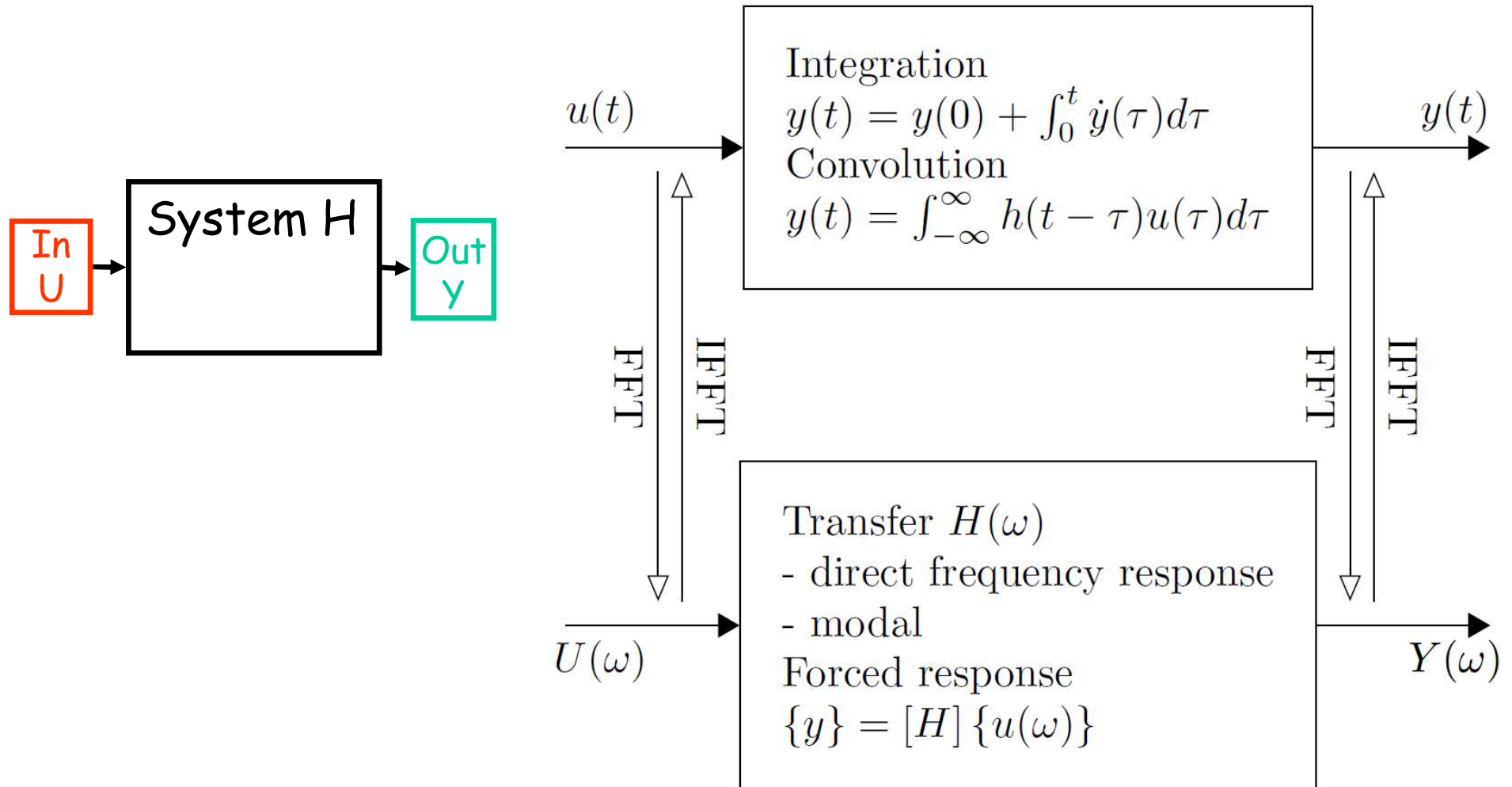


- Intro
- Signal processing basics
- FRF estimation
- Sensor/shaker technology

Course notes : chapter 4 : time and frequency domains, signal processing basics

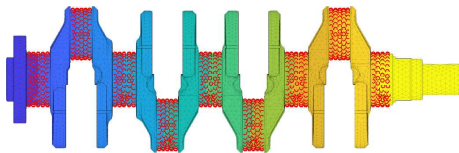
Why do we need frequency responses



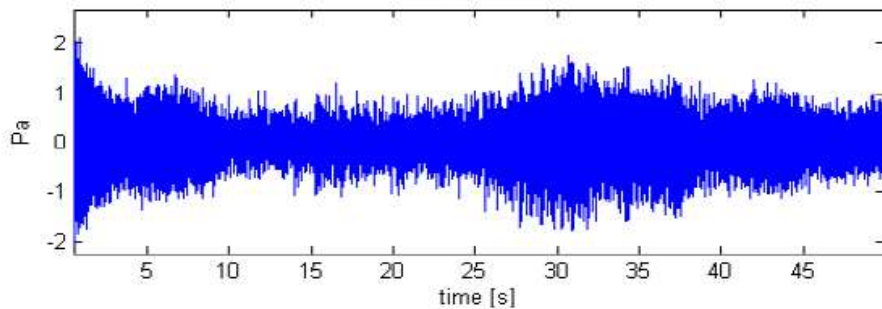
- Scenario 1 : run-up
- Scenario 2 : modal testing

Frequencies in a run-up

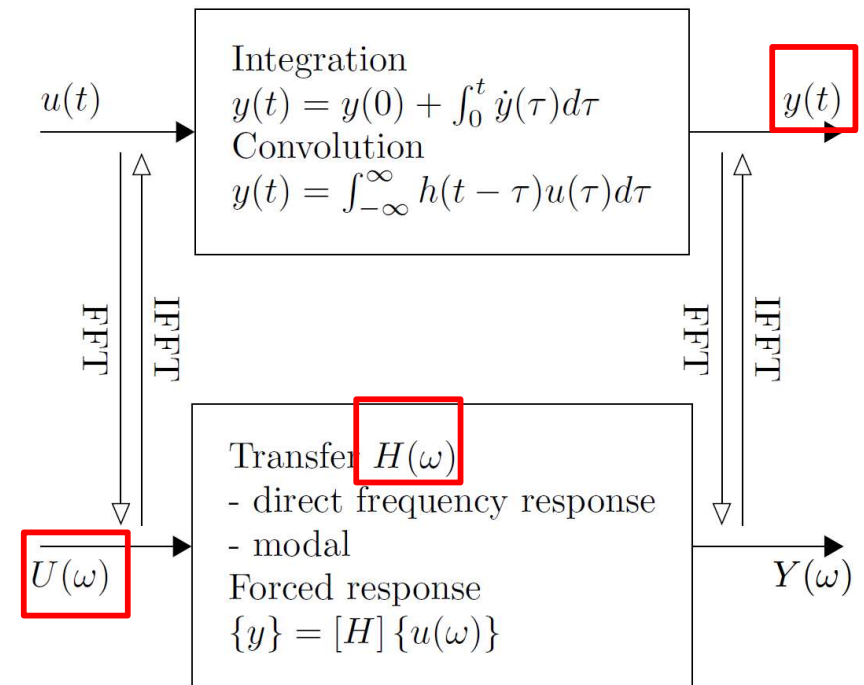
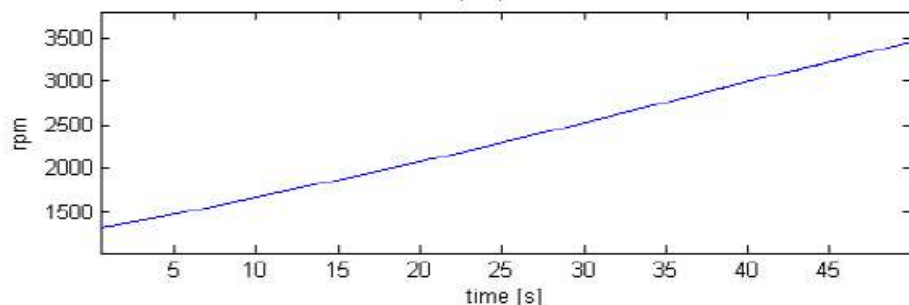
- Periodic excitation :
signal with harmonics $u(t) = \sum_n u_n(n\omega t)$
- Constant system $H(\omega)$
or time varying & NL
- Output contains
both harmonic & constant contributions



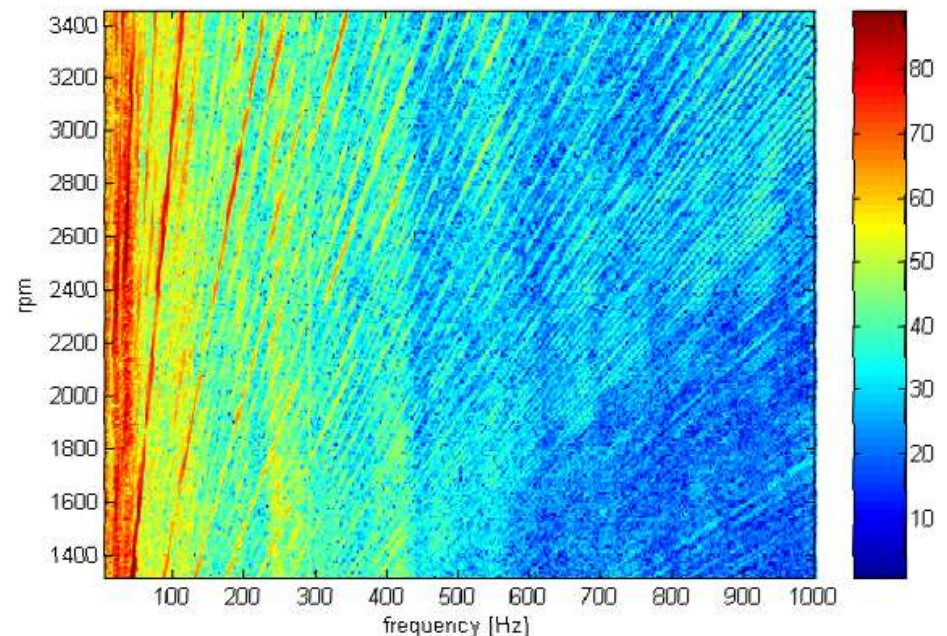
interior sound



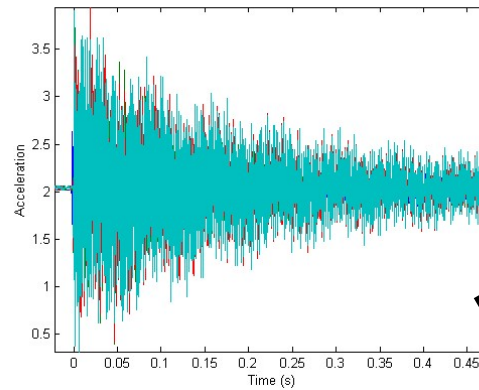
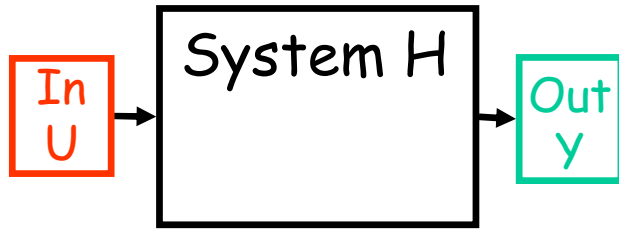
rpm profile



