Identification (notes chapters 6-7, slides Identification_EMA.pdf)

1. Identification demo
2. Inverse problems : model forms, data (s 7.1)
3. Model forms for identification, discussion of residual terms
4. Frequency domain least-squares solution, implicit NL
5. Evaluation of results (ch 8)

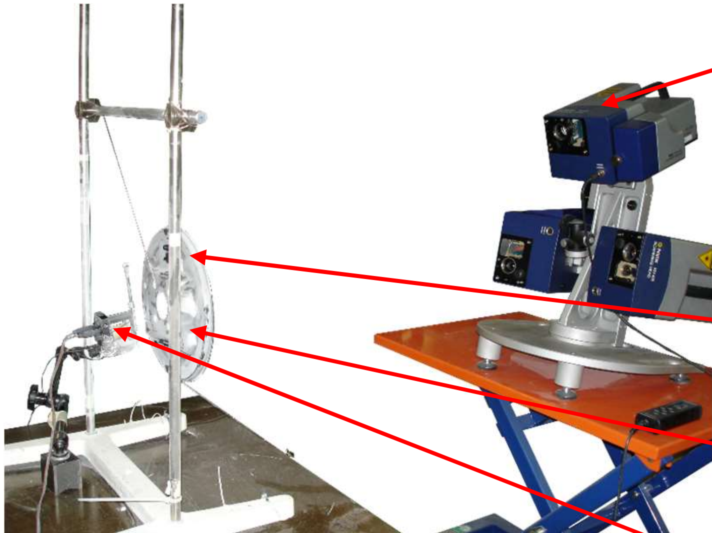
SD Tools VIBRATION SOFTWARE & CONSULTING

Arts et Métiers Institute of Technology

MS2SC http://savoir.ensam.eu/moodle/course/view.php?id=1874

PROVIR http://savoir.ensam.eu/moodle/course/view.php?id=490

Experimental modal analysis : data

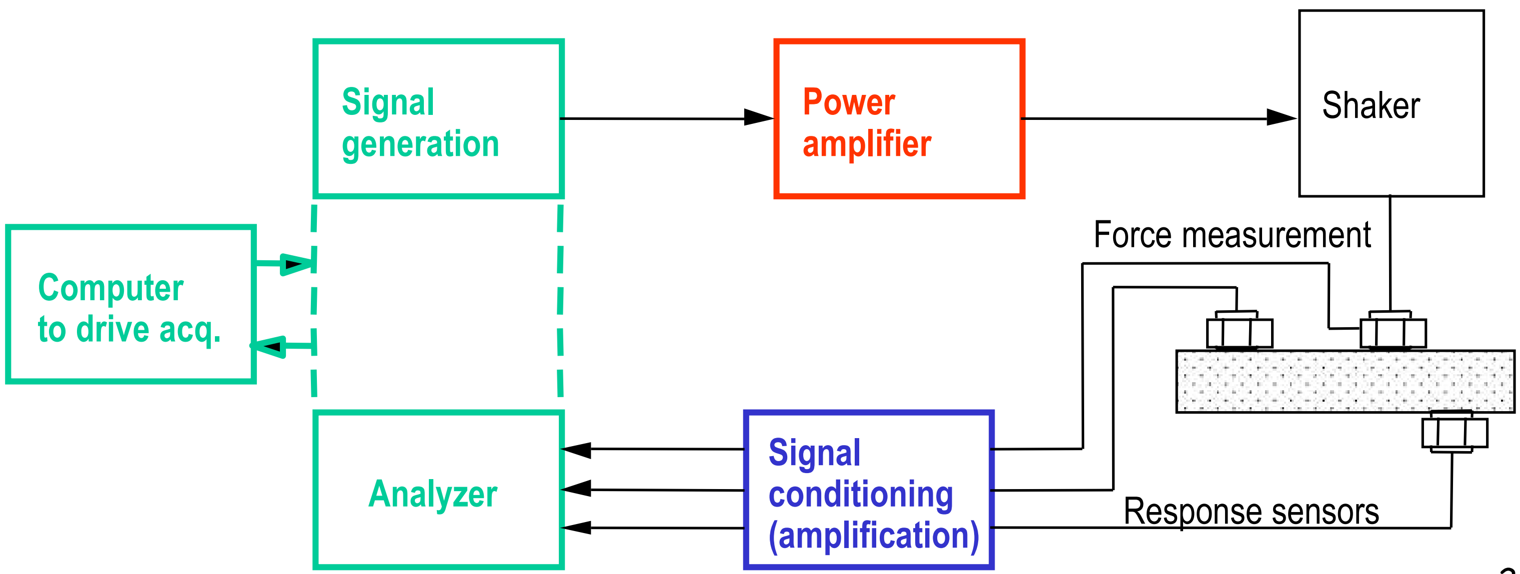


Laser vibrometer (measure)

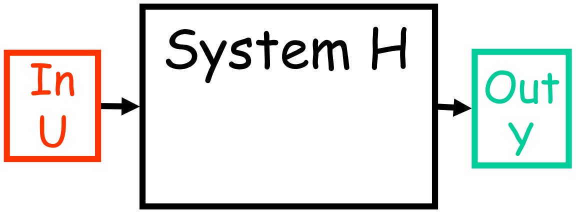
Measurement point

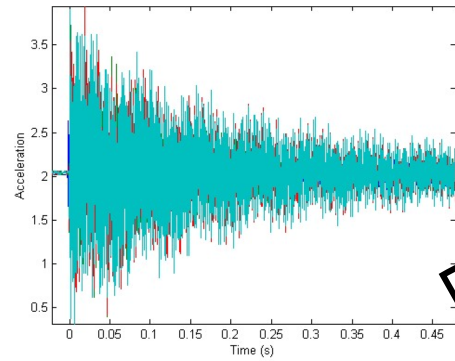
Plate

Shaker (excitation)



Assumption LTI system = transfers exist





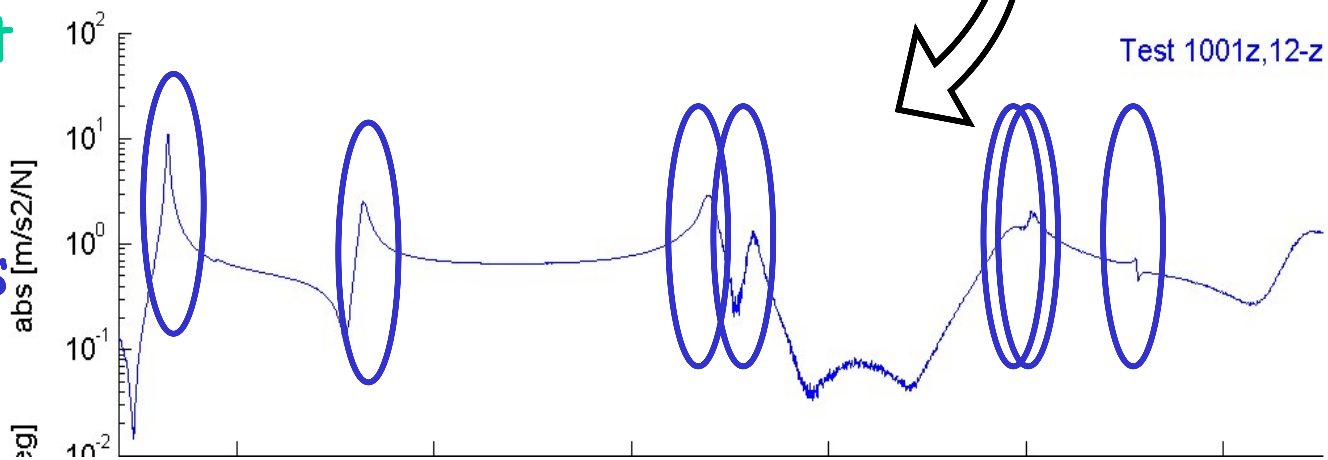
Transfers estimated from time response

ONE input

ONE output

$$\{Y(\omega)\}=[H(\omega)]\{U(\omega)\}$$

MANY resonances



Some standard texts for the vibration community

[1] D. J. Ewins, Modal Testing: Theory and Practice. John Wiley and Sons, Inc., New York, NY, 1984.

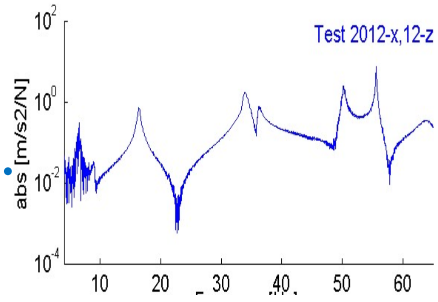
[2] W. Heylen et P. Sas, Modal analysis theory and testing. Katholieke Universteit Leuven, Departement Werktuigkunde, 2006

[3] K. G. McConnell, Vibration Testing. Theory and Practice. Wiley Interscience, New-York, 1995

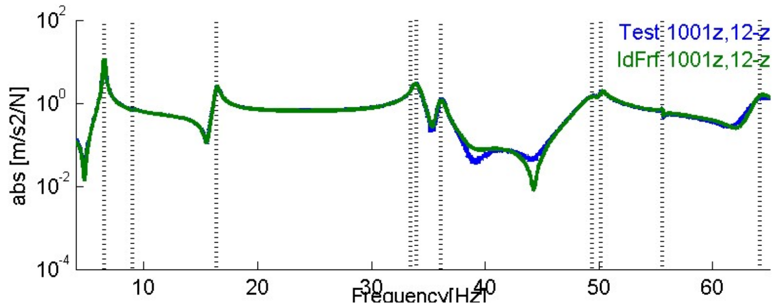
Experimental modal analysis: an inverse problem

See also chapter 5 (signal processing & non-parametric ID) & Signal_Test.pdf

Data
- Transfer
- Hankel mat.

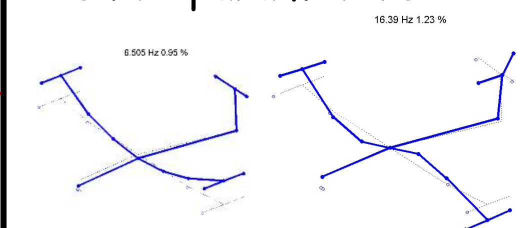


Objective H_{test}-H_{id}



Optimization

Result : parameters



Family of models

SVD $\sum_i \left\{ \begin{array}{c} U_i \\ \downarrow \\ sensors \end{array} \right\} \{a_i(t) \;\; \rightarrow \;\; time\}$

State space $\{\dot{x}\} = [A]\{x\} + [B]\{u\}$

Pole residue $\sum \frac{[R_j]_{NS\times NA}}{s-\lambda_j}$

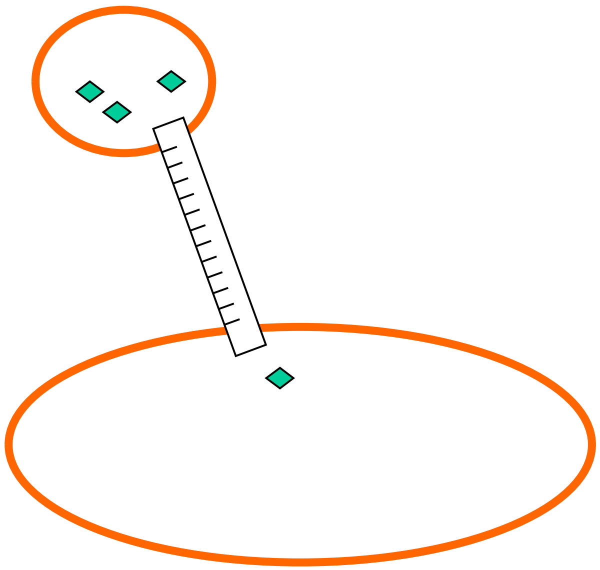
Rational fraction, modal model, second order …

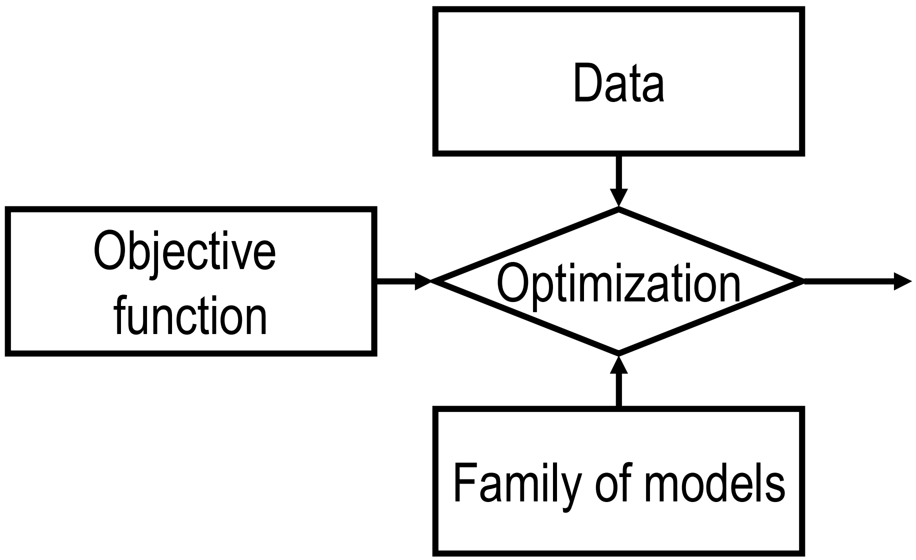
LTI model forms

- See MATLAB control or scipy.signal.lti
- ss state-space $$\{\dot{x}\} = [A]_{N\times N}\{x\} + [B]\{u\}_{NA\times 1}$$ $$\{y\}_{NS\times 1} = [c]\{x\} + D\{u\}$$
- Mechanical SS $$\left\{\begin{array}{c}\dot{p}\\ \ddot{p}\end{array}\right\} = \left[\begin{array}{cc}0 & I\\ -\left[\backslash\omega_j^2\backslash\right] & -\Gamma\end{array}\right]\left\{\begin{array}{c}p\\ \dot{p}\end{array}\right\} + \left[\begin{array}{c}0\\ \phi_j^T b\end{array}\right]\{u(t)\}$$ $$\{y(t)\} = \left[\begin{array}{cc}c\phi_j & 0\end{array}\right]\left\{\begin{array}{c}p\\ \dot{p}\end{array}\right\}$$
- tf $H = \frac{\sum_i [b_i]_{NS\times NA} s^i}{\sum_j a_j s^j}$ $$\left\{\begin{array}{c}\dot{q}\\ \ddot{q}\end{array}\right\} = \left[\begin{array}{cc}0 & I\\ -M^{-1}K & -M^{-1}C\end{array}\right]\left\{\begin{array}{c}q\\ \dot{q}\end{array}\right\} + \left[\begin{array}{c}0\\ M^{-1}b\end{array}\right]\{u(t)\}$$ $$\{y(t)\} = \left[\begin{array}{cc}c & 0\end{array}\right]\left\{\begin{array}{c}q\\ \dot{q}\end{array}\right\}$$
- zpk (zero pole gain) $H = k\frac{\prod_{k<N}(s-[z_j]_{NS\times NA})}{\prod_{j=1}^{N}(s-p_j)}$
- residue (partial fraction expansion = modal form)

$$[H(s)]_{NS\times NA} = \sum_{j=1}^{N/2}\left(\frac{[R_j]_{NS\times NA}}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}\right)$$

