

# Component mode synthesis

- Earlier (reduc.pdf)
  - Reduction principles
  - [Reduction illustrations](#)
- Now
  - coupling reduced models
  - Advanced reduction for coupling objectives

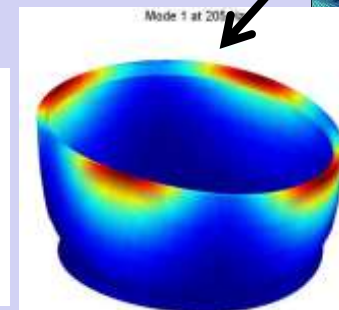
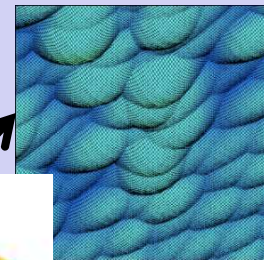
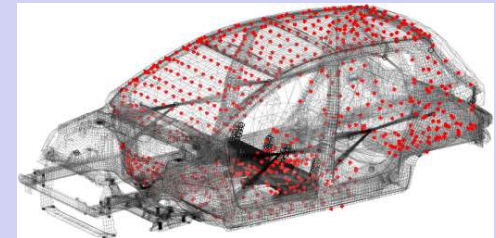
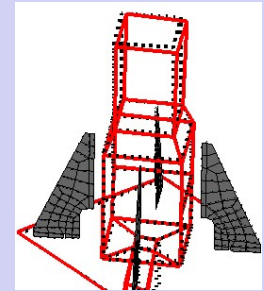
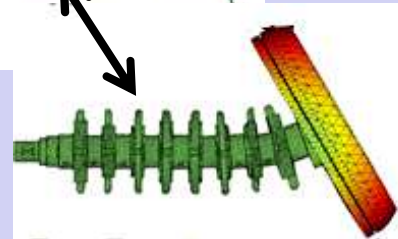
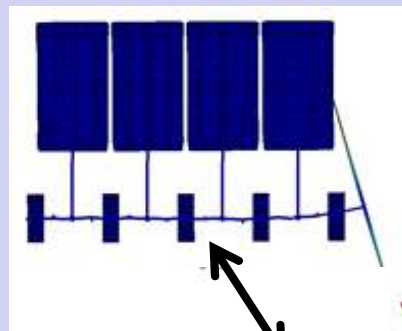
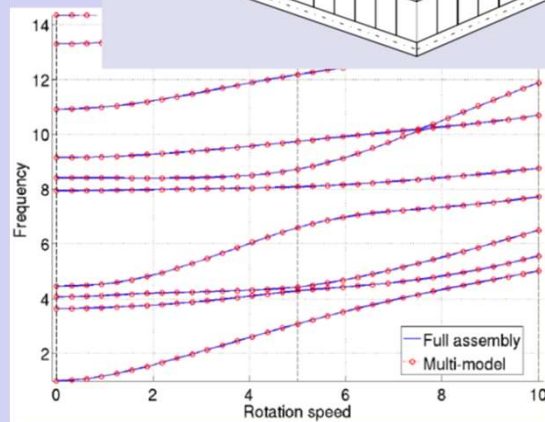
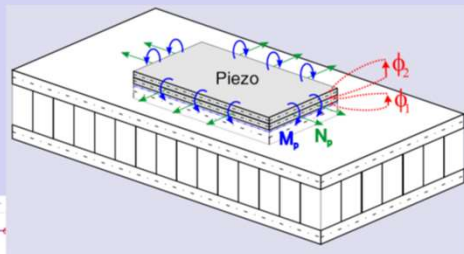
# Moving complexity in the coupling part