waterfall spectrum



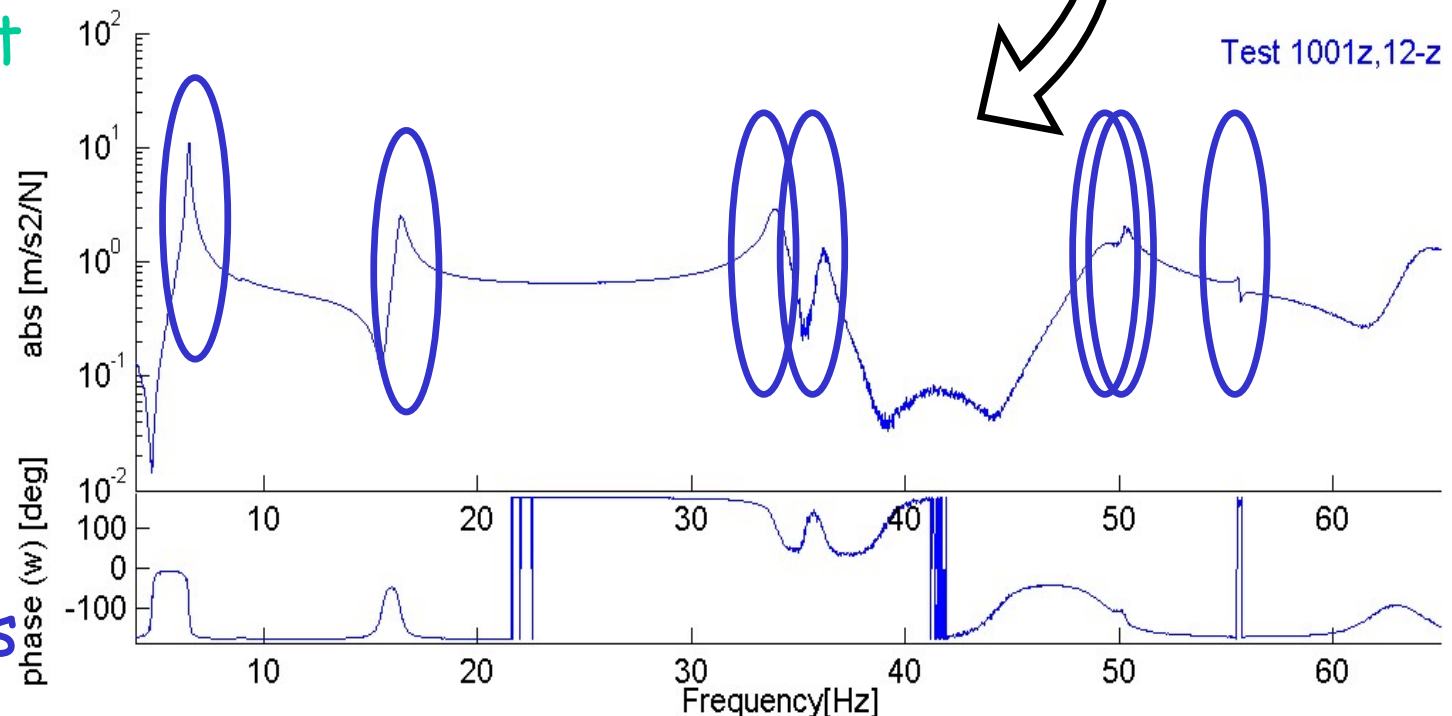
Modal analysis : transfers



Transfers estimated from time response

ONE input
ONE output

$$\{Y(\omega)\} = [H(\omega)]\{U(\omega)\}$$



MANY resonances

Bode plot : visualization of transfer function

Contents

- Signal processing basics
 - Continuous & discrete for different usage
 - Base problems : aliasing, leakage
- FRF estimation
- Sensor/shaker technology

Fourier transform : discrete & continuous

- Continuous $Y(\omega) = \int_{-\infty}^{+\infty} y(t)e^{-j\omega t} dt$ and $y(t) = \int_{-\infty}^{+\infty} Y(\omega)e^{j\omega t} d\omega$
- Periodic in time : discrete in frequency (series, example harmonic balance methods)

$$Y(k\Delta f) = \frac{1}{T} \int_0^T y(t)e^{-j2\pi\Delta f t} dt \quad \text{with} \quad y(t) = \sum_{k=-\infty}^{+\infty} Y(k\Delta f)e^{j2\pi k\Delta f t}$$

- Discrete in time : periodic in frequency (sample application periodic FEM)

$$Y(f) = \sum_{n=-\infty}^{+\infty} y(n\Delta t)e^{-j\omega n\Delta t} \quad \text{and} \quad y(n\Delta t) = \frac{1}{\omega_s} \int_{-\omega_s/2}^{\omega_s/2} Y(\omega)e^{j\omega n\Delta t} d\omega$$

- Finite length discrete time : finite frequency (measurements)

$$Y(k\Delta f) = \frac{1}{N} \sum_{n=0}^{N-1} y(n\Delta t)e^{j2\pi nk/N} \quad \text{and} \quad y(n\Delta t) = \sum_{k=0}^{N-1} Y(k\Delta f)e^{j2\pi nk/N}$$

Time and frequency

- Discrete transform only depends values
 $(e^{\frac{j\pi nk}{N}}$ regular on unit circle)

Time and frequencies are

- regularly spaced
- found by

$$\begin{aligned} k &= [1:N] \\ t_k &= [0:N-1]\Delta t \\ &= ([0:N-1]/N)T \\ f_k &= [0:N-1]/(N\Delta t) \\ &= [0:N-1]/(T) \end{aligned}$$

Impose	Consequence	Influence de N
Δt	$F_{Max} = \frac{1}{2\Delta t}$	$T = N\Delta t$ $\Delta f = \frac{1}{N\Delta t}$
F_{Max}	$\Delta t = \frac{1}{2F_{Max}}$	$T = N\Delta t$ $\Delta f = \frac{1}{N\Delta t}$
Δf	$T = \frac{1}{\Delta f}$	$\Delta t = \frac{T}{N}$ $F_{Max} = \frac{N}{2} \Delta f$
T	$\Delta f = \frac{1}{T}$	$\Delta t = \frac{T}{N}$ $F_{Max} = \frac{N}{2} \Delta f$

DFT Properties

Linearity

$$Y_1 + Y_2 = \mathcal{F}(y_1 + y_2) \text{ and } \alpha Y_1 = \mathcal{F}(\alpha y_1)$$

Base time change

$$\mathcal{F}(y(at)) = Y(f/a)/|a|$$

Time delay

$$\mathcal{F}(y(t - t_0)) = Y(f)e^{-j2\pi ft_0}$$

Time derivative

$$\mathcal{F}(\dot{y}) = i\omega \mathcal{F}(y)$$

Convolution

$$A(\omega)B(\omega) = \mathcal{F}\left[\int_{-\infty}^{+\infty} a(\tau)b(t - \tau)\right]$$

Energy (Parseval)

$$E = \int_{-\infty}^{+\infty} |g(t)|^2 dt = \int_{-\infty}^{+\infty} |G(f)|^2 df$$

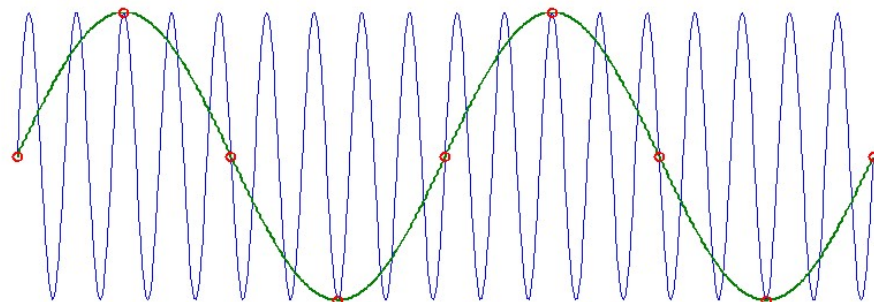
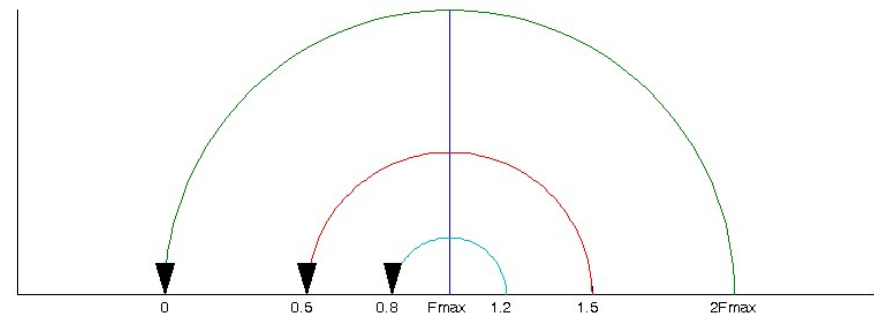
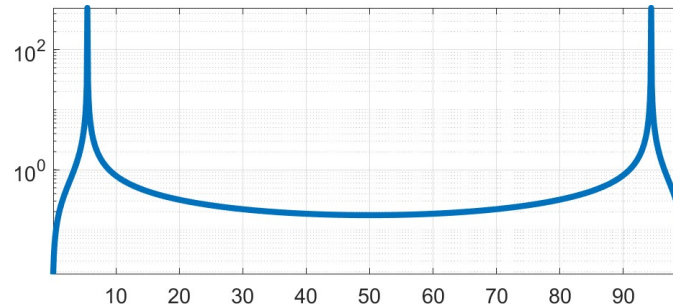
Continuous vs. DFT problem 1 : **aliasing**

Cannot distinguish first
and second half of
spectrum

Shannon : The signal
should not have
content above

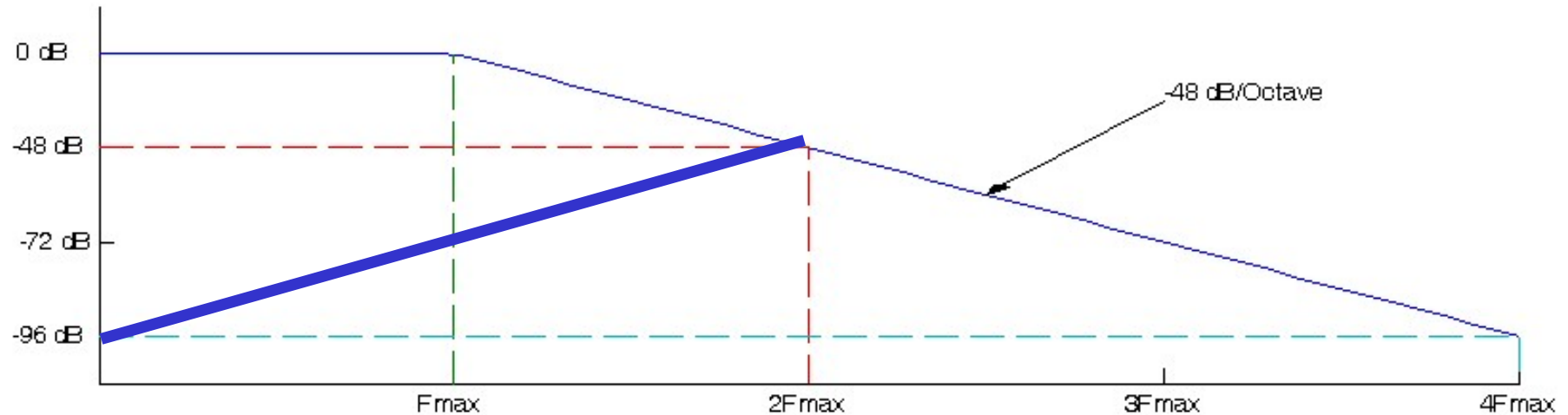
$$F_{\max} = F_{\text{sample}}/2$$

```
t=linspace(0,10,1000);  
u=sin(5.5*2*pi*t); U=fft(u); U(1)=0;  
f=(0:length(t)-1)/diff(t(1:2))/length(t);  
h=semilogy(f,abs(U));axis('tight');
```