Inverse problem





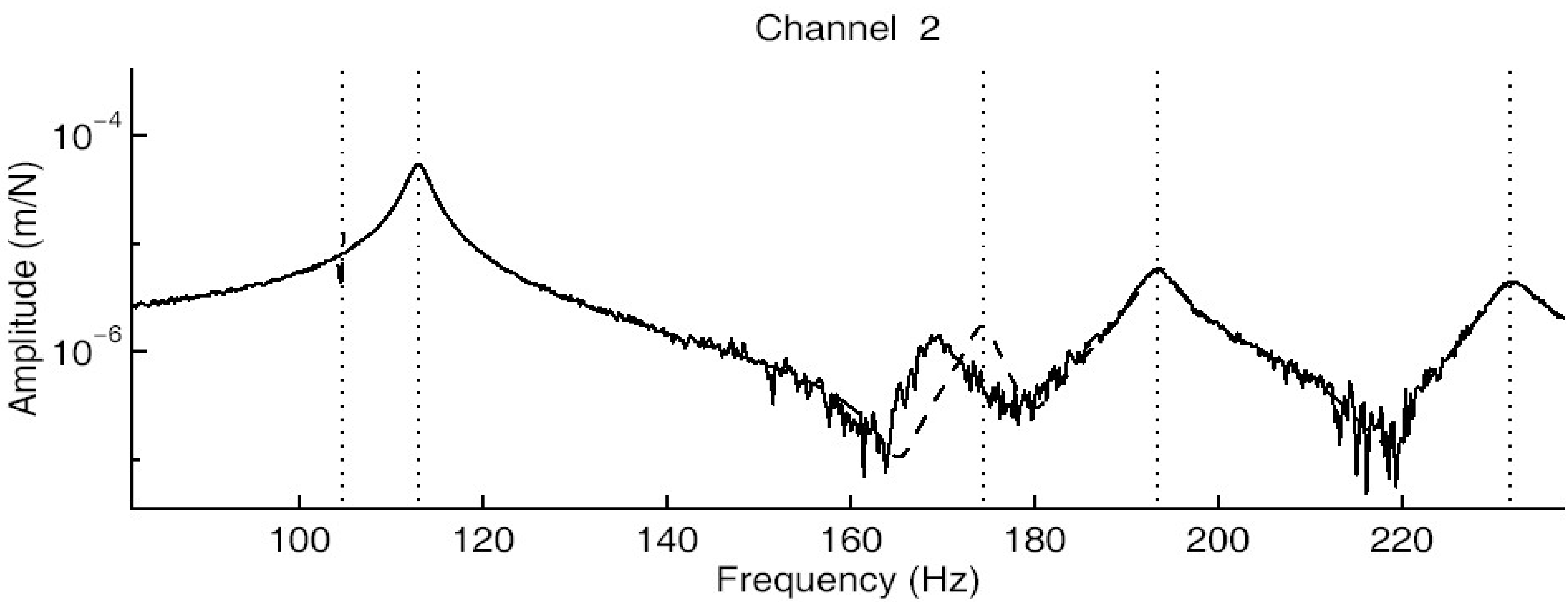
Objective function

- Least squares
  - Objective : minimize distance
  - Variants : Log-least squares, output-error, …
- Singular value decomposition
  - Objective : constrained rank approximation

This is done on the blackboard

Example : FD polynomial

- This is done on the blackboard
- Section 7.4.1 (frequency domain ARX)



Circle fit / -3 dB

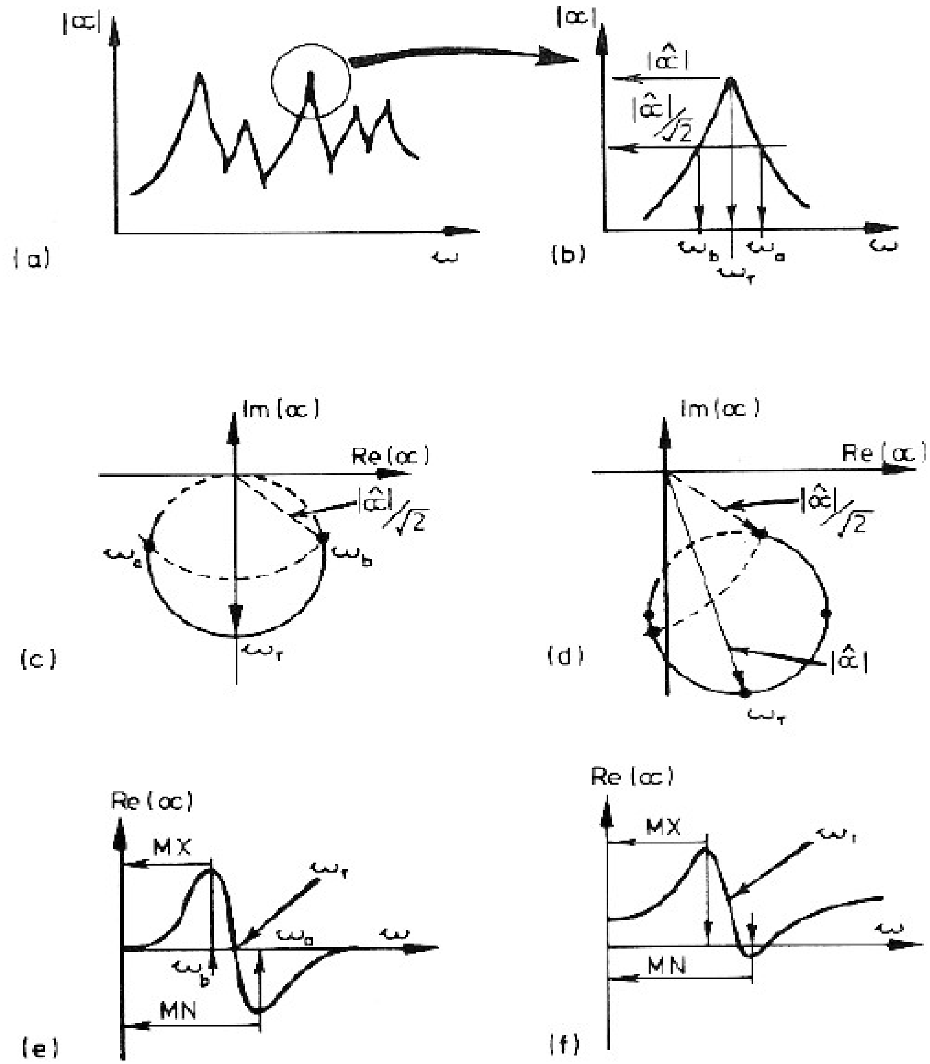
- Assumed model form

$$[H(s)] \approx \frac{A_j}{s^2 + 2\zeta_j \omega_j s + \omega_j^2}$$

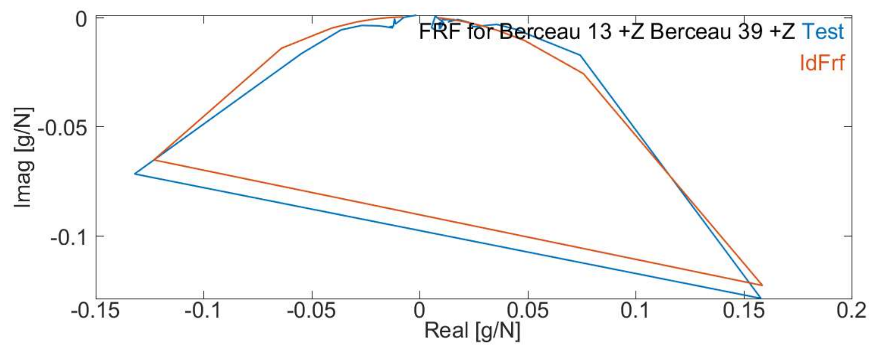
- Geometrical properties of Nyquist give ω,ζ

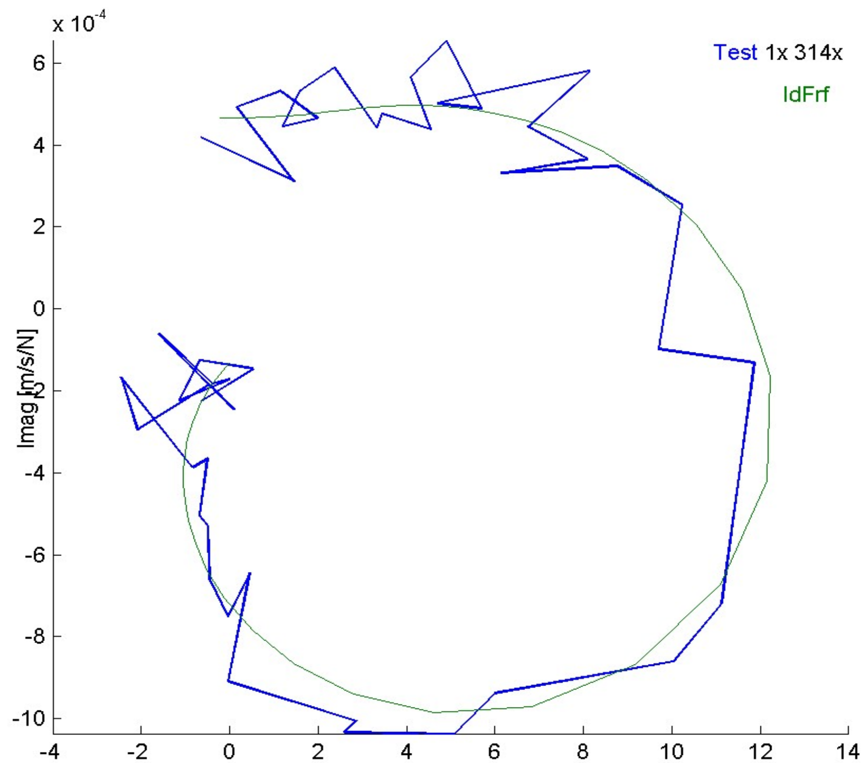
$$\zeta_j \approx \frac{\omega_b - \omega_a}{2\omega_n}$$

