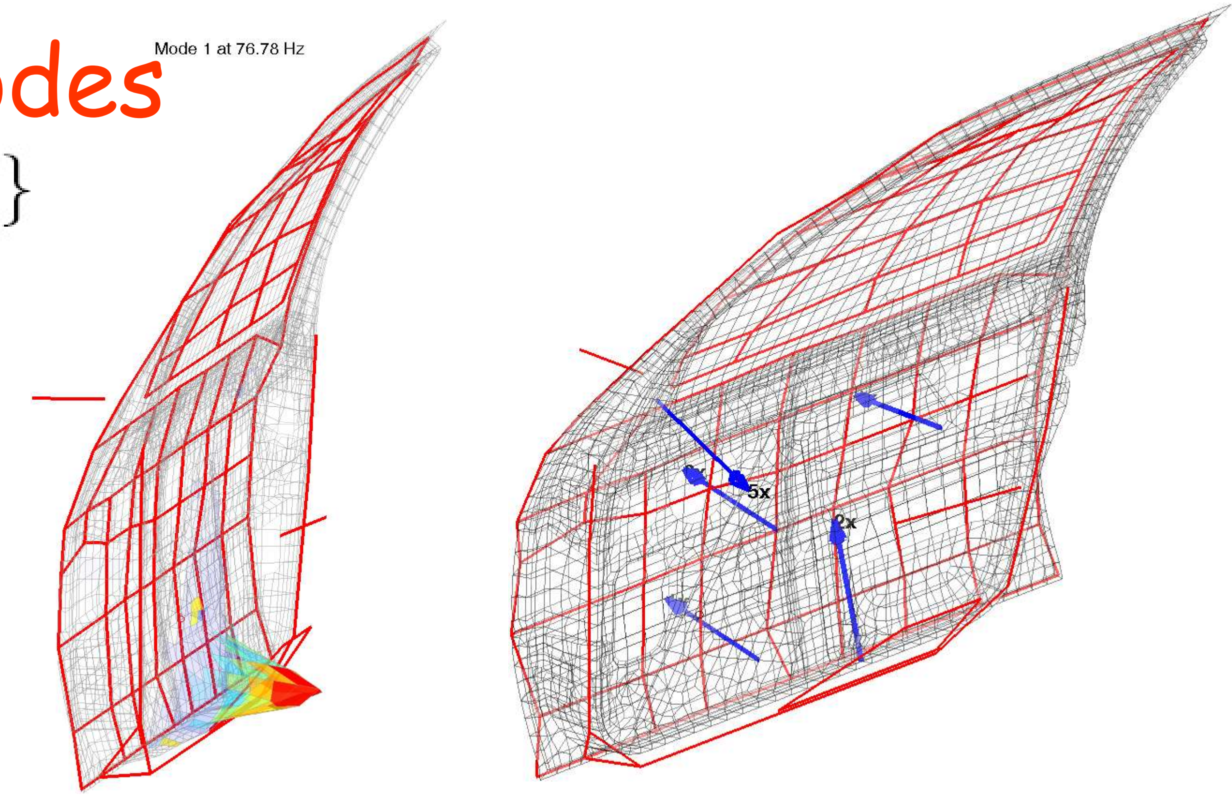
Fixed sensor mode

Fixed sensor modes

$$\left[K-\omega_j^2 M\right]\{\phi_j\}=\{0\}$$

With

$$[c_m]_{m\leq k}\{\phi_j\}=0$$



Uses

- *Analyze static expansion validity*
- Place additional sensors to extend frequency band (IMAC 05)