Application : lab 1DOF

Anti-aliasing filters



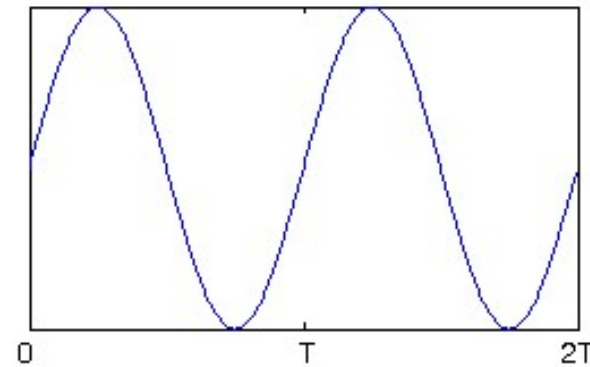
Objective : **aliased signal** reduced by resolution at $F_{\max} = F_{\text{sample}} / (2 + \alpha)$

Example

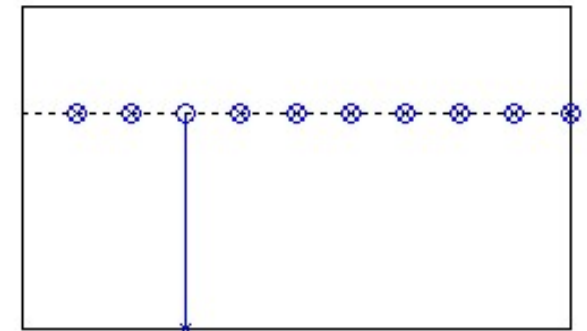
- resolution 12 bits = 72 dB
- Filter -48 dB/octave
- Reduction $F_{\max} = F_{\text{sample}} / 4$

Modern analyzers mix analog and digital anti-aliasing

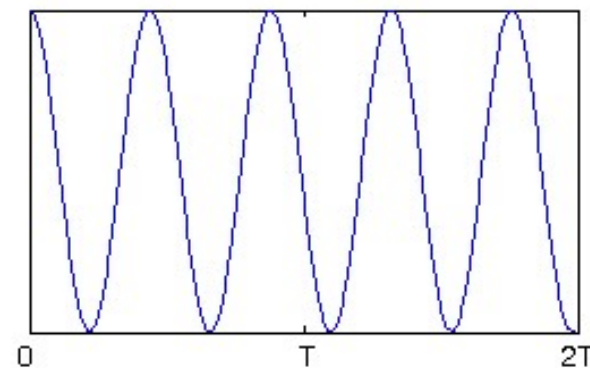
Simple signals



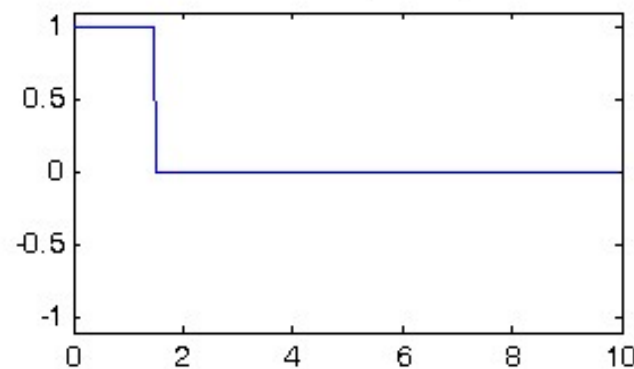
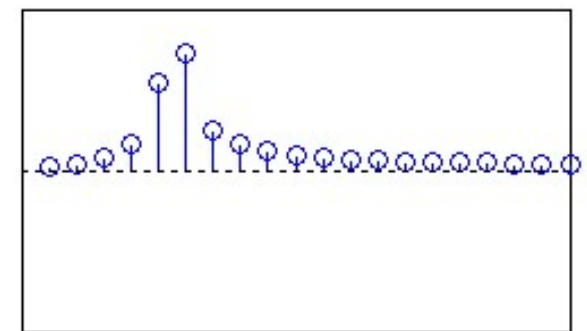
sinus periodique



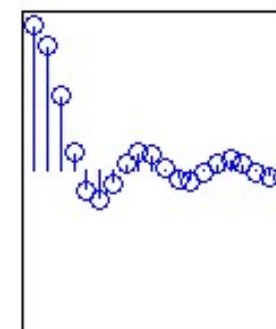
o reel, x imaginaire



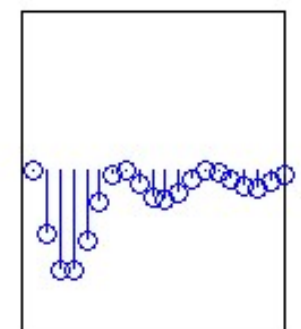
cosinus non-periodique



signal rectangulaire



Reel



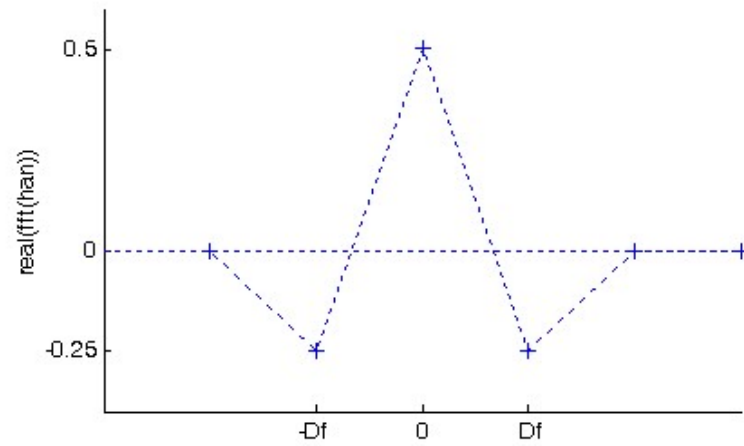
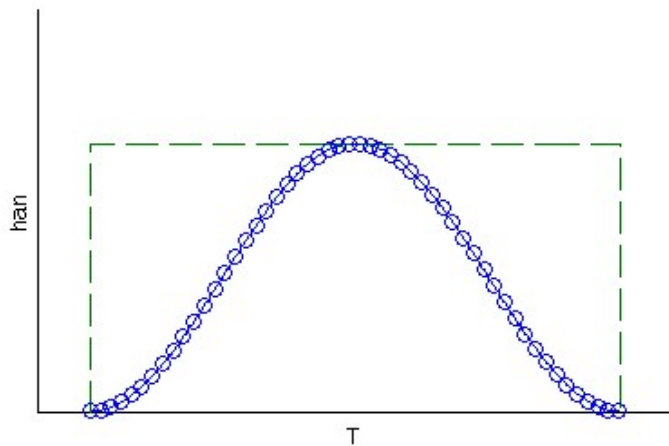
Imag

Continuous vs. DFT problem 2 : leakage

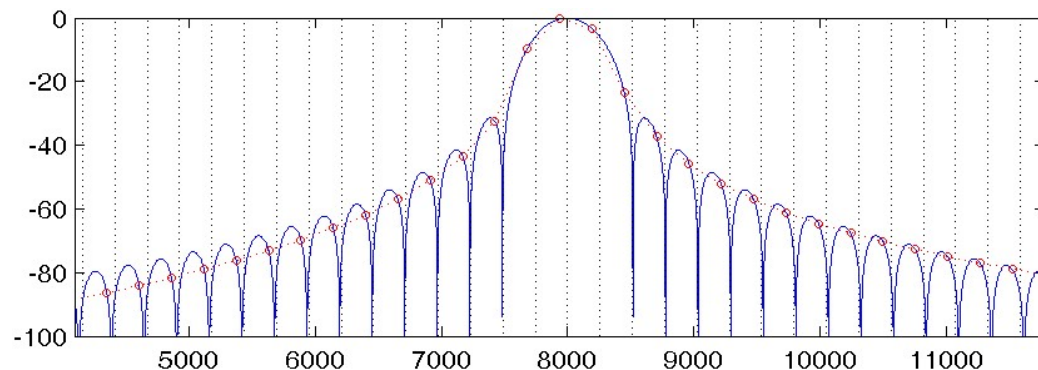
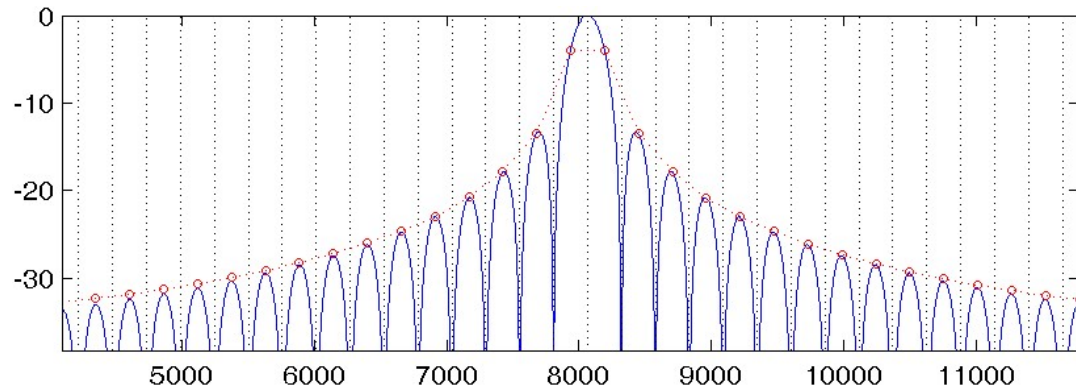
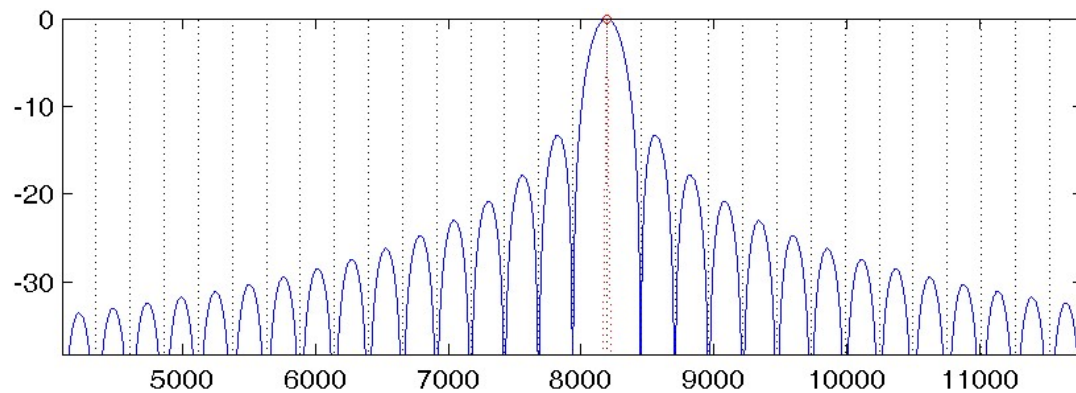
DFT = continuous transform of windowed signal

$$y_{Test}(t) = y(t)w(t) \iff y_{Test}(\omega) = \int_{-\infty}^{+\infty} w(k)y(\omega - k)$$