- Poor accounting of other contributions



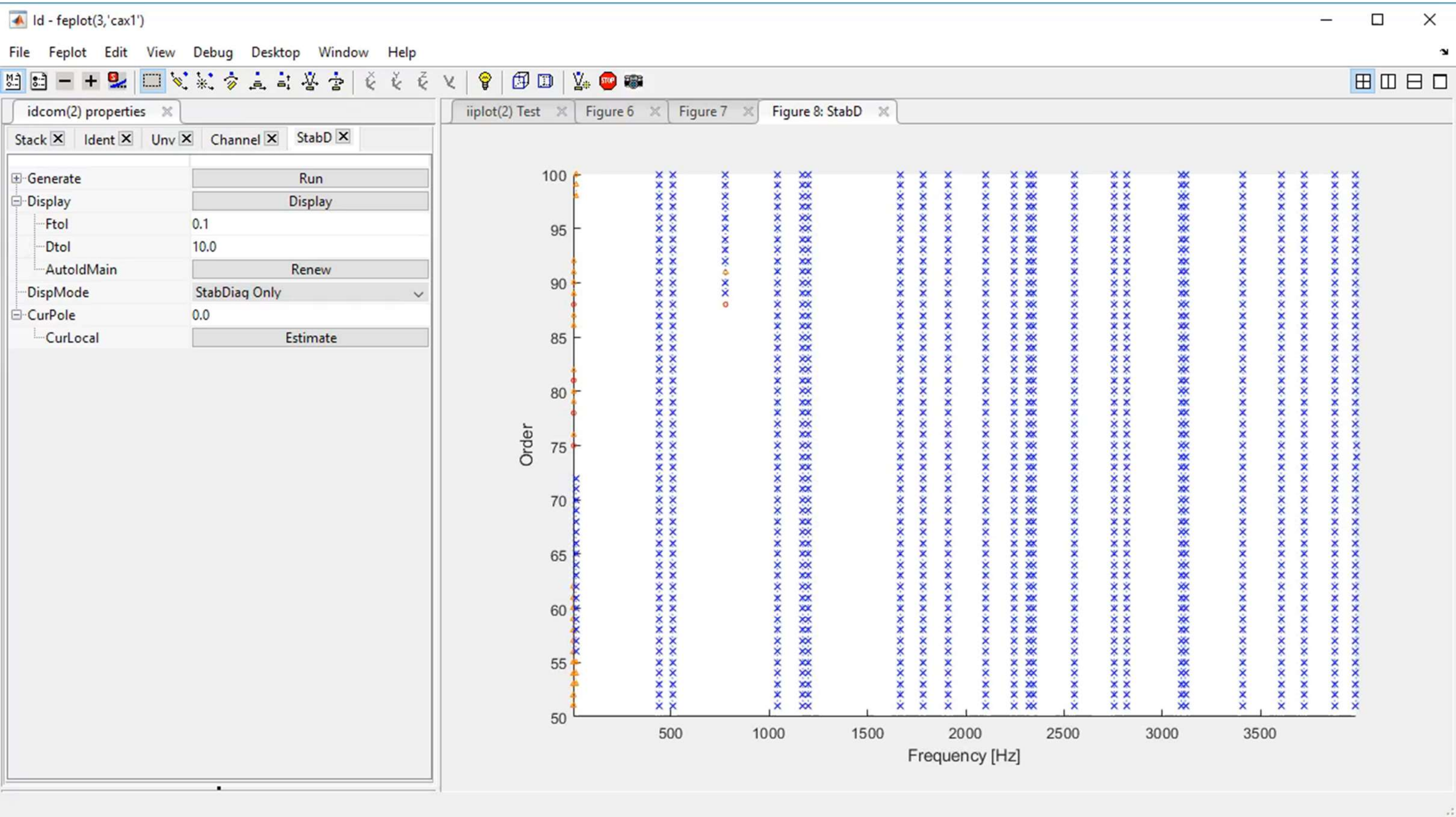
-3dB failures





This is done on the blackboard 10

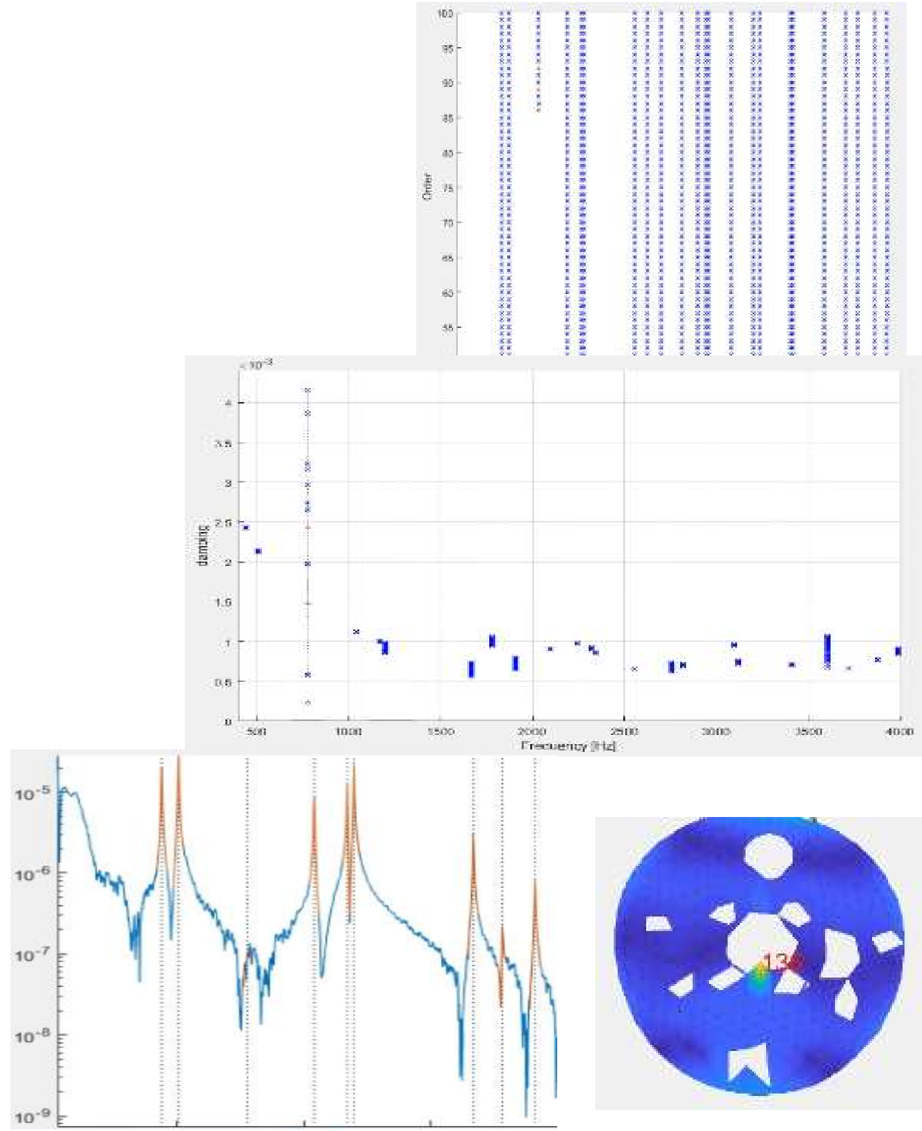
Classical linear system Id



https://www.youtube.com/channel/UCy5PHLw70NqfDDvg24tXJWg

Identification phases

- Initialization pick from :
  - Single pole estimate [1]
  - Stabilization diagram [2]
- Estimate by band (why ? [1,3])
- How can problems be detected ? [3-4]
- Re-optimize poles (why ?)



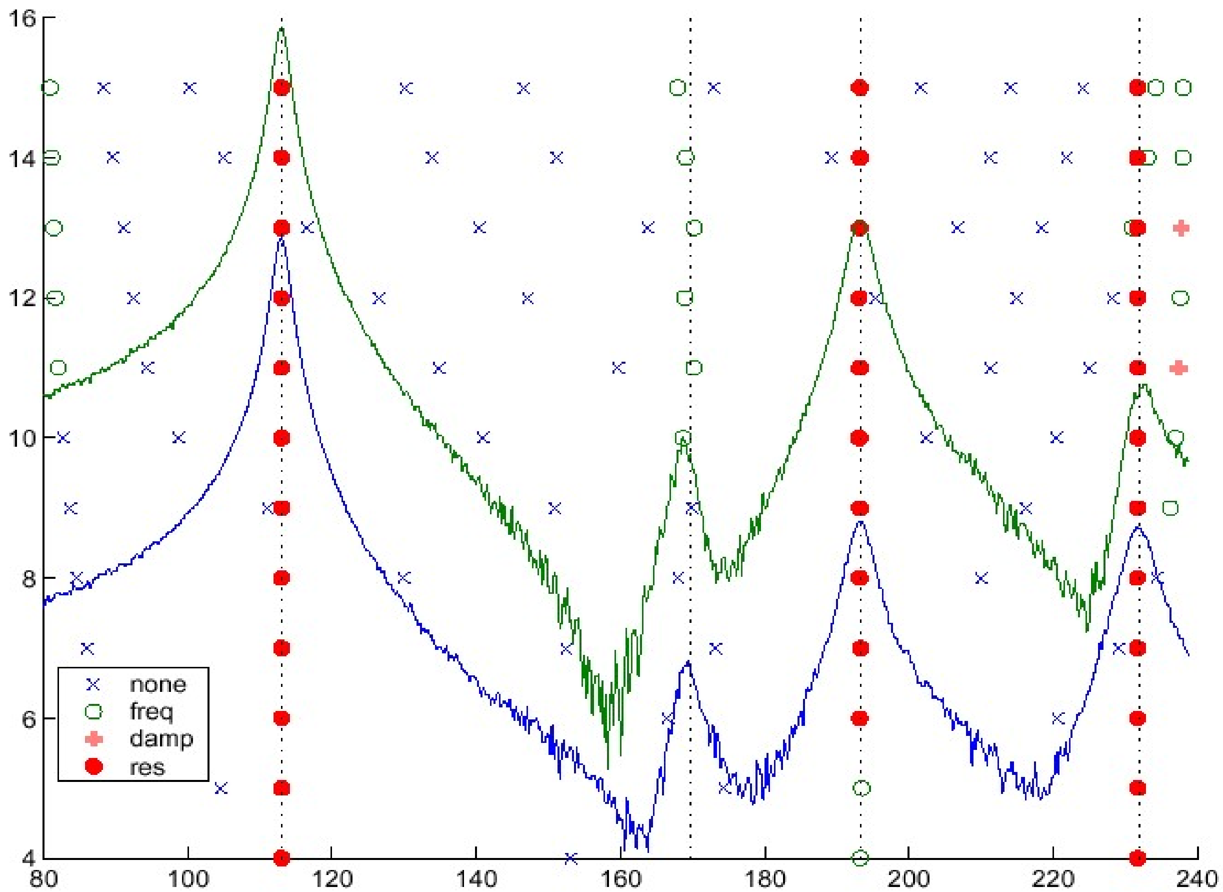
[1] E. Balmes, « Frequency domain identification of structural dynamics using the pole/residue parametrization », IMAC 1996

[2] P. Verboven, « Frequency-domain system identification for modal analysis », Ph.D. thesis, 2002.

[3] G. Martin, « Méthodes de corrélation calcul/essai pour l'analyse du crissement », Ph.D. CIFRE SDTools, Arts et Metiers ParisTech, Paris, 2017