In

Reduced model

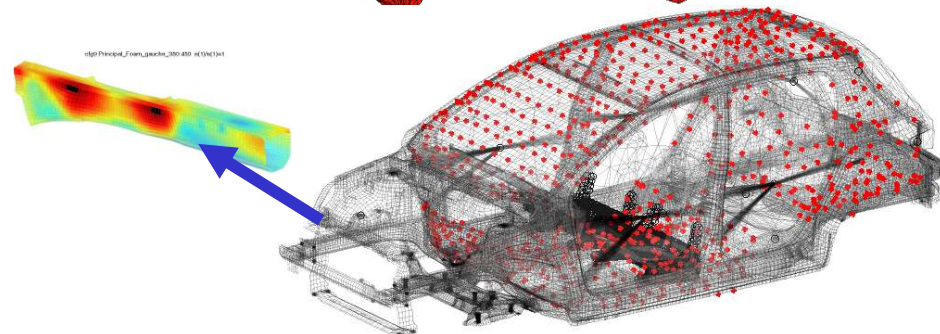
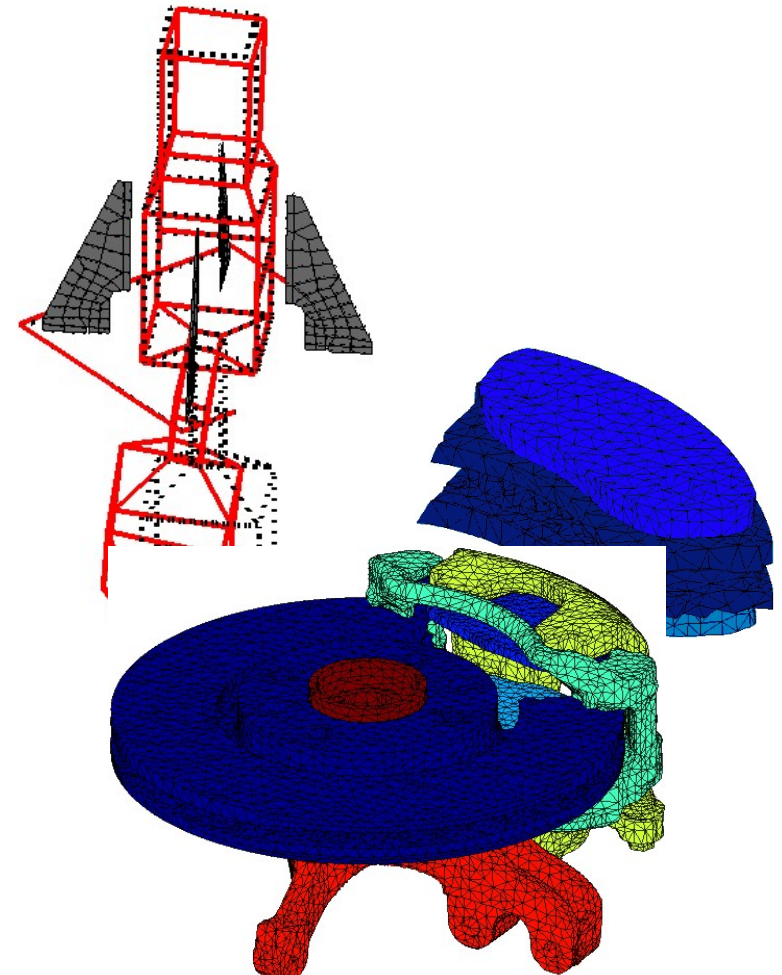
Sensors

- Coupling : test/FEM, fluid/structure active control, ...
- Local non-linearities : machining, bearings, contact/friction, ...
- Optimization / uncertainty



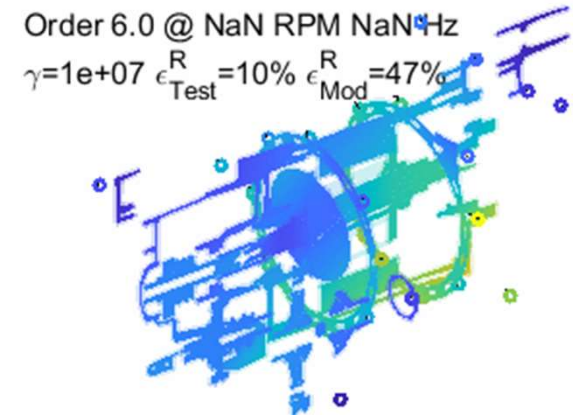
# Sample CMS problems

1. Acoustic prediction from test shapes
2. Fluid structure interaction (in particular with heavy fluids)
3. Structural Dynamics Modification
4. Reduce a brake model while keeping
  - all elements of NL contact area
  - exact modes of linear model
5. Design of damping treatment for structure borne transfer
6. Non-linearity (contact on tip blade)



# Why CMS ?

- A reason of **procedure**
  - Represent **linear structural dynamics** for coupling **in another code** (hybrid test/FEM, acoustics, multi-body dynamics, control, local non-linearity, ...)
  - Transmit a **compact/confidential model** to another group/company
  - Understand effects of **components**
  - Reduced data output
- For **computational cost** objectives
  - One step approximations (low cost linear model)
  - Iterative (often parallel) solution of exact problem