This is equivalent to a weighted averaging by $w(k)$ in the frequency domain



Leakage (df offset)



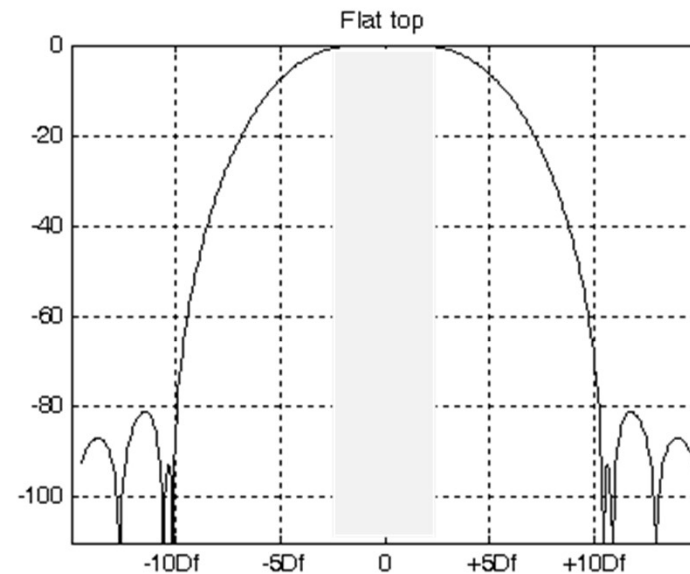
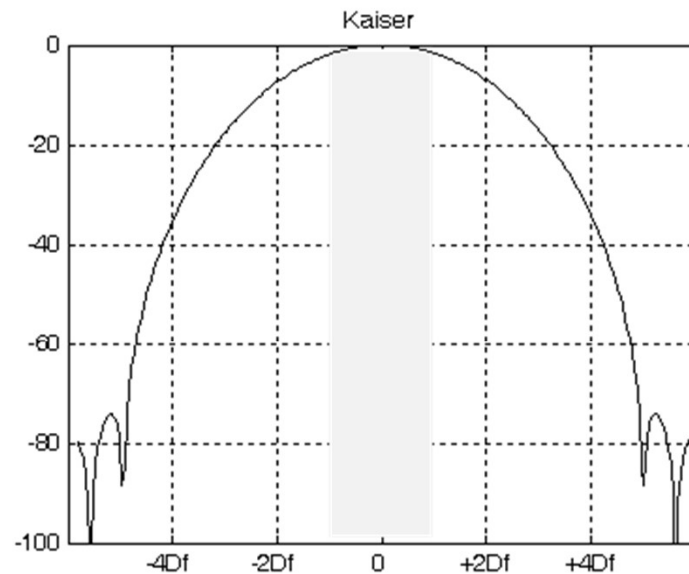
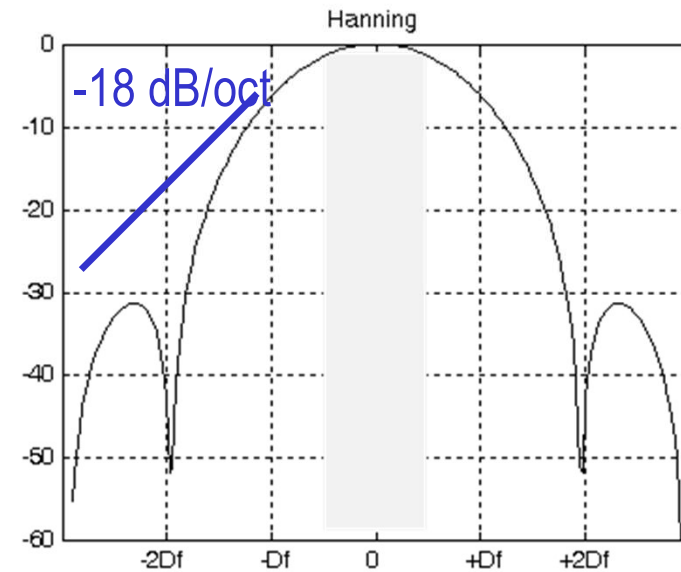
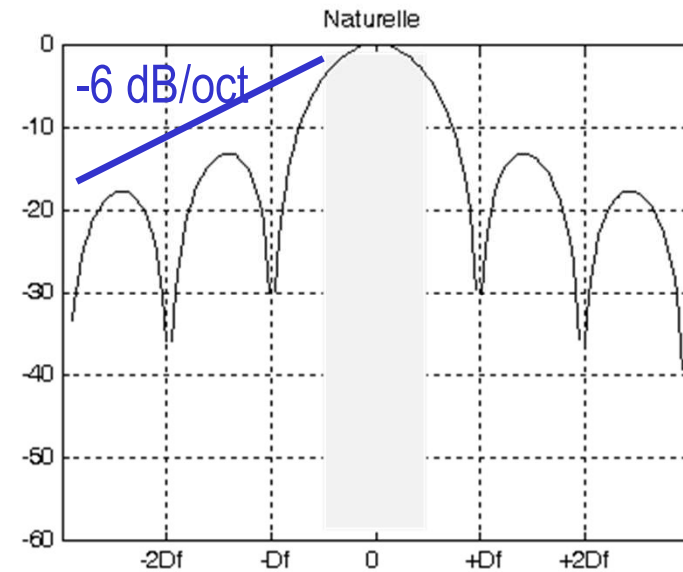
$$y(n\delta\omega) = \int_{-\infty}^{\infty} w(k) Y_{CFT}(\omega - k) dk$$

For sine

$$y(n\delta\omega) = w(n\delta\omega + \omega_{in})$$

- Natural matching freq :
no leakage
- Natural $\Delta f/2$:
maximum error
- Hanning $\Delta f/4$
spread close but
decrease faster

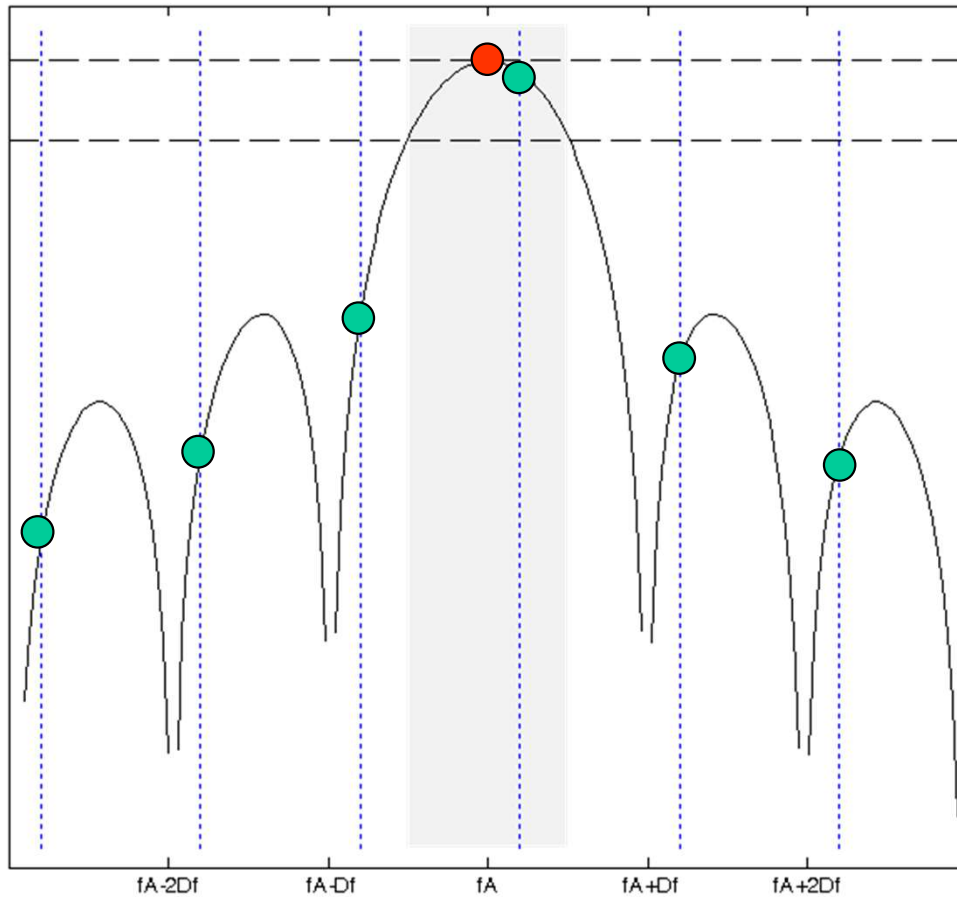
Time window properties



[1] K. G. McConnell, *Vibration Testing. Theory and Practice*. Wiley Interscience, New-York, 1995.

[2] W. Heylen et P. Sas, *Modal analysis theory and testing*. KUL, 2006

Leakage

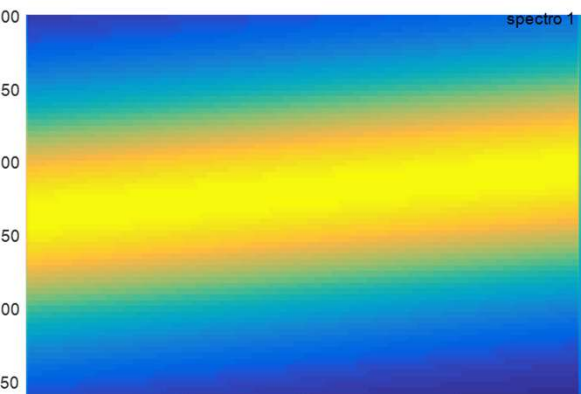


Non coincident
Sine freq.
● DFT values

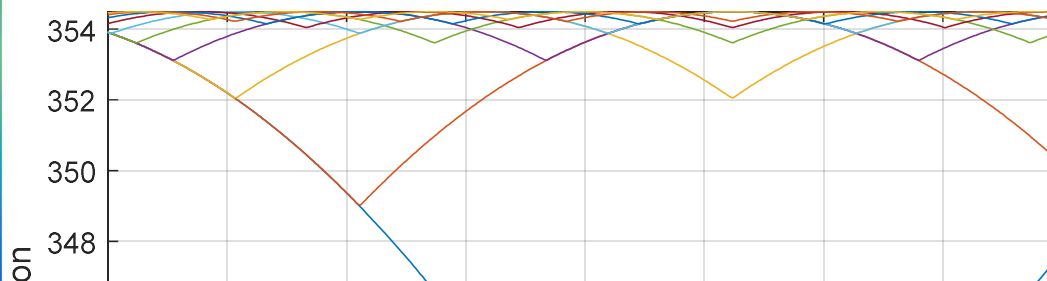
Coincident sine
● DFT values

Leakage =

- energy at other f
- error on frequency
- error on amplitude



Below example with sine sweep



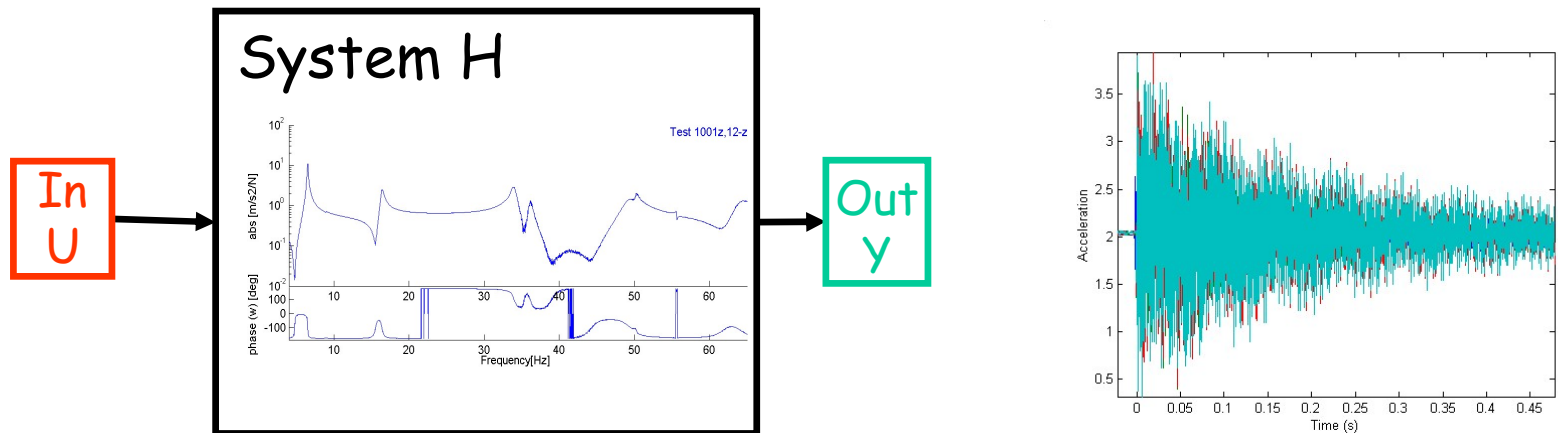
Time windows

$$w(t) = \left(\sum_{j=0,2} a_j \cos\left(\frac{2\pi t}{N\Delta t} j\right) \right) \left(\sum_{k=1,N} \delta(t - k\Delta t) \right)$$