[4] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

Stabilization diagrams

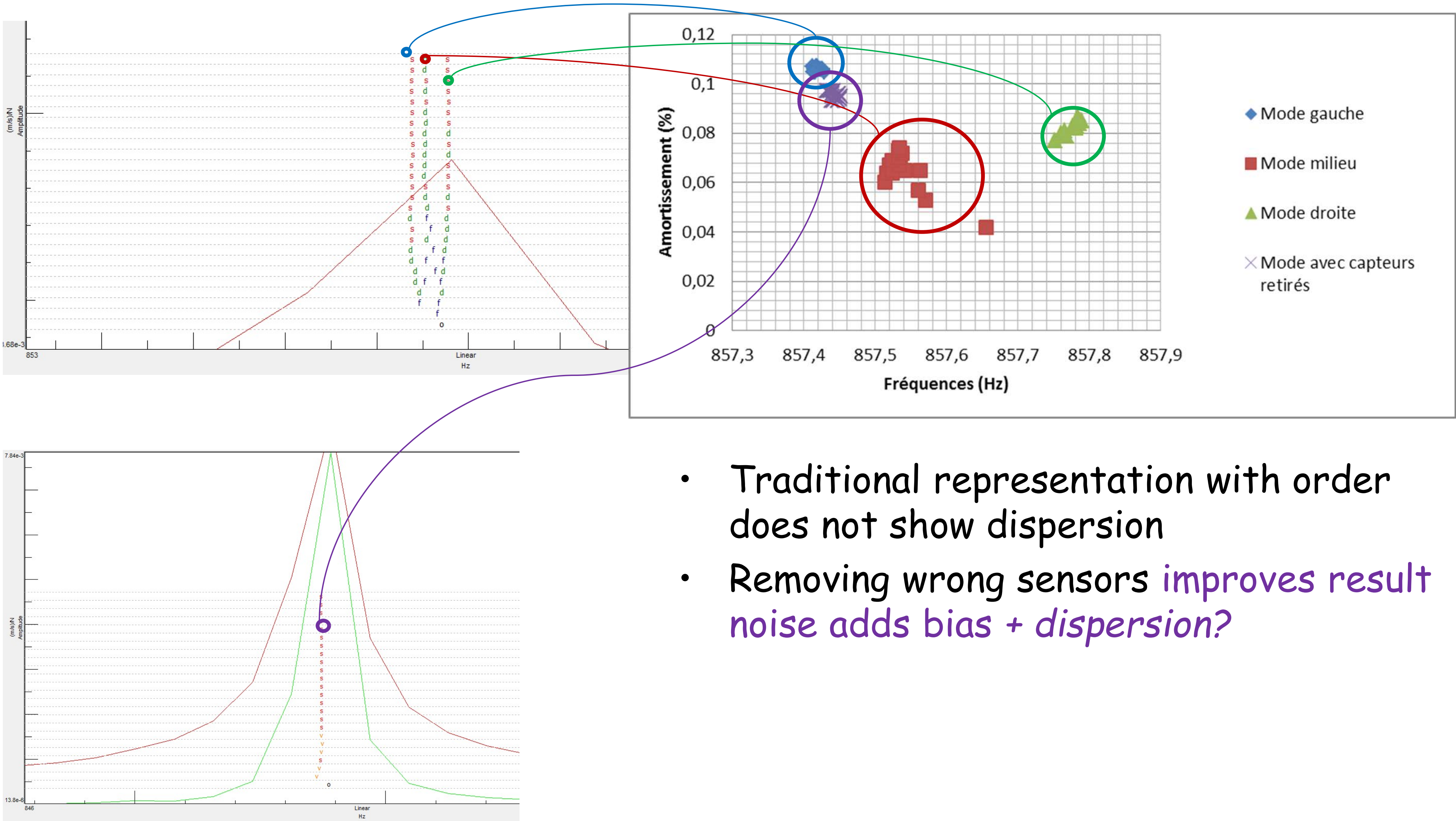


$$\Delta\omega_j/\omega_j < 1\%$$

$$\Delta\zeta_j/\zeta_j < 10\%$$

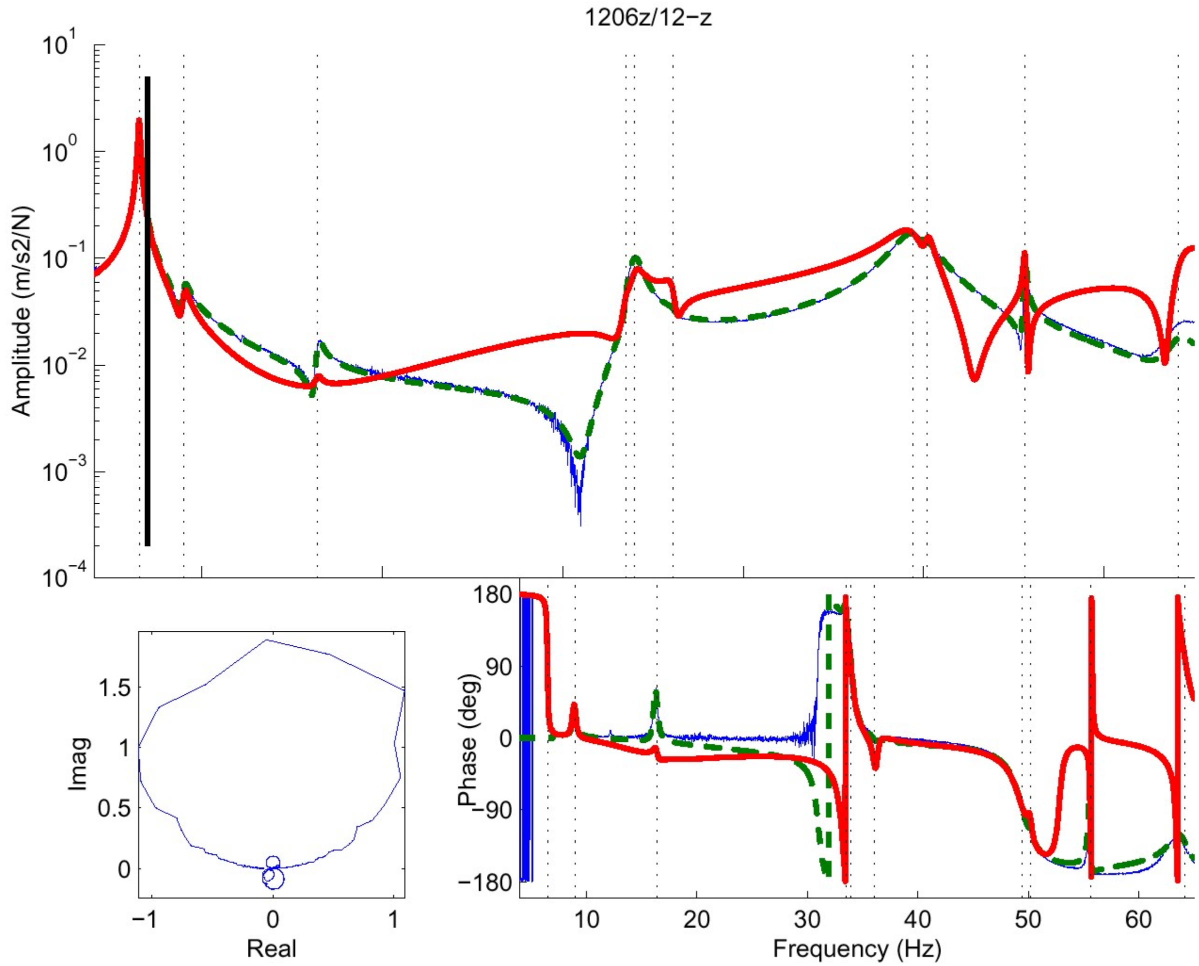
$$\Delta R_j/R_j < 10\%$$

Stabilization diagrams & bias



- Traditional representation with order does not show dispersion
- Removing wrong sensors improves result noise adds bias + *dispersion?*

Noise induced bias -> work by band

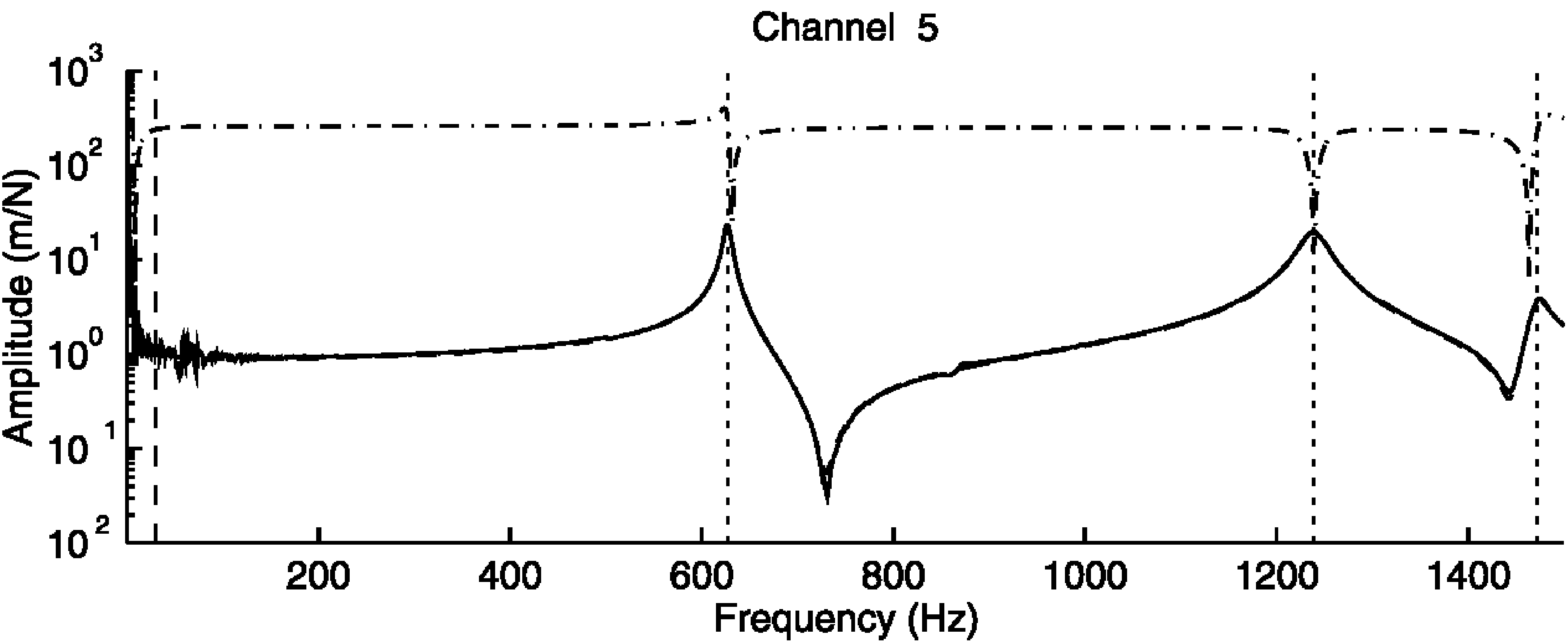


Inconsistence between data and family of models induces bias / variability

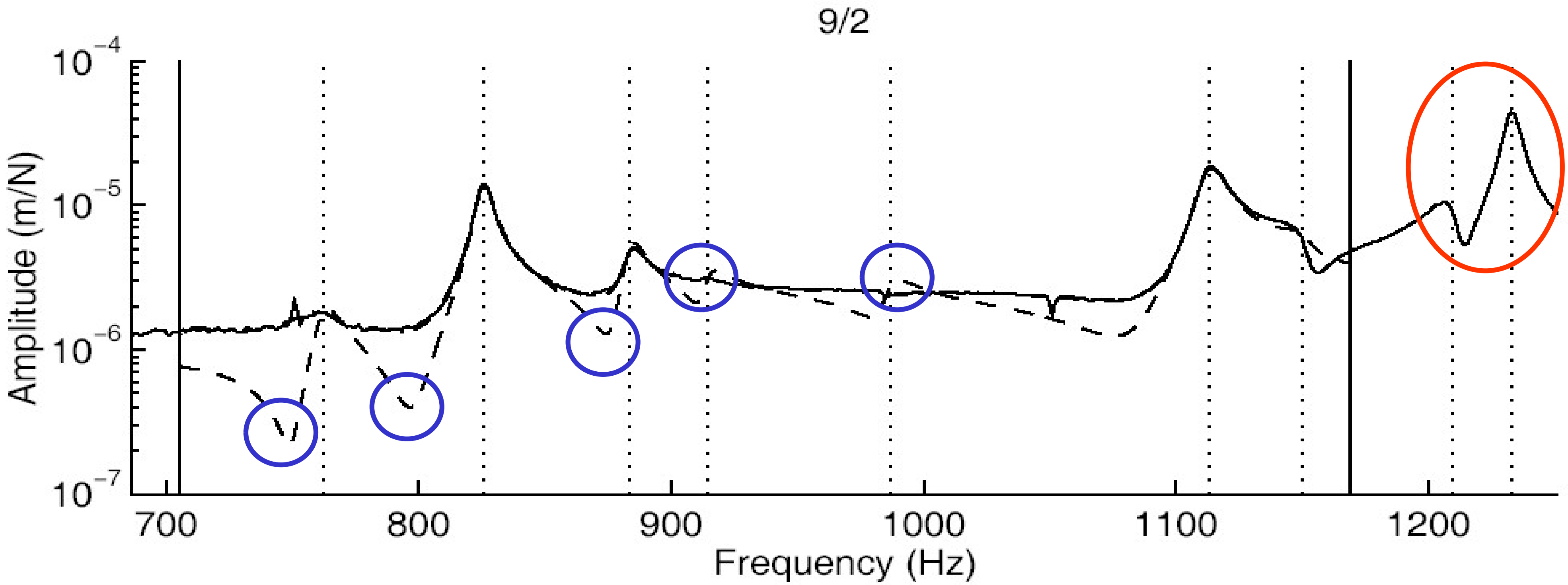
[1] M. Böswald, « Analysis of the bias in modal parameters obtained with frequency-domain rational fraction polynomial estimators », Proceedings of ISMA 2016

[2] El-kafafy, De Troyer, Peeters, Guillaume, « Fast maximum-likelihood identification of modal parameters with uncertainty intervals: A modal model-based formulation », MSSP, 2013

Low frequency noise bias



Effects of residual terms



Here : ignoring two nearby modes induces bias

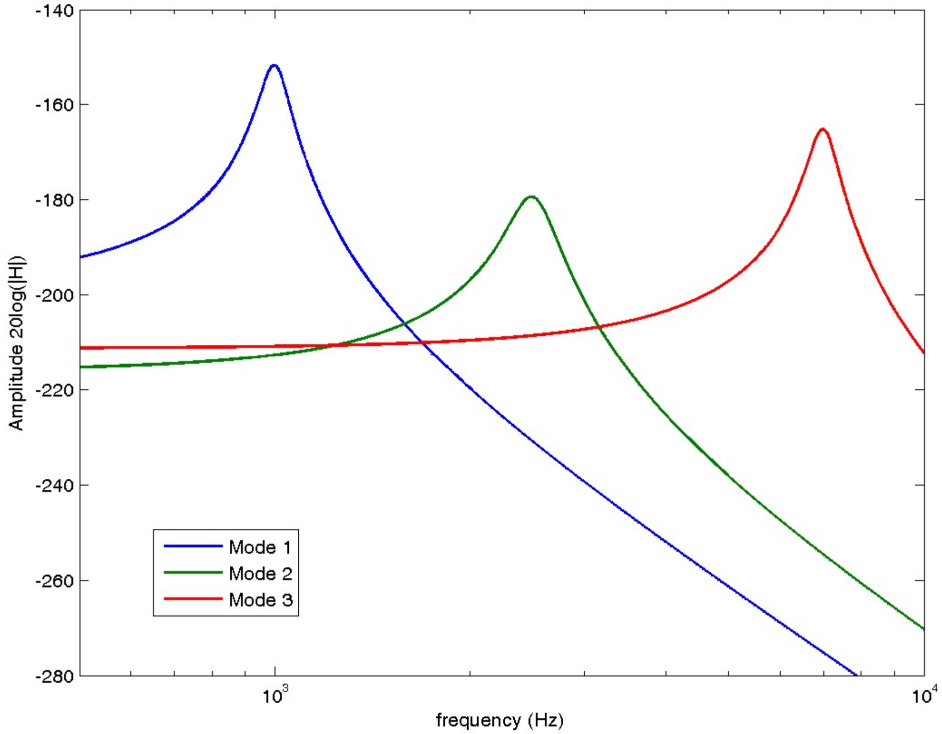
In general : account for flexibility and inertia effects (lower/upper residual)