# Blackboard discussion

- Draw non conform contact case,  
gauss points (nodes special case)  
gap and sliding observation  
contact/friction constitutive law (surface laws)  
model loads
- Energy coupling (surface constitutive laws)
- Mathematically idealized bonding (constraints, Lagrange)
- $1 - \epsilon$  compatibility
- Kinematic reduction for coupling
  - Remind McNeal & Craig-Bampton
  - Interface modes
  - Learning using exact solutions (CMT)

# Incompatible mesh contact

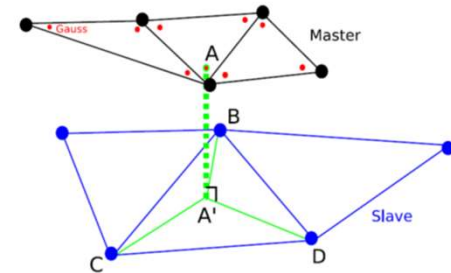
- Gap (out of plane) incompatible

- Define **contact points** matched
- Match **slave elements**

$$g_g = [c_g]\{q\} = [N_{master}(r, s) - N_{slave}(r, s)]\{q\}$$

- Associate integration rule and compute work

$$\{q^*\}^T \{P_{contact}\} = \sum_g \{q^*\} [c_g]^T w_g J_g P_g$$



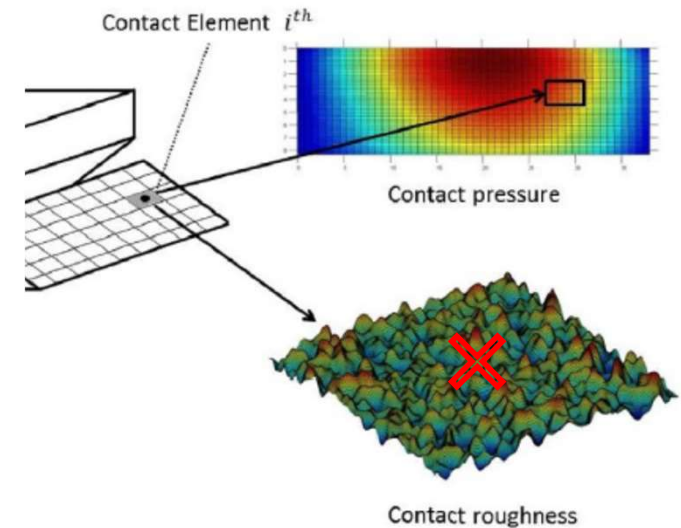
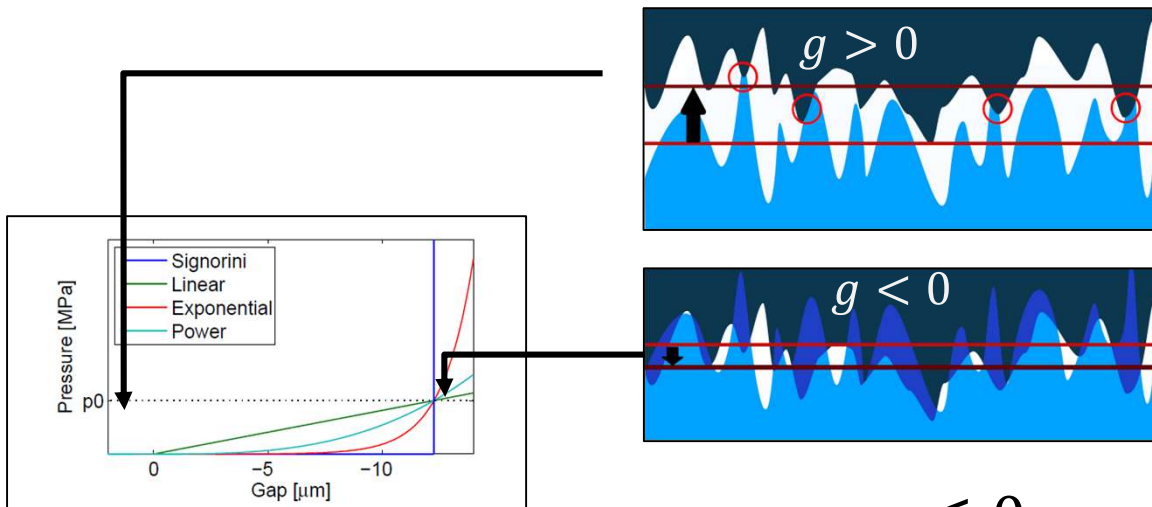
- **Gap for compatible mesh**

- Use nodal displacement
- Define gap at gauss points (zero thickness cohesive element)