Type	a0	a1	a2	a3	
Rectangular	1	0	0	0	Periodic signals
Hanning	0.5	-0.5	0	0	Continuous random signals
Hamming	0.54	-0.46	0	0	
Flat Top	0.281	-0.521	0.198	0	Low amplitude error (calibration)
Kaiser-Bessel	1	-1.298	0.244	0.003	Separate close components
Exponential					Transients of length $> T$

Contents

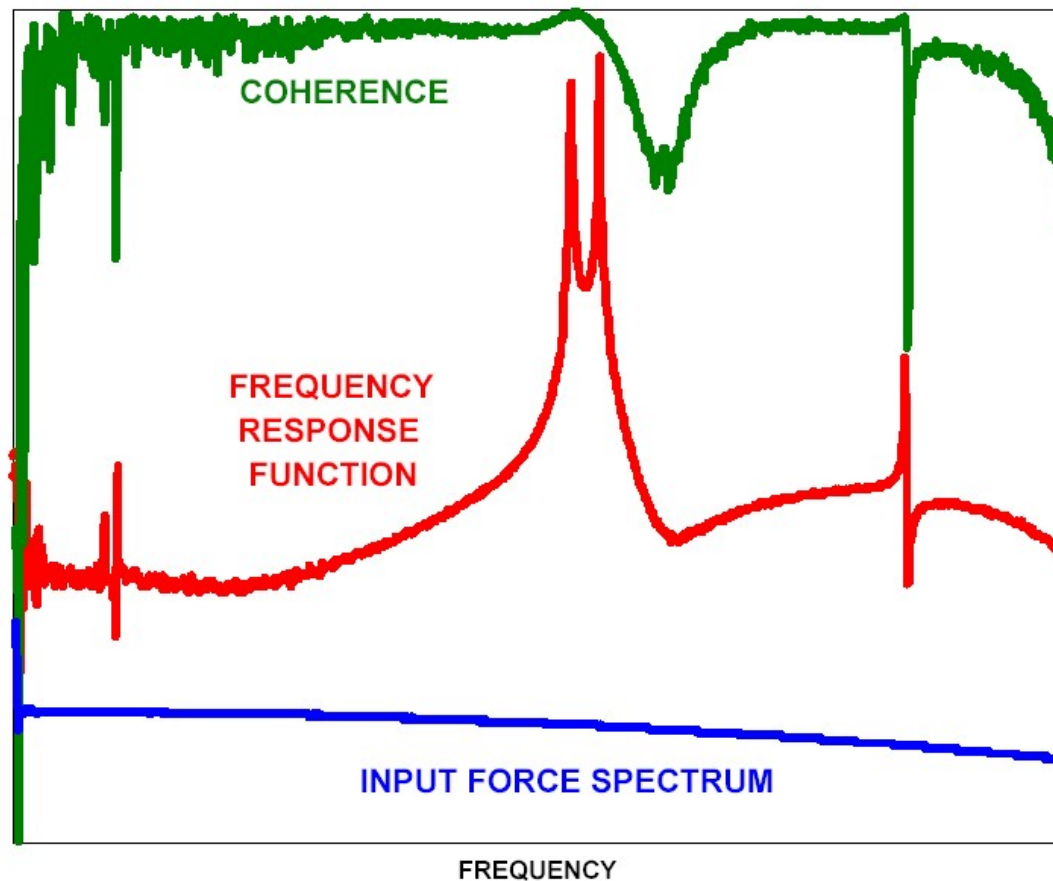
- Signal processing basics
- FRF estimation



- Sensor/shaker technology

Coherence

$$\left[\hat{H}_1(f) \right] = \frac{\sum_{n=1}^N y_n(f)^H u_n(f)}{\sum_{n=1}^N u_n(f)^H u_n(f)} = \frac{\sum_{n=1}^N (G_{yu})_n}{\sum_{n=1}^N (G_{uu})_n} = \frac{\hat{G}_{yu}}{\hat{G}_{uu}}$$



$$\gamma^2 = |\hat{G}_{uy}|^2 / (\hat{G}_{uu} \hat{G}_{yy})$$

[1] <http://macl.caeds.eng.uml.edu/umlspace/apr03.pdf>

[2] <https://www.uml.edu/Research/SDASL/Education/Modal-Space.aspx>

[3] W. Heylen, & al., Modal Analysis Theory and Testing. KUL Press, Leuven, 1997.

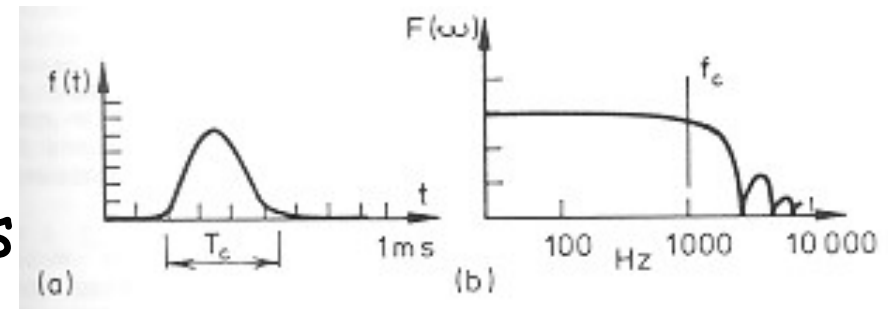
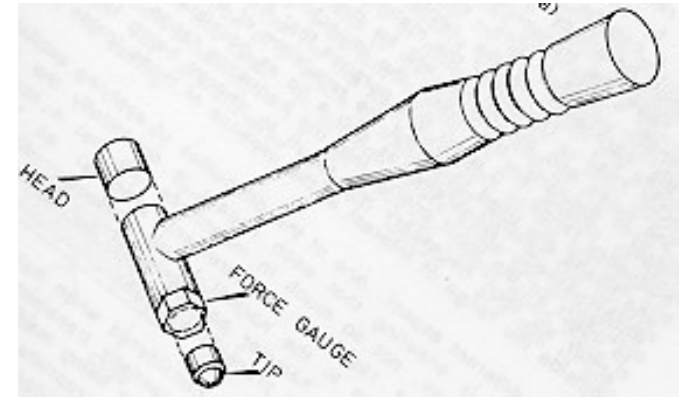
Data acquisition and processing

Contents

- Sensor/shaker technology
- Analyzer
- Signal processing basics
- FRF estimation

Hammer

- + Easy to use
- Master single impact
- Low energy level
- Trigger problems
- Triboelectric problems in cables
- Max 10 kHz



Related : cord cutting, pyrotechnic

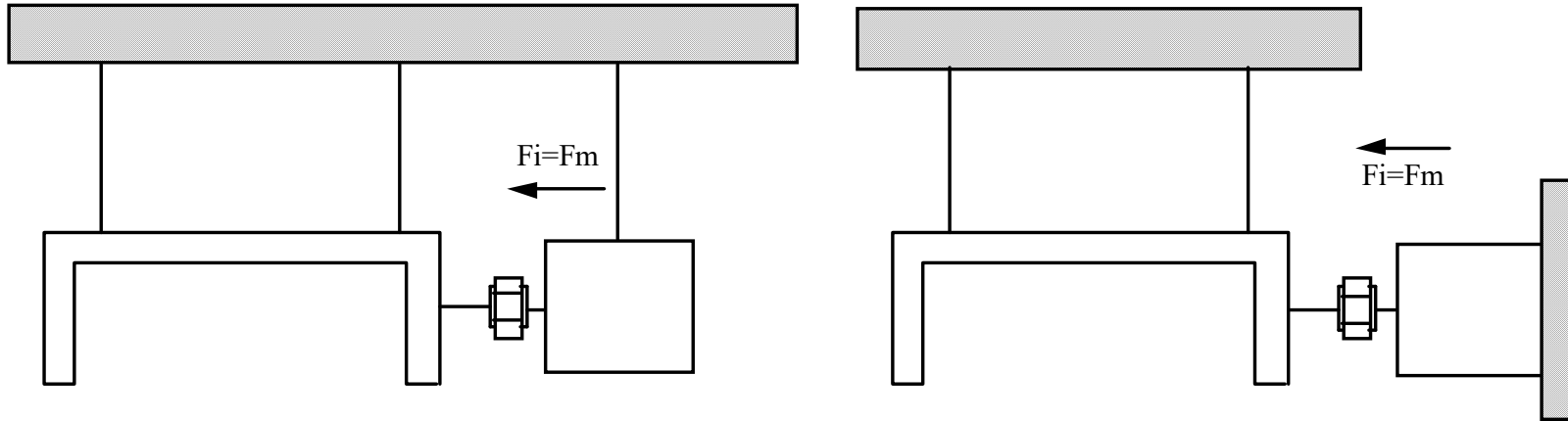
Shakers

- + High energy level
- + Sustained excitation
- + Repeatable input signal
- Need attachement (modifies response)
- Enforcing acceleration is difficult

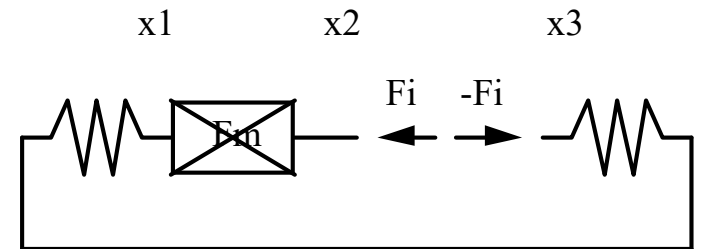
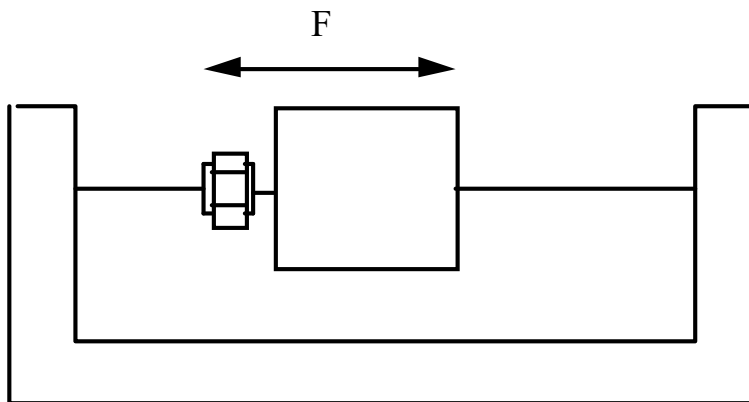
Related : piezo inertial stack & patch