Identification model

- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$

$\boxed{[H_1(s)]}$ $\boxed{[H_2(s)]}$ $\boxed{[H_3(s)]}$

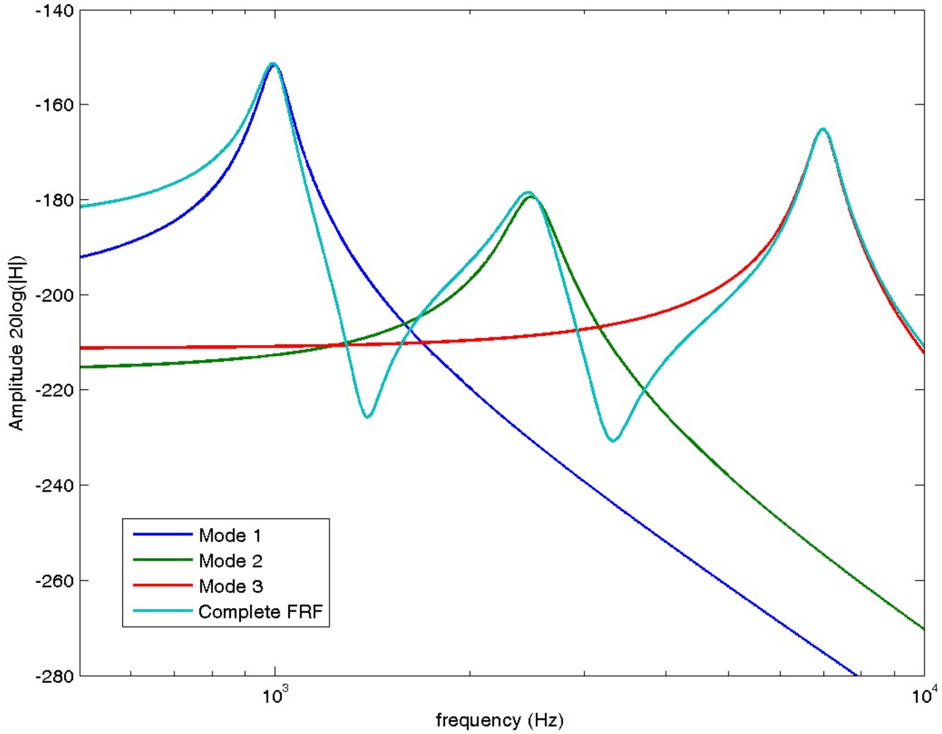


Identification model

- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$

$$\boxed{[H_1(s)]} \quad \boxed{[H_2(s)]} \quad \boxed{[H_3(s)]}$$



$$[H_{model}(s)] = \boxed{[H_1(s)]} + \boxed{[H_2(s)]} + \boxed{[H_3(s)]}$$

Identification model

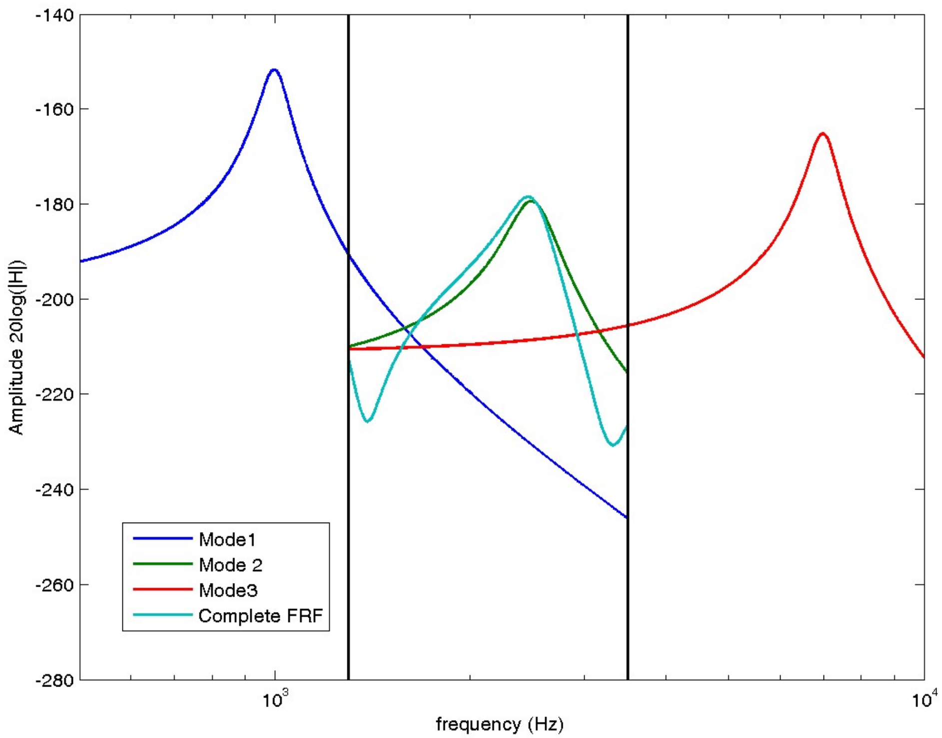
- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$

Bandwidth

Low residual | in | High residual

$\boxed{\cancel{[H_1(s)]}}$ $\boxed{[H_2(s)]}$ $\boxed{\cancel{[H_3(s)]}}$



$$[H_{model}(s)] = \boxed{\cancel{[H_1(s)]}} + \boxed{[H_2(s)]} + \boxed{\cancel{[H_3(s)]}}$$

Identification model

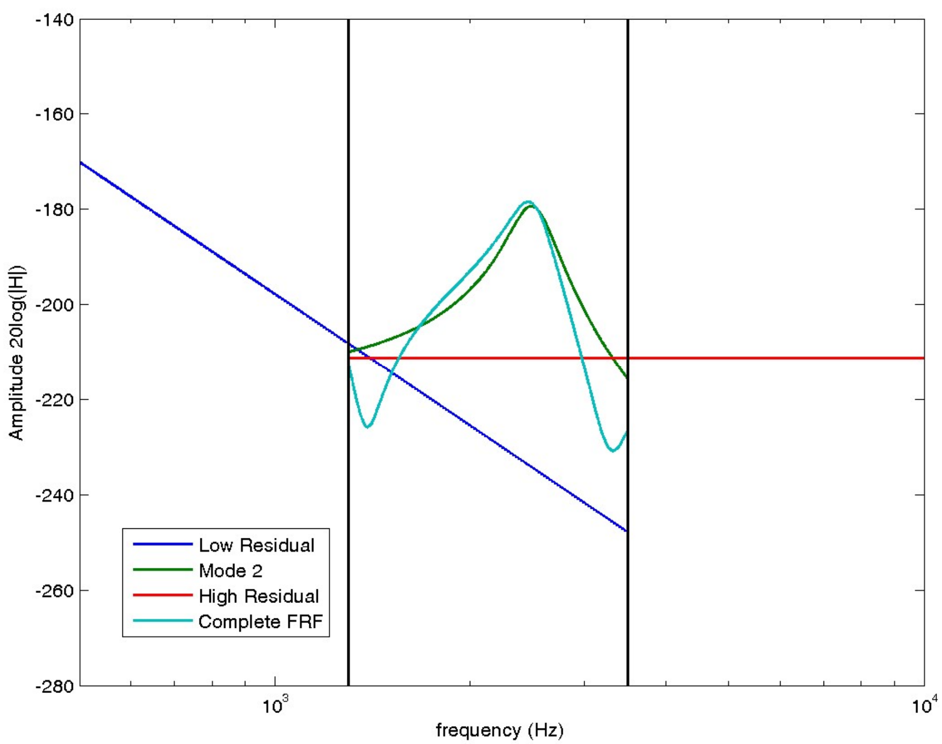
- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$

Bandwidth

Low residual — in — High residual

$$\boxed{\cancel{[H_1(s)]}} \rightarrow \boxed{\frac{[E]}{s^2}} \qquad \boxed{[H_2(s)]} \qquad \boxed{\cancel{[H_3(s)]}} \rightarrow \boxed{[F]}$$

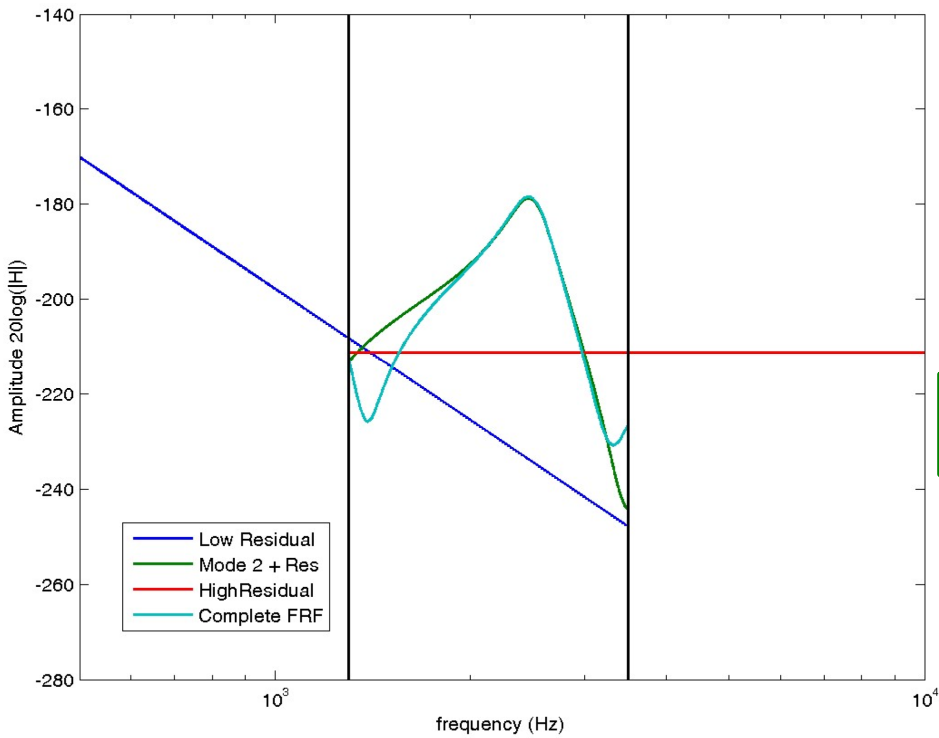


$$[H_{model}(s)] = \boxed{\cancel{[H_1(s)]}} + \boxed{[H_2(s)]} + \boxed{\cancel{[H_3(s)]}}$$

Identification model

- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$



Bandwidth

Low residual | in | High residual

$\boxed{\cancel{[H_1(s)]}}$ $\rightarrow$ $\boxed{\dfrac{[E]}{s^2}}$

$\boxed{[H_2(s)]}$

$\boxed{\cancel{[H_3(s)]}}$ $\rightarrow$ $\boxed{[F]}$

$$\boxed{[H_{model}(s)]} = \boxed{\frac{[E]}{s^2}} + [H_2(s)] + \boxed{[F]}$$

Identification strategies

$$\underbrace{\min_{\lambda_j} \min_{R_j,E,F} \left| [\alpha(s)]_{test} - \sum_{j\in\texttt{identified}} \left(\frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j} \right) + [E(s)] \right|^2}$$

Freq. domain linear least squares with residuals
almost always used
Unknowns λ_j (2*Np), R_j (Ns*Na*2*Np), E,F (Ns*Na*2)

Determination of poles : main differentiator between methods

Two main strategies

- select in over-specified set
- build gradually

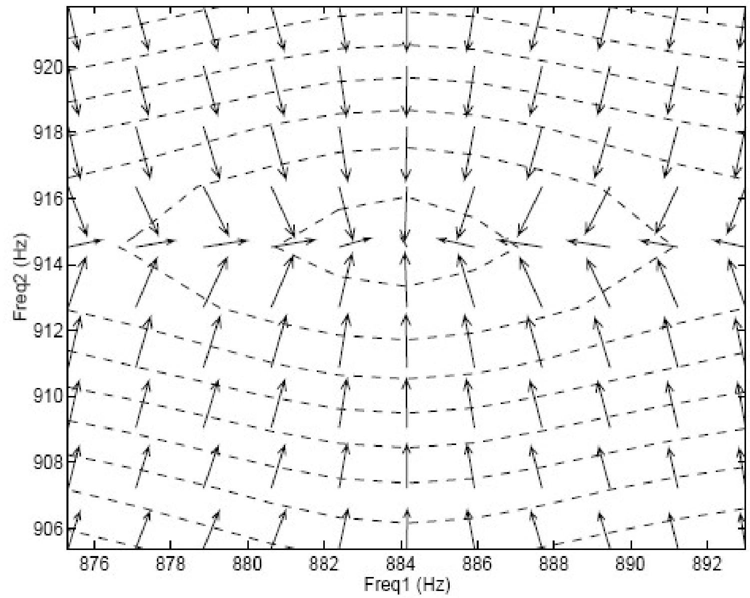
Frequency domain output error

$$\min_{\lambda_j} \min_{R_j, E, F} \left| [\alpha(s)]_{test} - \sum_{j \in \mathtt{identified}} \left(\frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j} \right) + [E(s)] \right|^2$$

- R_j,E,F implicit functions of λ_j
- $\frac{\partial J}{\partial \lambda_j} = \frac{\partial R^T R}{\partial \lambda_j} = 2R^T \frac{\partial R}{\partial \lambda_j}$ can be computed analytically
- around a pole gradient not very affected by error on other poles

SDT implements

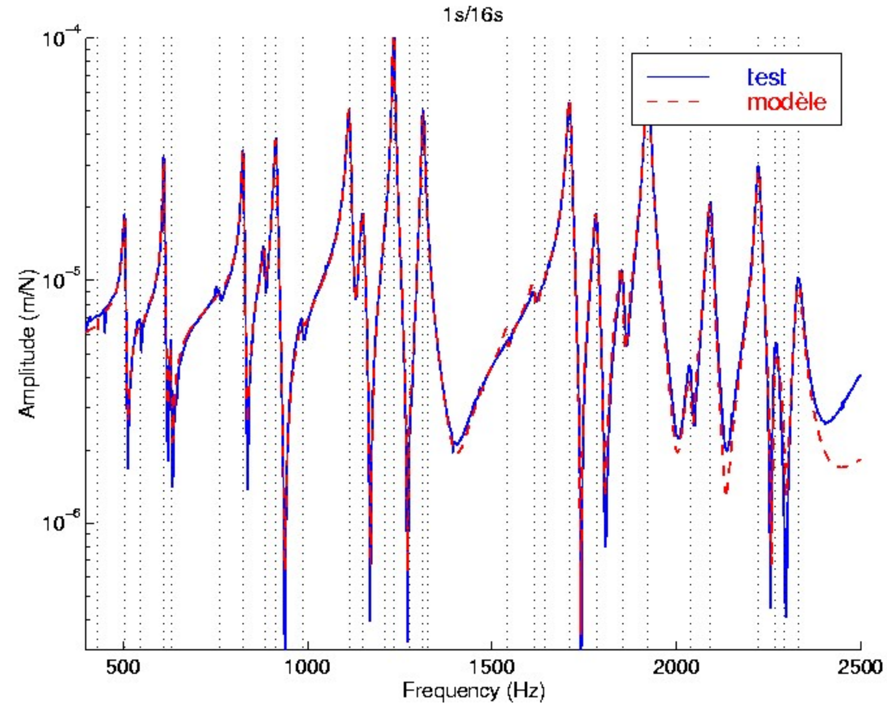
- Fast optimization
- All λ_j changed at each step using gradient only