- Extension in plane : adhesion/sliding/friction

# Contact constitutive law

- Macro-scale surface not flat (**1 gauss**)
- Macro load function of gauss strain

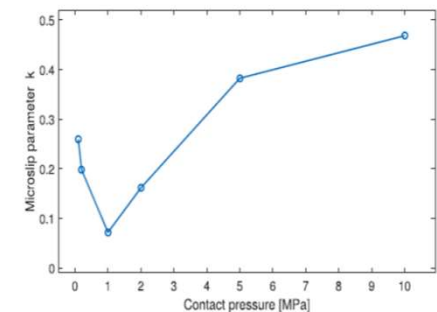
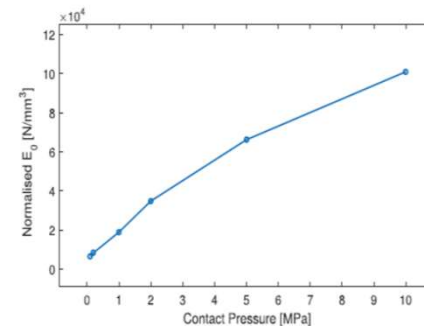


From : L. Pesaresi. JSV 2018

- Idealization Signorini
- Reality  $p(g)$

$$\begin{aligned} g &\leq 0 \\ p &> 0 \\ pg &= 0 \end{aligned}$$

- Friction : coulomb  $\sigma_t = -\mu p \frac{v_t}{\|v_t\|}$



# Stiffness/energy coupling

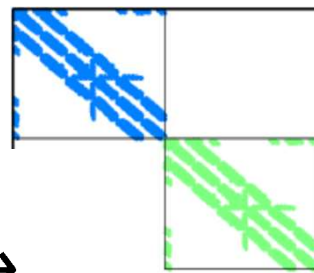
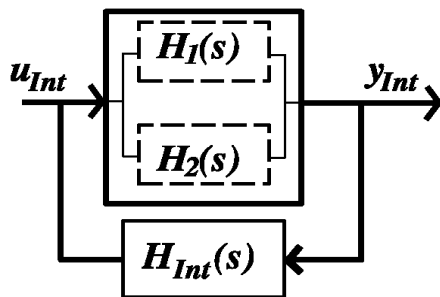
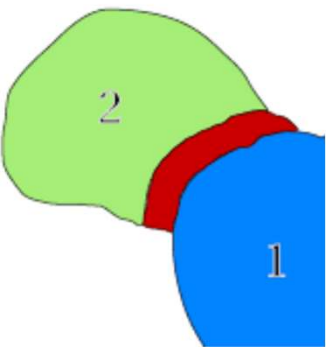
- Interface motion
- Interface stiffness (cohesive elements)

$$\{y_j(X, s)\} = [c_{j\text{int}}(X)] \{q_j(s)\}$$

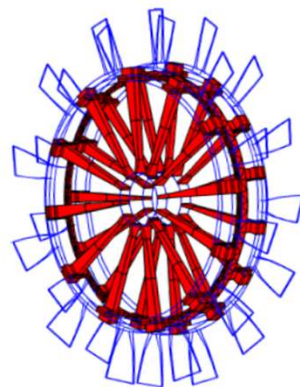
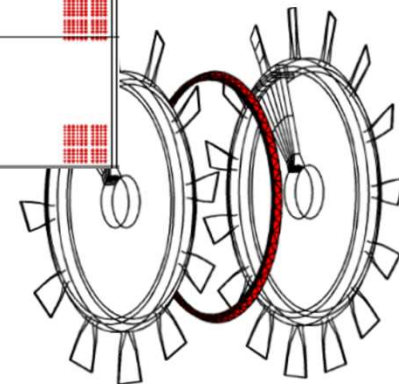
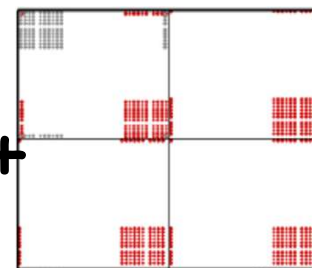
$$\begin{bmatrix} Z_{jj\text{int}} & \cdots & Z_{cj\text{int}} \\ \vdots & \ddots & \vdots \\ Z_{jc\text{int}} & \cdots & Z_{cc\text{int}} \end{bmatrix} \left\{ \begin{bmatrix} [c_{\text{int}}] \{q_j\} \\ \vdots \\ \{q_{\text{int}}\} \end{bmatrix} \right\} = \begin{bmatrix} F_{\text{int}} \\ \vdots \\ \{0\} \end{bmatrix}$$