Shakers setup

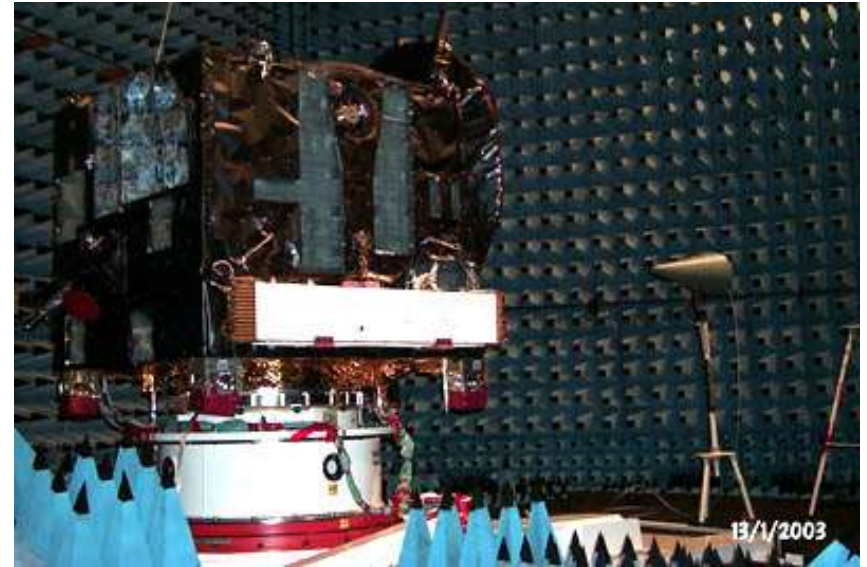
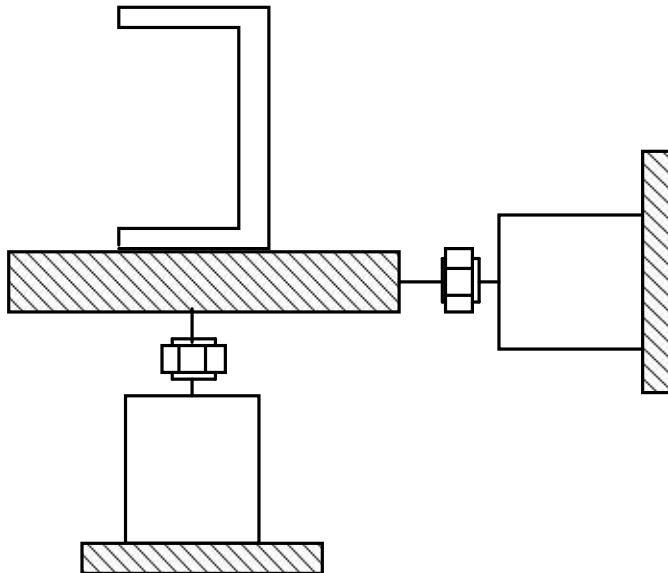


F measured $\Rightarrow$ Shaker boundary conditions indifferent



Internal force application $\Rightarrow$ improper test

Shaker table



$$\begin{bmatrix} K_{ii}(s) & K_{ic}(s) \\ K_{ci}(s) & K_{cc}(s) \end{bmatrix} \begin{Bmatrix} \langle q_i \rangle \\ q_c \end{Bmatrix} = \begin{Bmatrix} f_i \\ 0 \end{Bmatrix}$$

$$y = c_i q_i + c_c K_{cc}^{-1} [-K_{ci} q_i] = c_i q_i + c_c \left[\sum_j \frac{\phi_{cj} \phi_{cj}^T}{s^2 + 2s\zeta\omega_{cj} + \omega_{cj}^2} \right] [-s^2 M_{ci} q_i]$$

Accelerometers

Piezoelectric: measure deformation which is linked to acceleration of seismic mass

Capacitive: measure change in gap

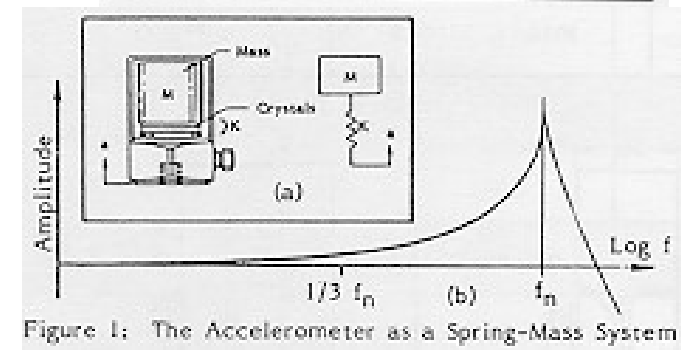
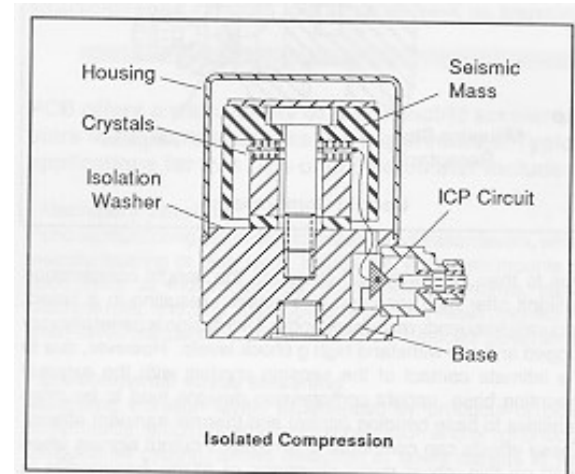
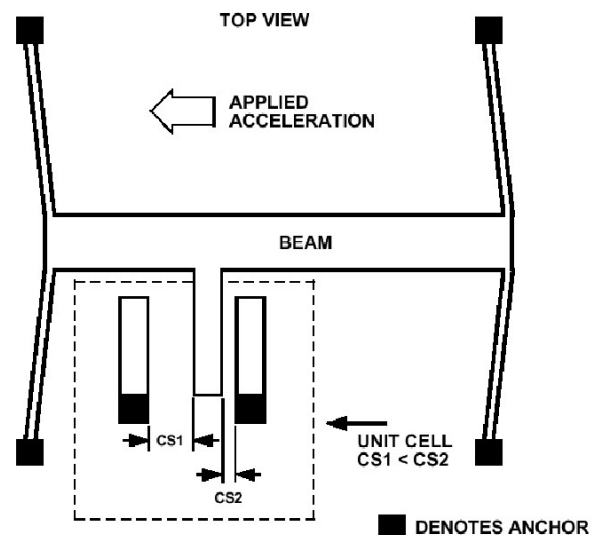
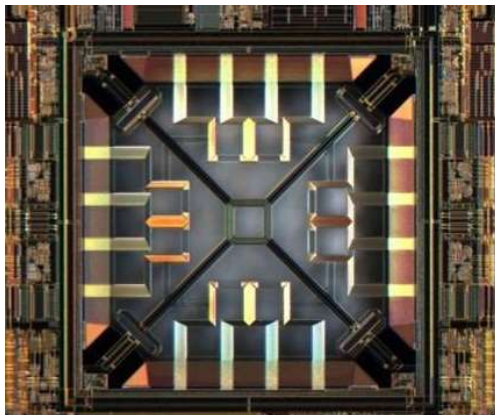


Figure 1: The Accelerometer as a Spring-Mass System

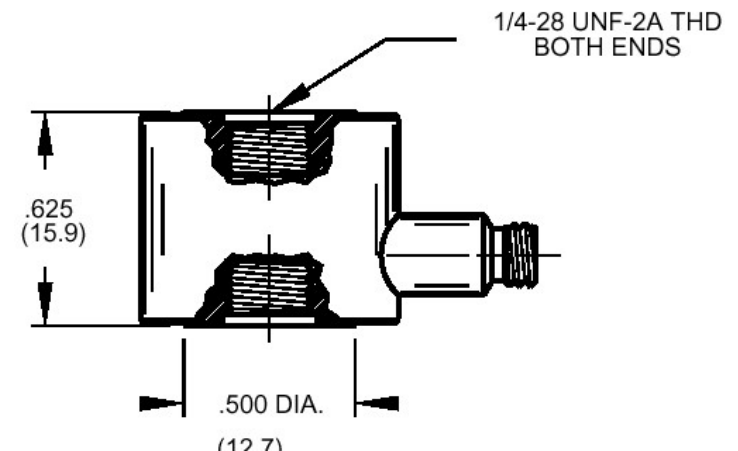
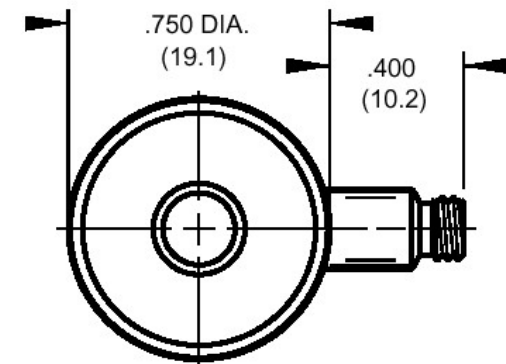
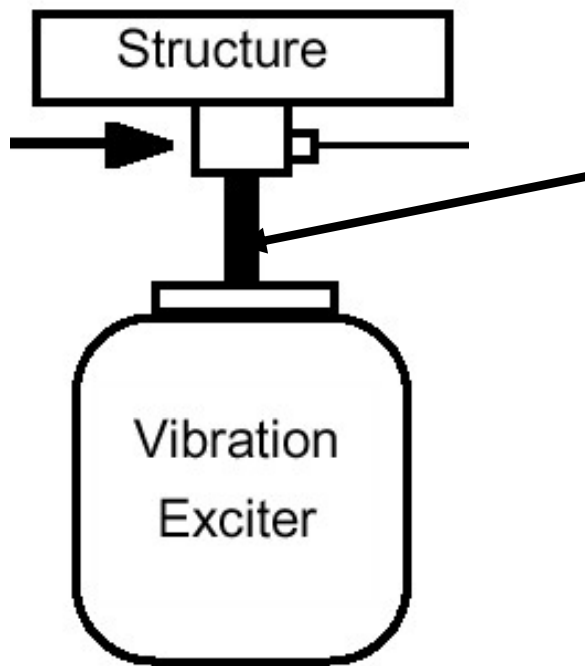