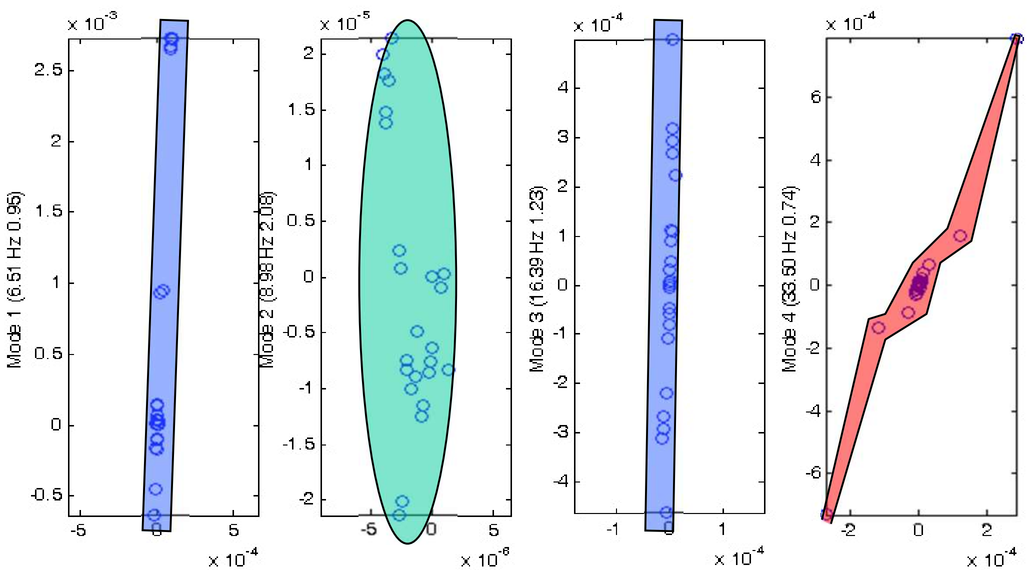


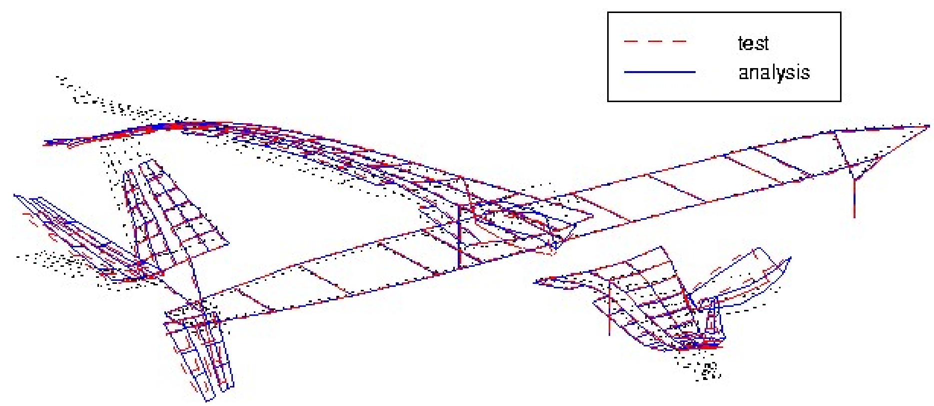
Balmes, PhD MIT 93 & http://www.sdtools.com/pdf/IMAC96id.pdf

Validations of results

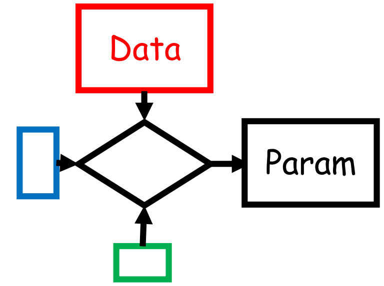
- FRF Superposition
- Modeshape visualization
- Residue properties





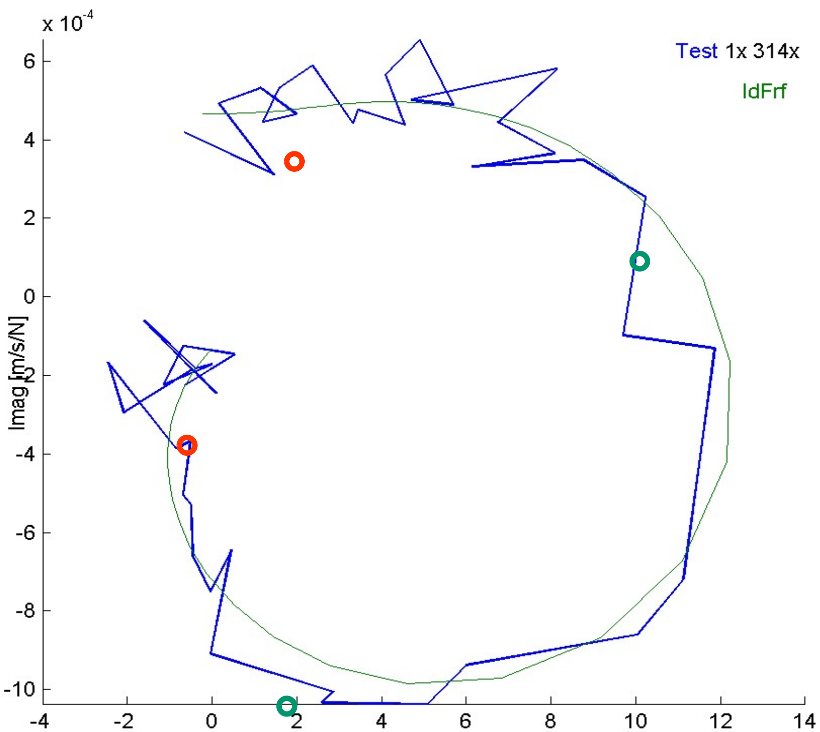


Quality : an error criterion



Is the model well identified?

- Superpose measured and identified FRF for each sensor c, around each mode j
- Compute error on Nyquist



$$e_{jc} = \frac{\int_{\omega_j(1-\alpha\zeta_j)}^{\omega_j(1+\alpha\zeta_j)} |H_{cTest} - H_{cId}|^2}{\int_{\omega_j(1-\alpha\zeta_j)}^{\omega_j(1+\alpha\zeta_j)} |H_{cTest}|^2}$$

"Around each mode" :