Load cells



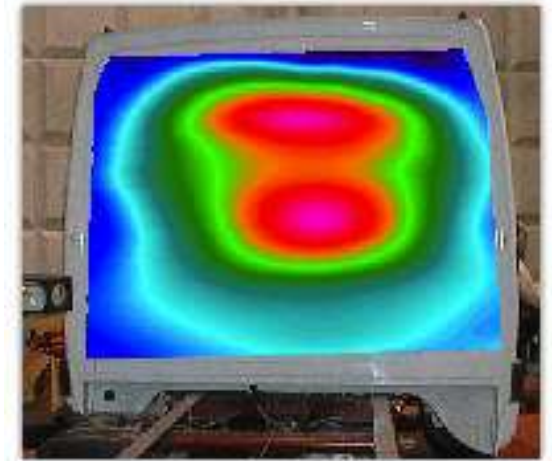
Actual size

- Measure deformation to estimate force
- Rod to minimize moment transmission



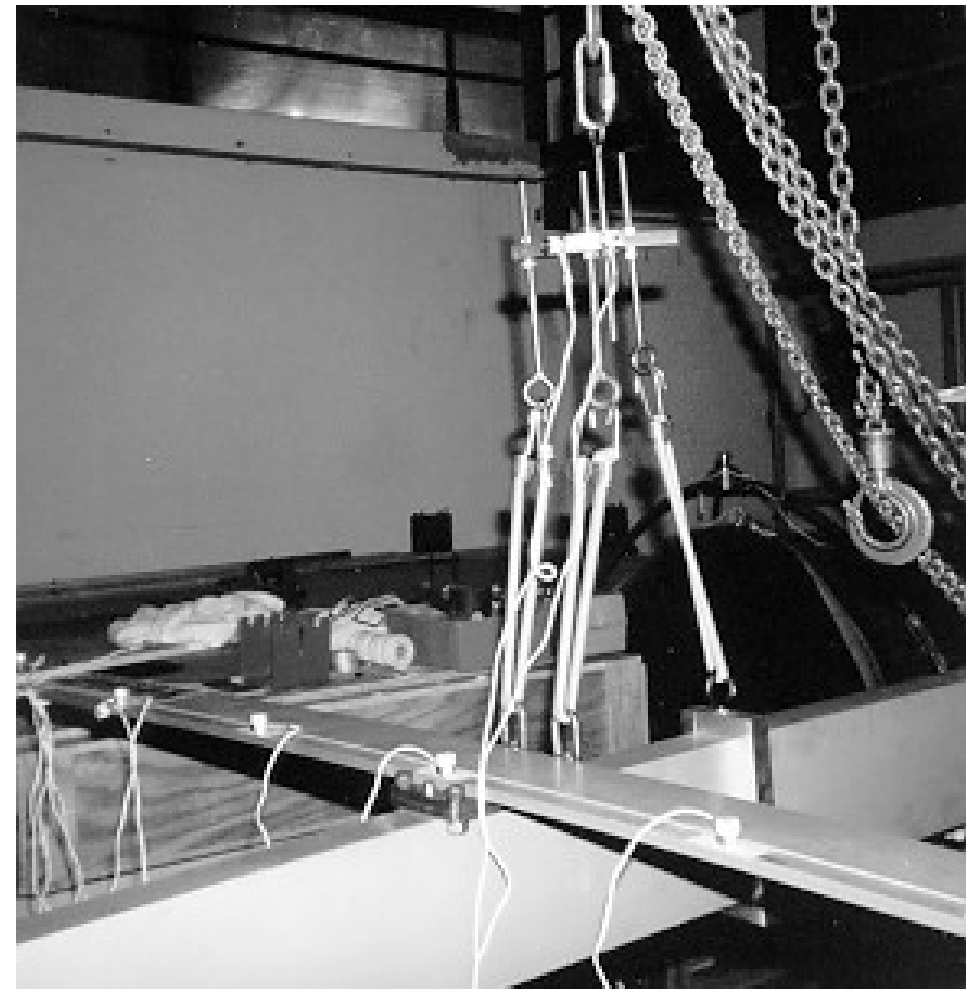
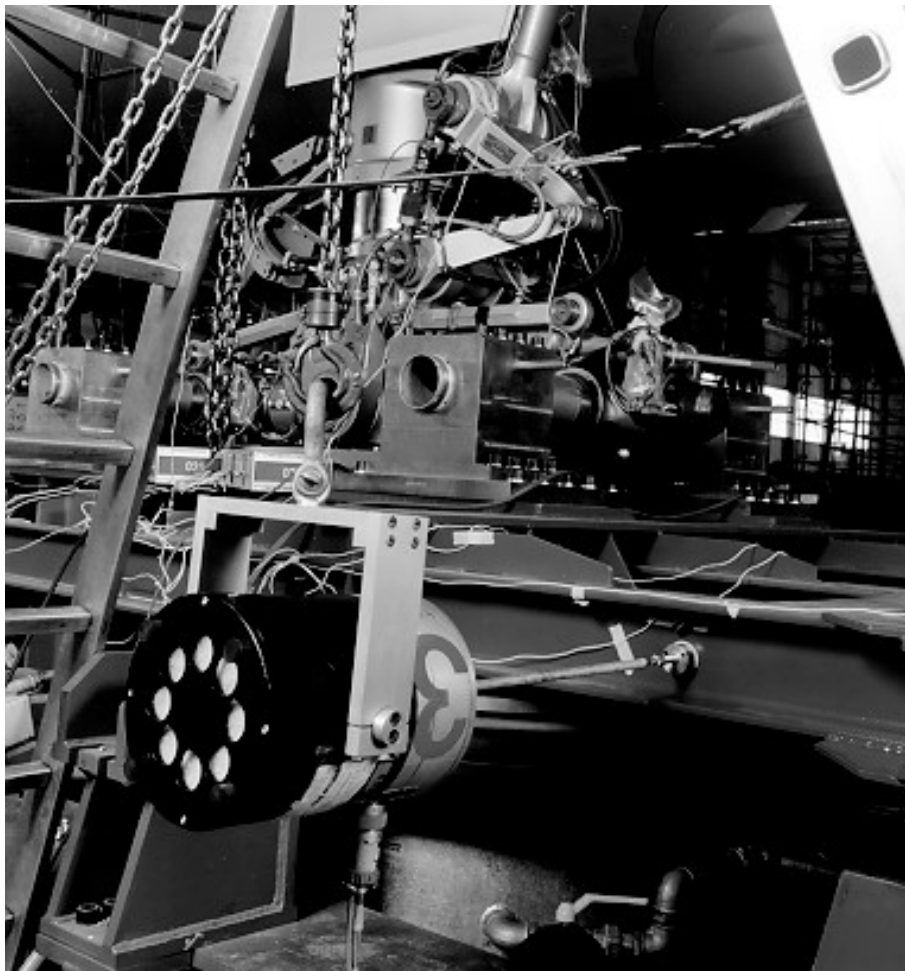
Optical techniques

- Laser Doppler Vibrometers
high amplitude resolution, point scanning
- Images (high number of simultaneous points, smaller amplitude resolution)
 - ESPI (Electronic Speckle Pattern Interferometer)
 - Image correlation white light



Boundary conditions

Suspended, fixed ...

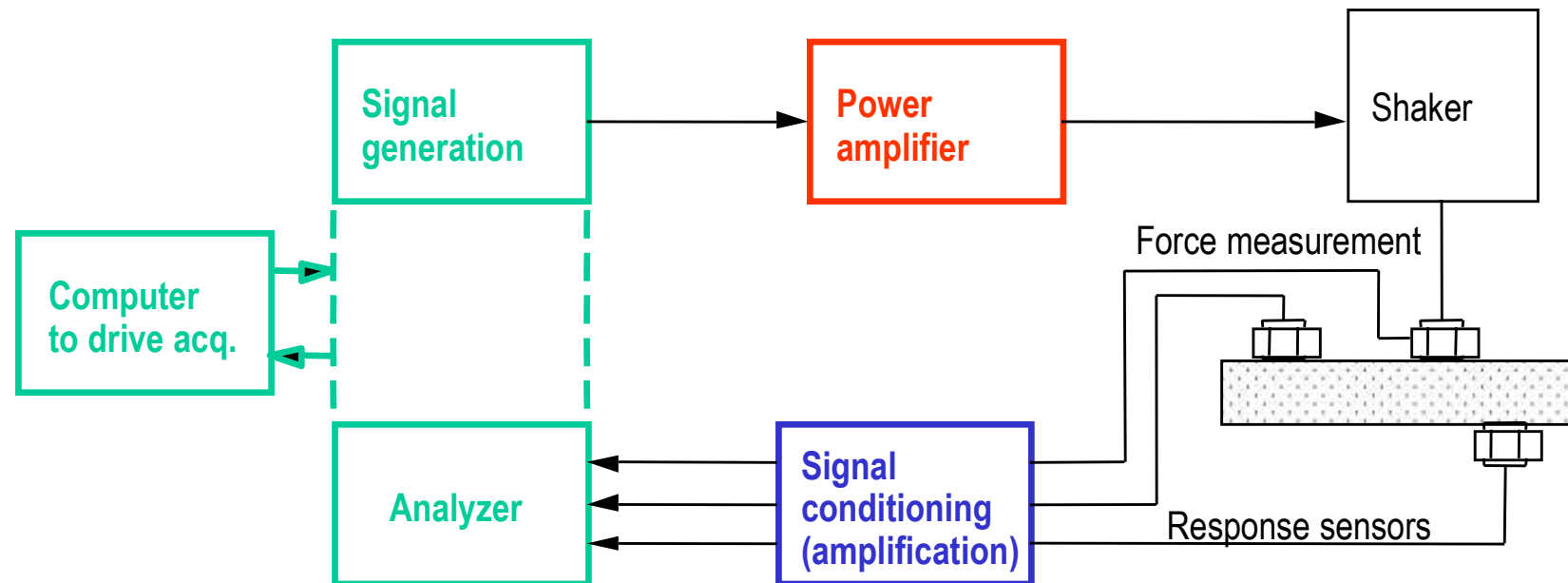


Signal conditioning

Analyzers measure/generate voltages

Sensors generate charges (charge amplifier external or ICP)