- Half power : +/- $\zeta_j\omega_j$
- Visible peak : +/- $5\zeta_j\omega_j$

[1] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

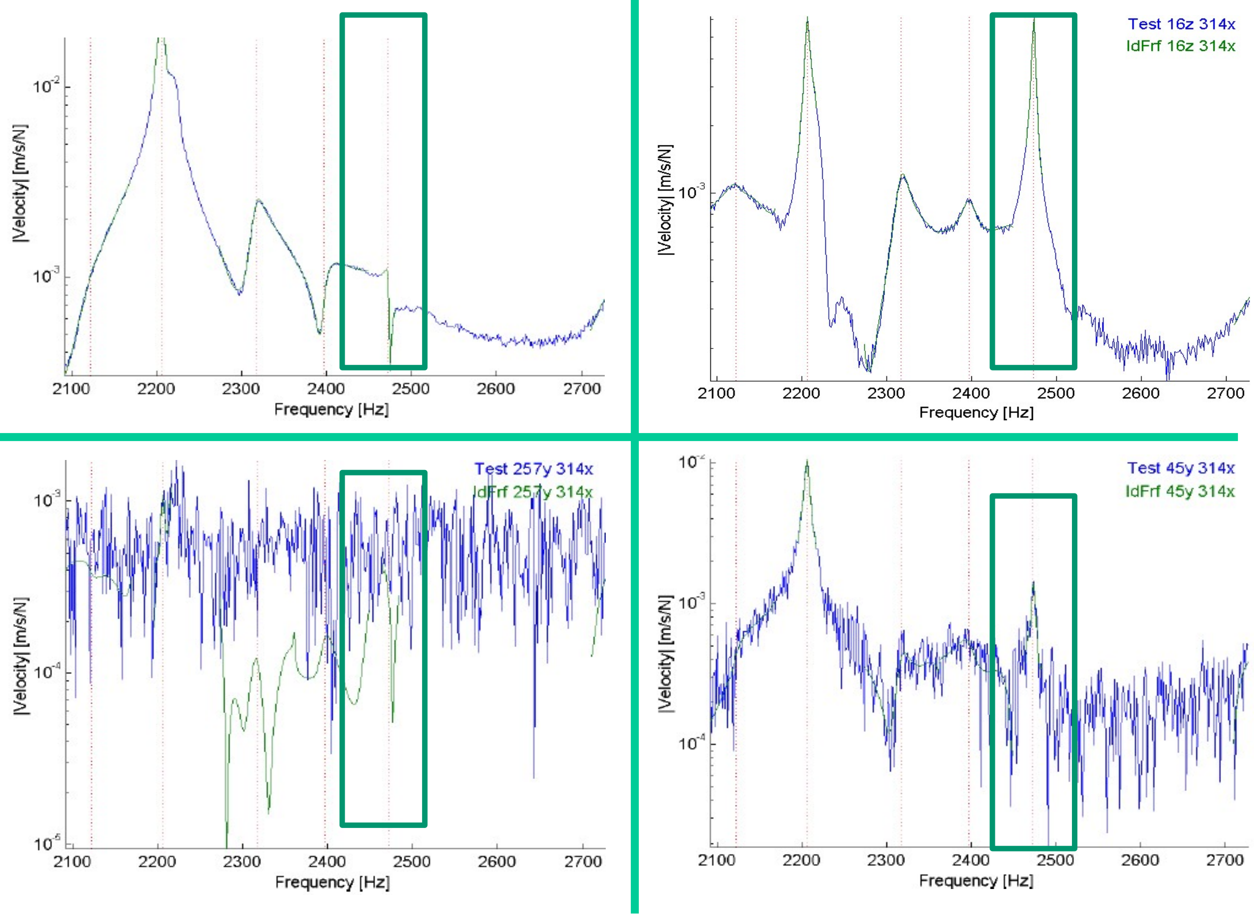
Test/identification error

Contribution --

Contribution ++

Error --

Error ++

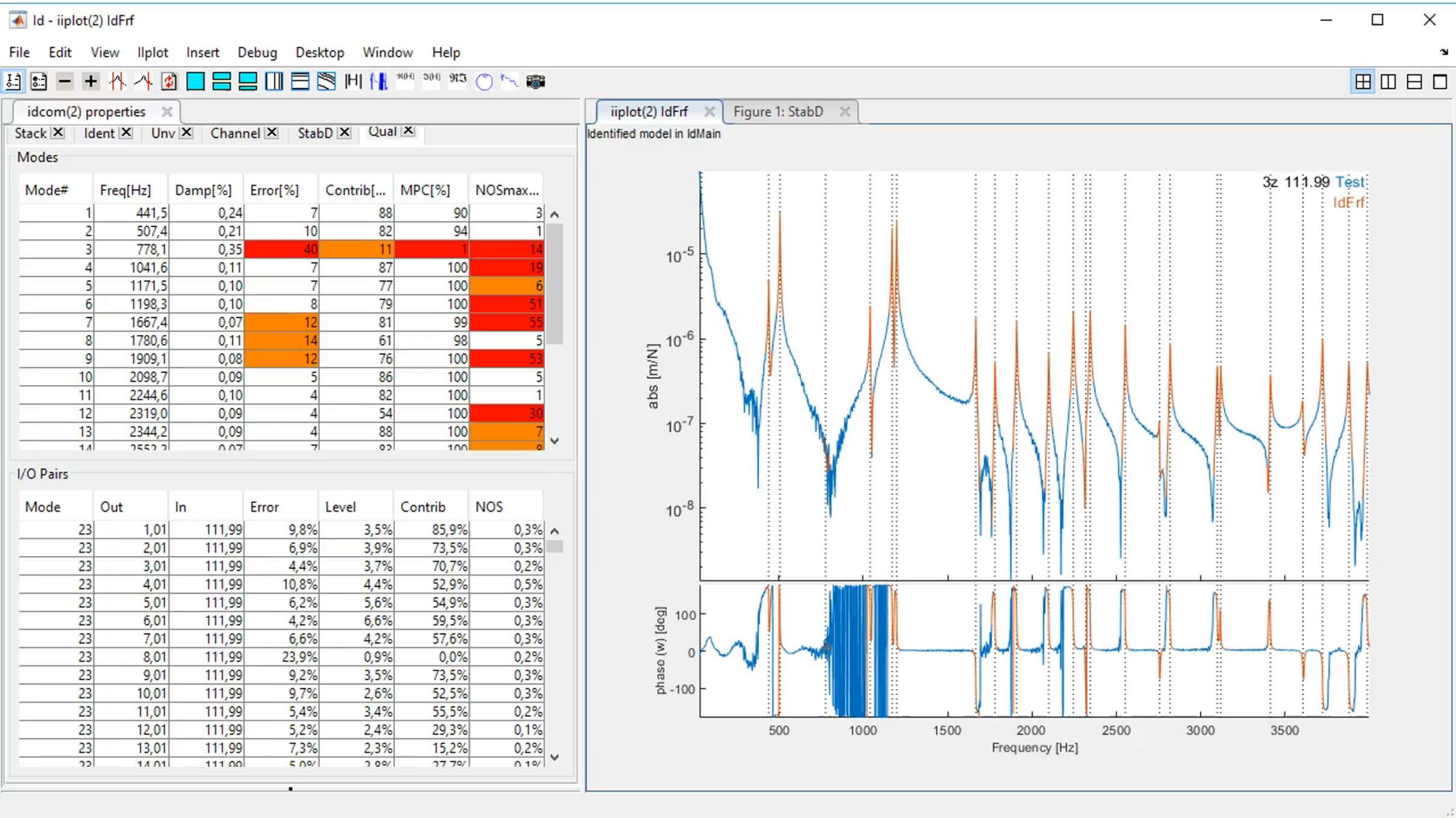


For each sensor, each mode may have strong error/noise & low contribution

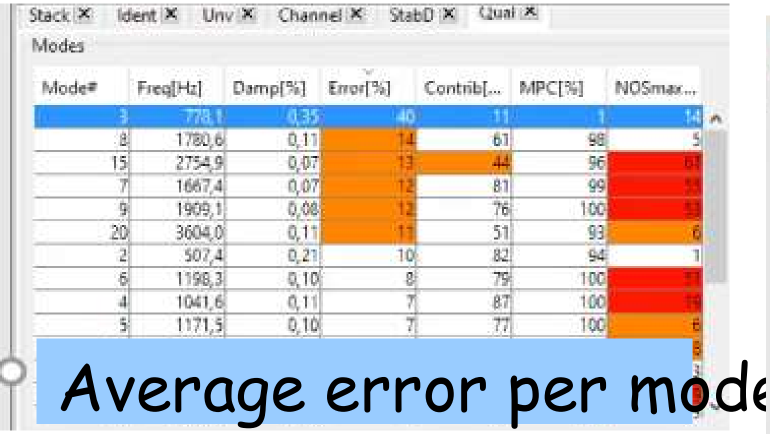
Test error per mode/sensor

[1] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

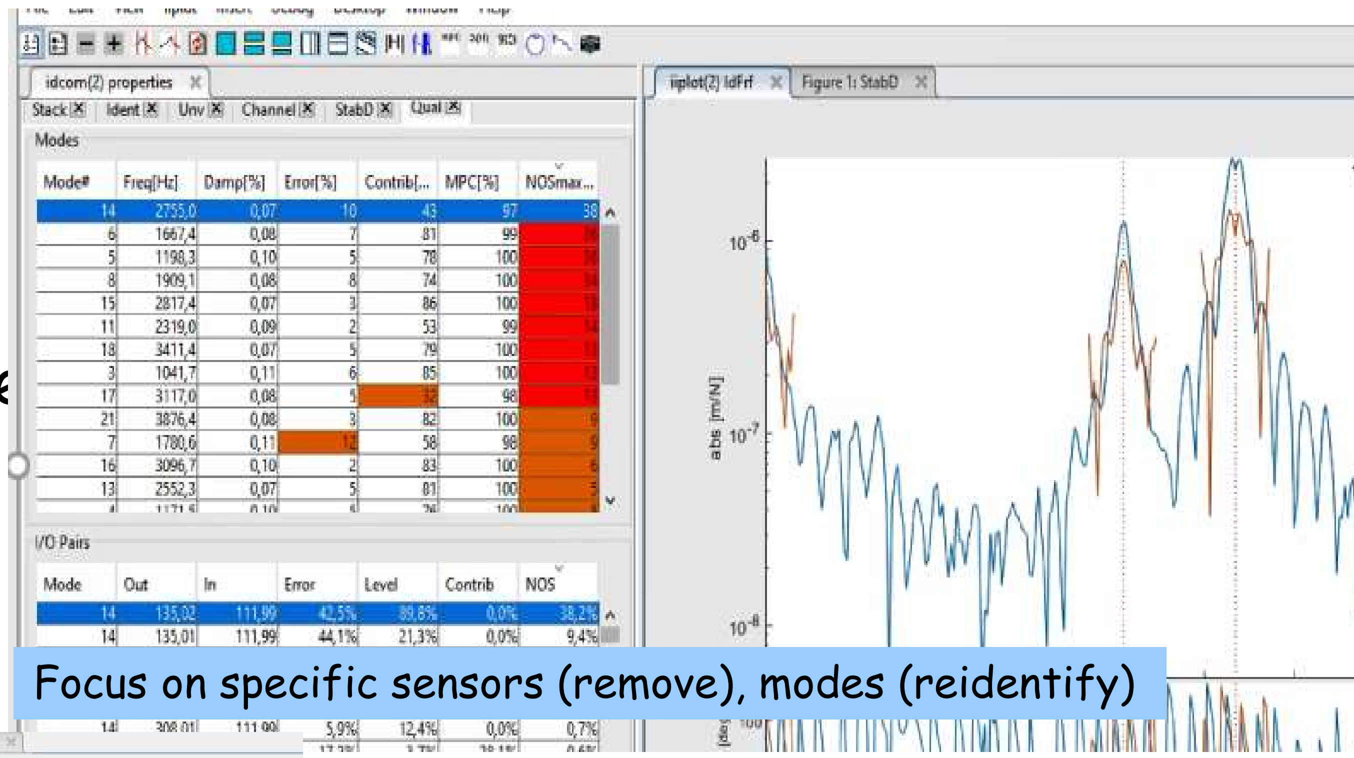
Evaluate quality of identification



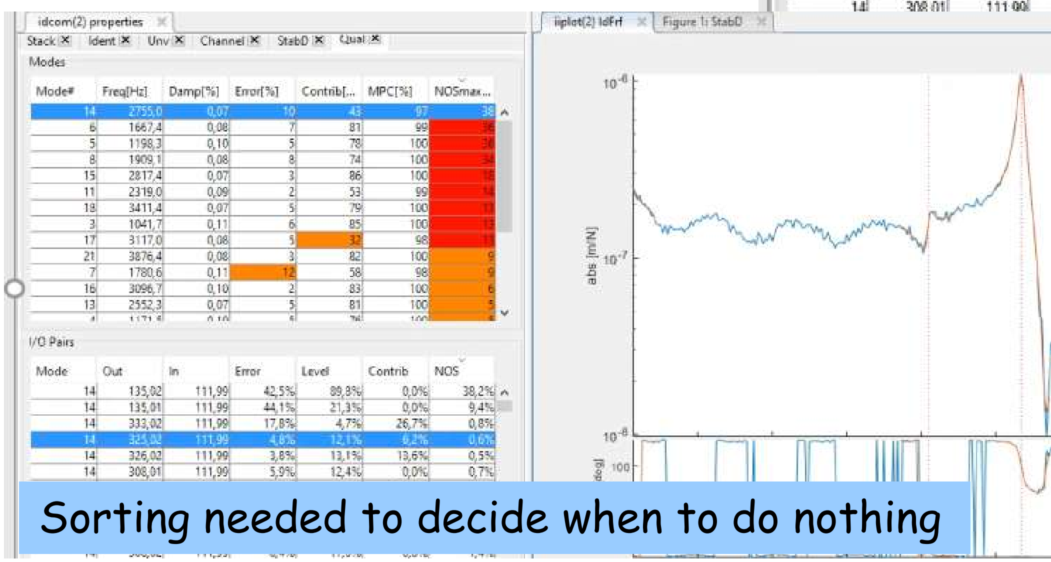
Error analysis usage



Average error per mode



Focus on specific sensors (remove), modes (reidentify)

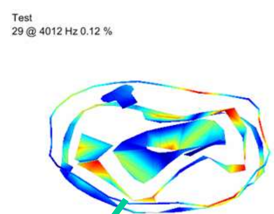


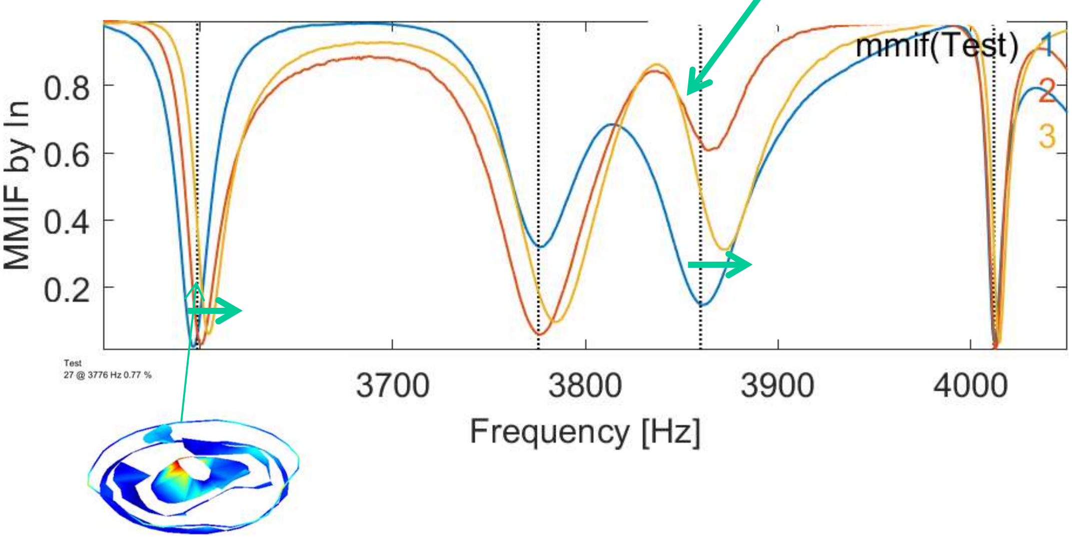
Sorting needed to decide when to do nothing

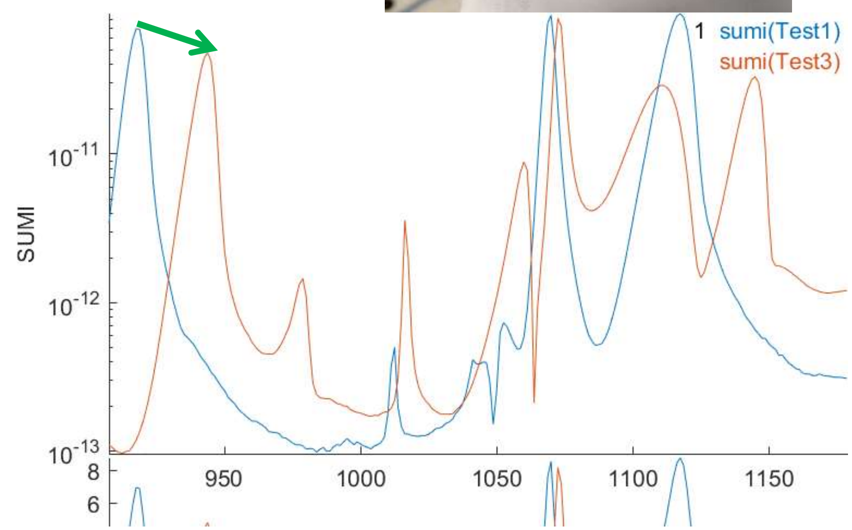
Other errors : NL/LPV ≠ one system

- Non linear system : resonance dependent on input point / accurate positioning

- MMIF or identification per impact location shows significant dispersion
- Multiple identification results are not perfectly coherent

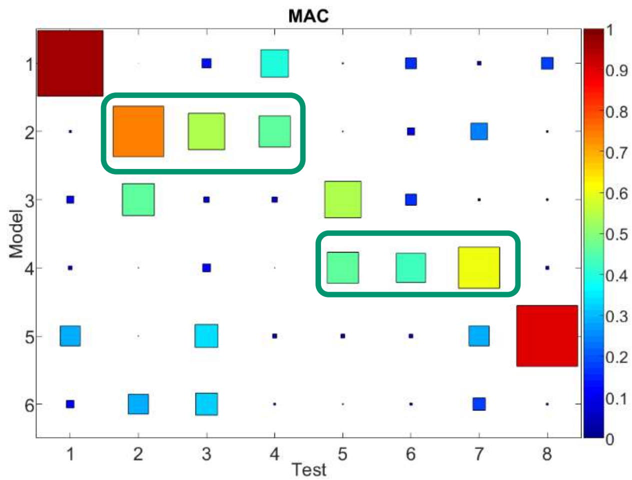




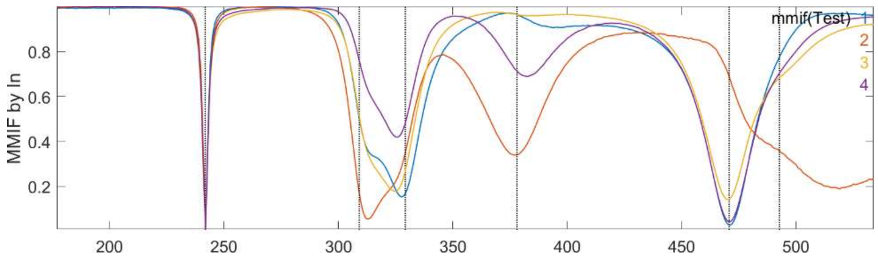


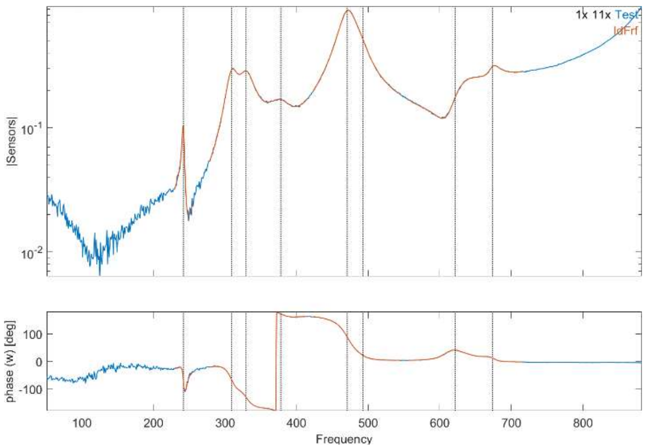
Other errors : local modes

- « Same mode » multiple times



- non structural masses generate global mode duplicates
- « small mass » ≠ « tiny peak » (if this were true proof mass dampers would not work)





Subspace methods : IO (ERA)

$$[h(k\Delta t)]_{pq} = \begin{bmatrix} [y(k)] & [y(k+j_1)] & \dots & [y(k+j_{q-1})] \\ [y(k+i_1)] & [y(k+i_1+j_1)] & \dots & [y(k+i_1+j_{q-1})] \\ \vdots & \vdots & \ddots & \vdots \\ [y(k+i_{p-1})] & [y(k+i_{p-1}+j_1)] & \dots & [y(k+i_{p-1}+j_{q-1})] \end{bmatrix}$$

$$[h(k\Delta t)]_{pq} = [C]_p \, [A]^{k-1} \, [B]_q = \begin{bmatrix} [C] \\ \vdots \\ [C]\,[A]^{j_{p-1}} \end{bmatrix} [A]^{k-1} \left[[B] \;\; \dots \;\; [A]^{j_{q-1}}\,[B] \right]$$

$$[h(k\Delta t)]_{pq} \approx [U] \left\lceil {}^{\setminus} S_{\setminus} \right\rfloor [V]^T$$

$$[A] = \left[\left\lceil {}^{\setminus} S_{\setminus} \right\rfloor^{-1/2} [U]^T \, [h(2\Delta t)]_{pq} \, [V] \left\lceil {}^{\setminus} S_{\setminus} \right\rfloor^{-1/2} \right]$$

$$[B] = \left\lceil {}^{\setminus} S_{\setminus} \right\rfloor^{1/2} [V]^T \begin{bmatrix} \left\lceil {}^{\setminus} I_{\setminus} \right\rfloor \\ [0] \\ [0] \end{bmatrix}_{qNA \times NA}$$

$$[C] = \left[\left\lceil {}^{\setminus} I_{\setminus} \right\rfloor \, [0] \, [0] \right]_{NS \times pNS} [U] \left\lceil {}^{\setminus} S_{\setminus} \right\rfloor^{1/2}$$

[1] J. N. Juang and R. S. Pappa, "An Eigensystem Realization Algorithm (ERA) for Modal Parameter Estimation and Model Reduction," *J. Guidance, Control, and Dynamics*, vol. 8, no. 5, pp. 620–627, 1985.

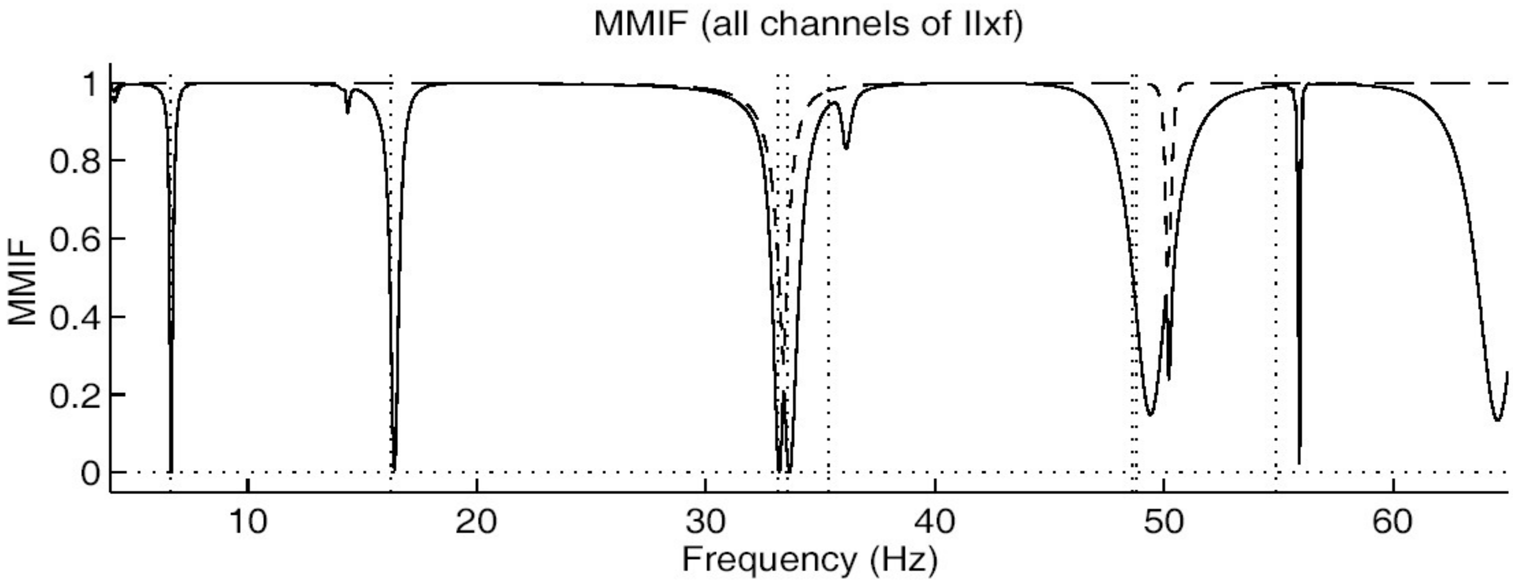
[2] E. Reynders, "System identification methods for (operational) modal analysis: review and comparison," Archives of Computational Methods in Engineering, vol. 19, no. 1, pp. 51–124, 2012.

InverseSVD

Order selection : MMIF

- Mode indicator functions
- Related to force appropriation (do by hand)
- Appropriation excites a single mode (allow picture taking)

$$MMIF_i = min_{u \perp u_{j<i}} \frac{u^T [Re(H)^T Re(H)] u}{u^T [H^H H] u}$$

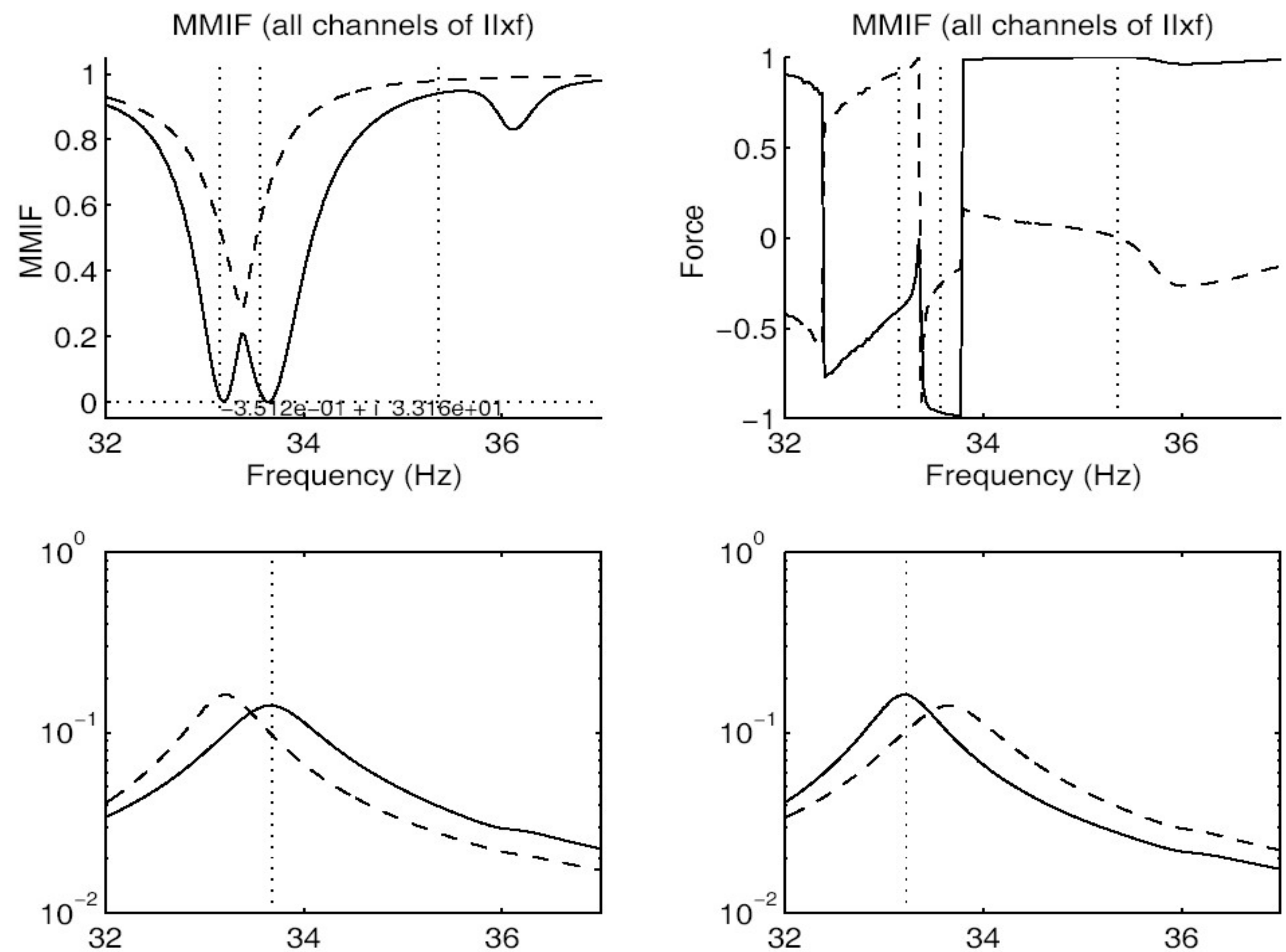


Order selection : MMIF

- Multi exciter allows spatial filter $\{u_i(t)\}_{NA} = \{w_i\}_{NS} a(t)$
- Appropriation : choose $\{w_i\}_{NS}$ to make other modes unobservable
- Keywords

  Modal coordinates

  Modal filter

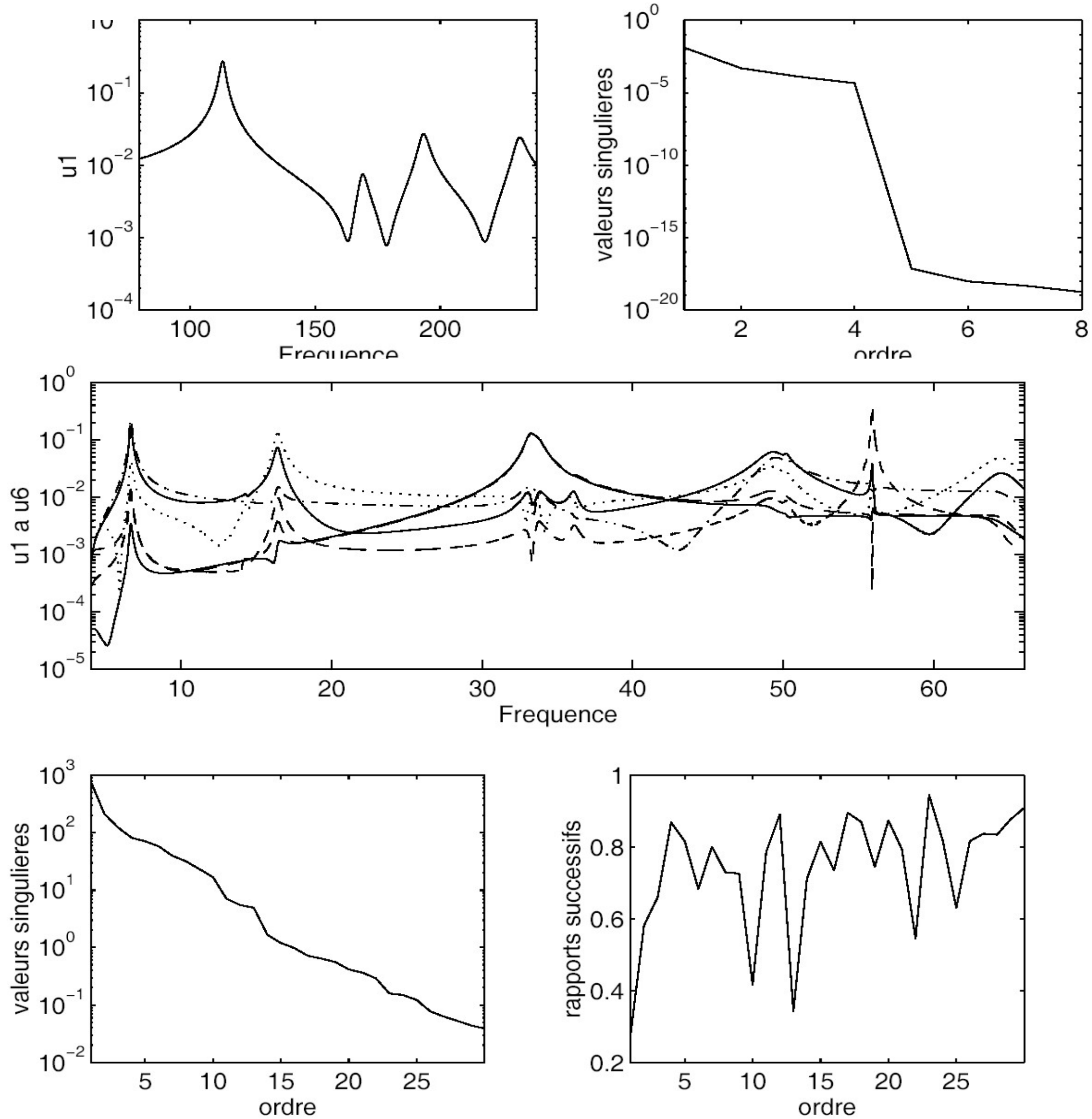


Order selection : SVD

$$[H_{kl}(\omega)]_{NW\times N_{kl}} = \left[\frac{1}{\omega_{1:NW}-\lambda_{1:NM}}\right]_{NW\times NM}[R_{j,kl}]_{NM\times N_{kl}}$$

- Rank of H is limited
- Perfect for simulation
- Problem for real data
- More useful for data volume reduction

P. Guillaume & al. 2005 Improved techniques for outlier detection



Properties of residues

- Minimal

$$\frac{R_j}{s-\lambda_j} = \frac{\{c\psi_j\}\left\{\psi_j^T b\right\} + \{c\psi_{j+1}\}\left\{\psi_{j+1}^T b\right\}}{s-\lambda_j}$$

- Reciprocal

$$\left([c_{\texttt{col}}]\{\psi_j\}\right)^T = \{\psi_j\}^T [b_{\texttt{col}}]$$

- Second order model (modal damping)

$$\frac{[R_j]}{s-\lambda_j} + \frac{\left[\bar{R}_j\right]}{s-\bar{\lambda}_j} = \frac{2(s+\zeta_j\omega_j)Re(R_j) + 2\omega_j\sqrt{1-\zeta_j^2}Im(R_j)}{s^2+2\zeta_j\omega_j s+\omega_j^2}$$

$$[R_j] = \frac{T_j}{i\omega_j\sqrt{1-\zeta_j^2}} = [c]\,\frac{\phi_j}{\sqrt{i\omega_j\sqrt{1-\zeta_j^2}}}\frac{\phi_j^T}{\sqrt{i\omega_j\sqrt{1-\zeta_j^2}}}\,[b]$$

- Modal mass (reciprocal, second order)

$$\frac{1}{\mu_j(c_{col})} = c_{col}\phi_j\phi_j^T b_{col} = (c_{col}\phi_j)^2 = T_{jcol} = i\omega_j\sqrt{1-\zeta_j^2}R_{jcol}$$

- Second order non proportional damping