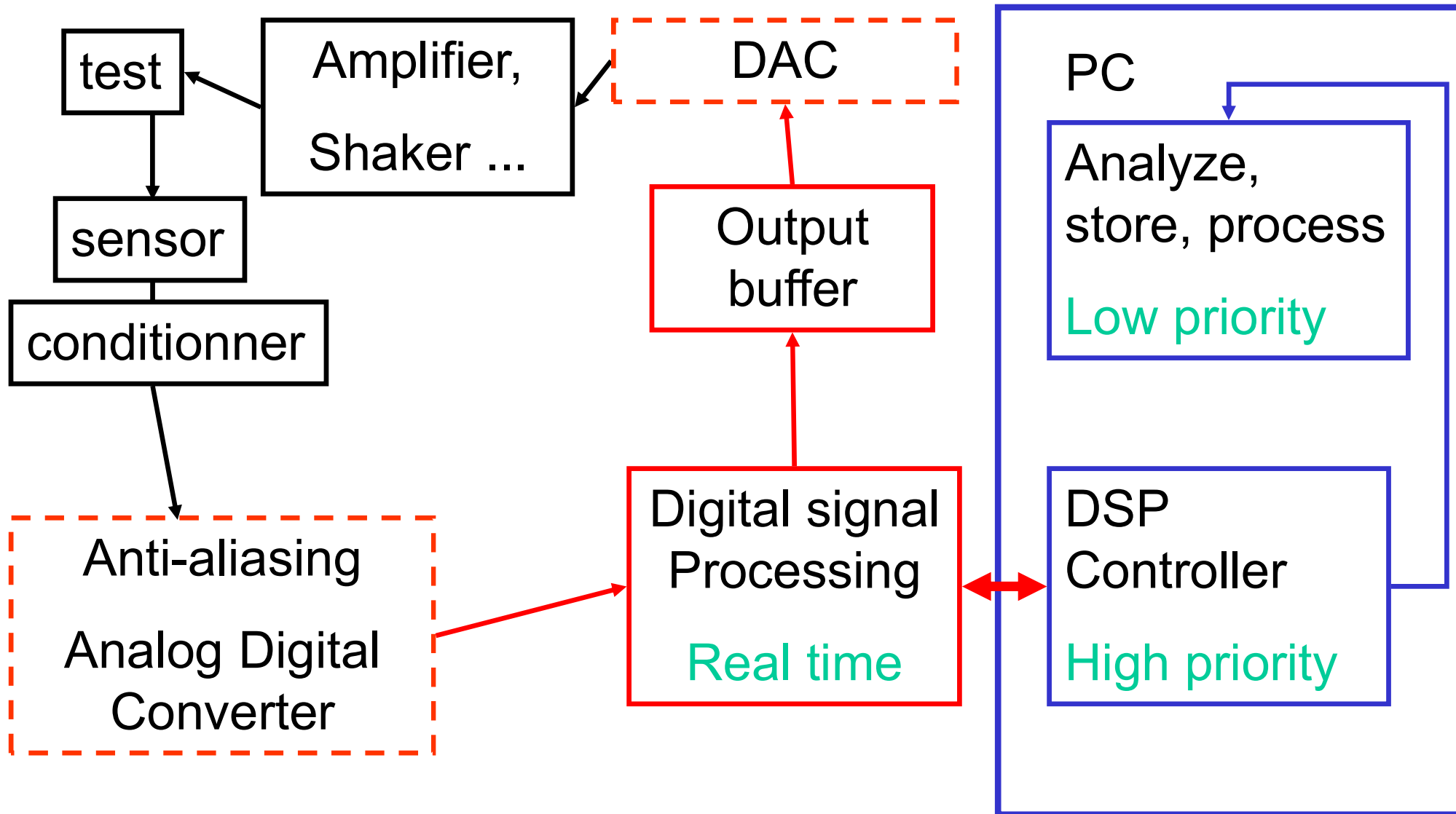
Shakers need power



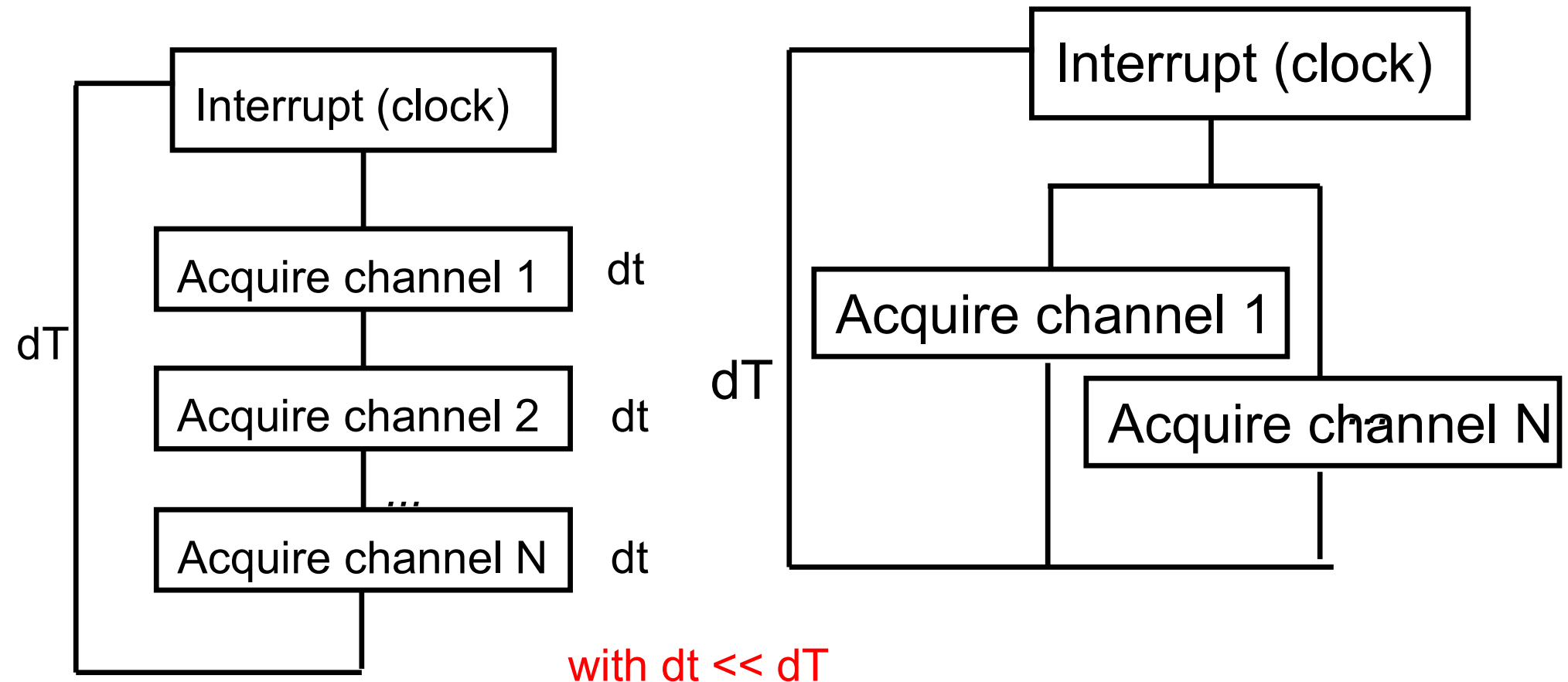
Contents

- Sensor/shaker technology
- Analyzer
- Signal processing basics
- FRF estimation

Acquisition and processing



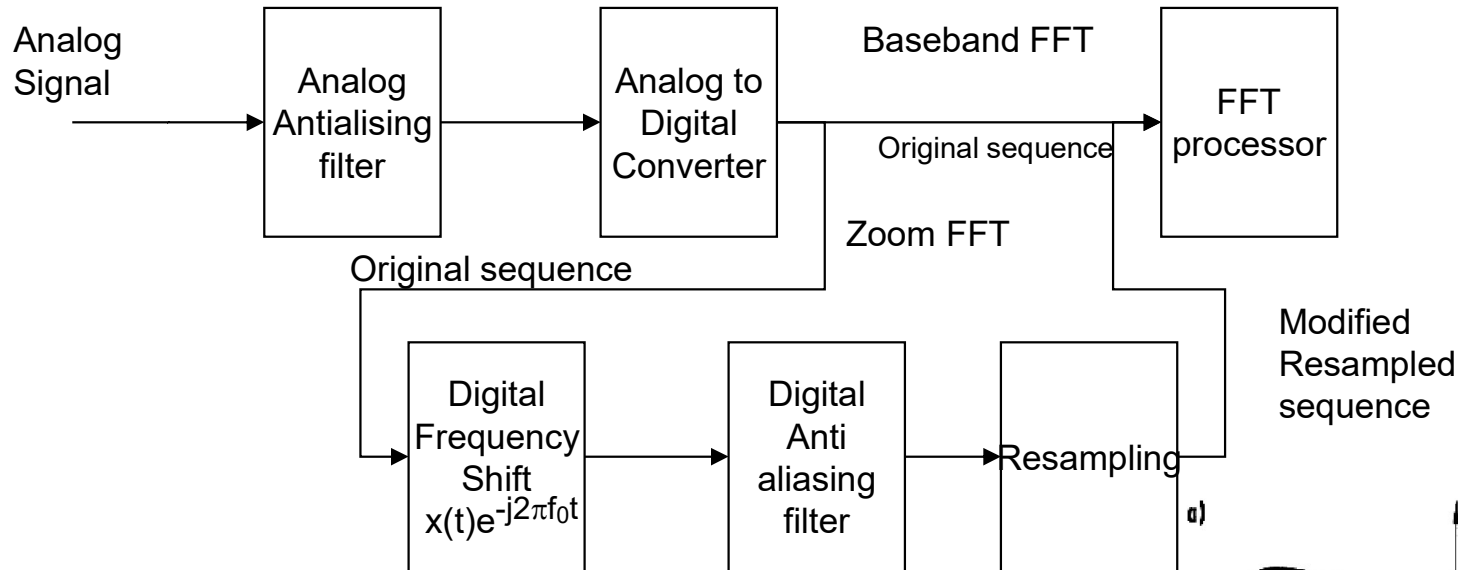
Sampling technology



DAC : Precision = $\text{RangeV} / 2^n \text{Bits}$

Example : 12 bits $P = 20\text{V}/4096$, 24 bits $P = 20/16777216$

FFT Zoom

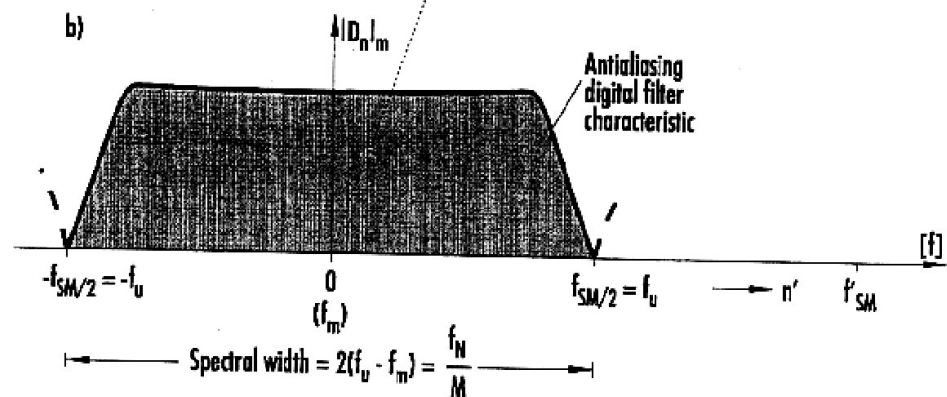
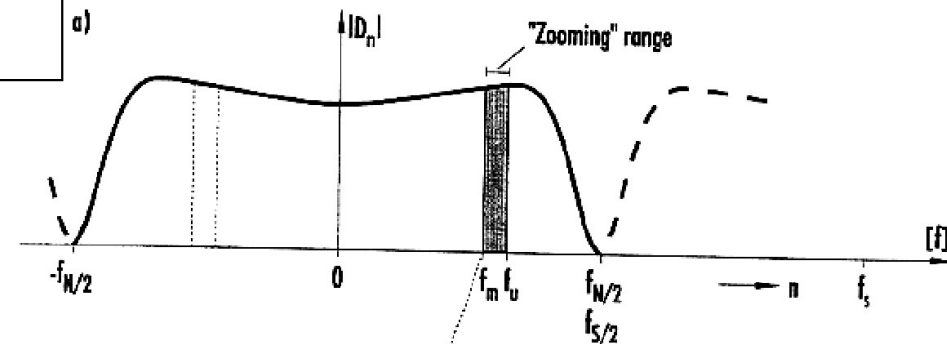


Based on spectrum shift property

$$\mathcal{F}(y(t)e^{-j2\pi f_0 t}) = Y(f - f_0)$$

```

Y=fft(y.*exp(-sqrt(-1)*2*pi*ftarg*t)); % shift and transform
N=length(t)/Nsub; Y(N:end-N)=0; % filter
Y(end:-1:end-N+1)=conj(Y(2:N+1)); % make symmetric
Y(1)=real(Y(1));
x=real(ifft(Y));
tx=t(1:Nsub:end); x=x(1:Nsub:end); % resample
fx=[1/diff(tx(1:2))*[0:length(tx)-1]'/length(tx)]+ftarg;
Y=fft(x)*Nsub;
  
```



a) Ordinary (baseband) spectrum
b) Zoomed spectrum

Hardware / software