- Coupled equations (sum of energies)

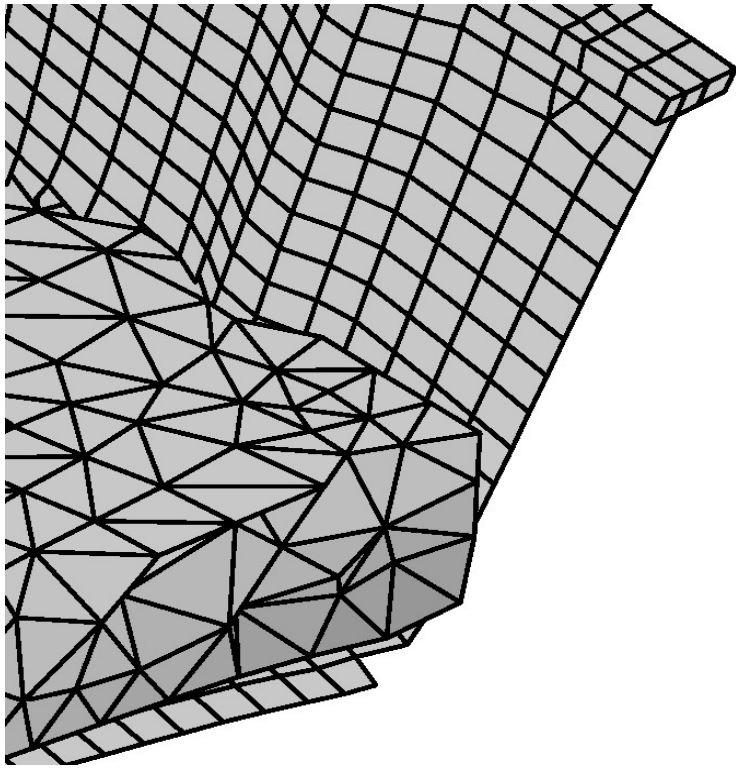
$$\left( \begin{bmatrix} \boxed{Z_1} & 0 \\ 0 & \boxed{Z_2} \end{bmatrix} + \begin{bmatrix} c_1^T & 0 \\ 0 & c_2^T \end{bmatrix} \boxed{Z_{\text{int}}} \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [b] \{u(s)\}$$



+

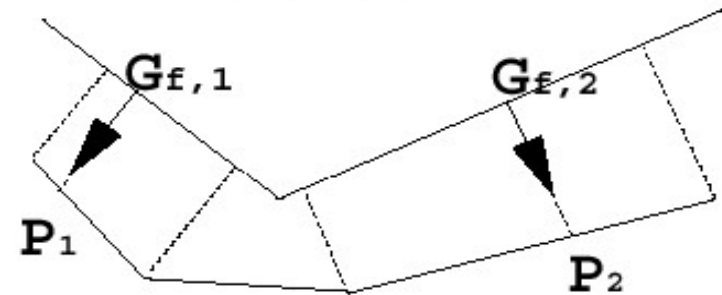


# Incompatible mesh : fluid/structure

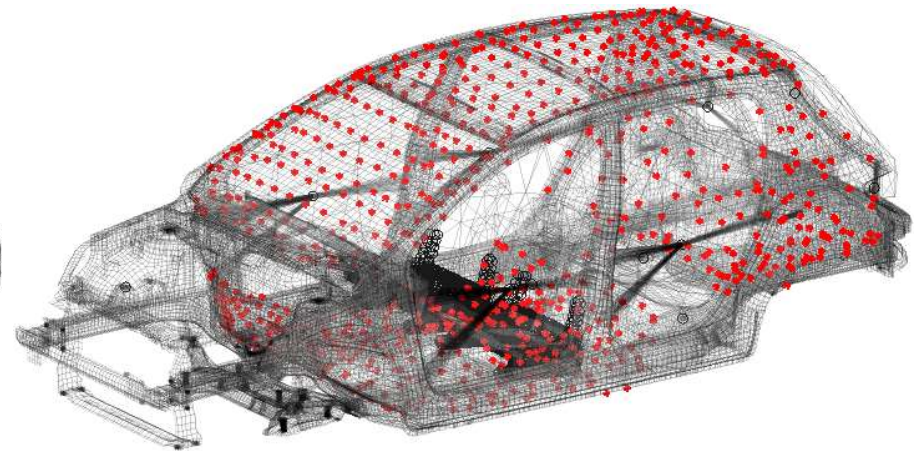
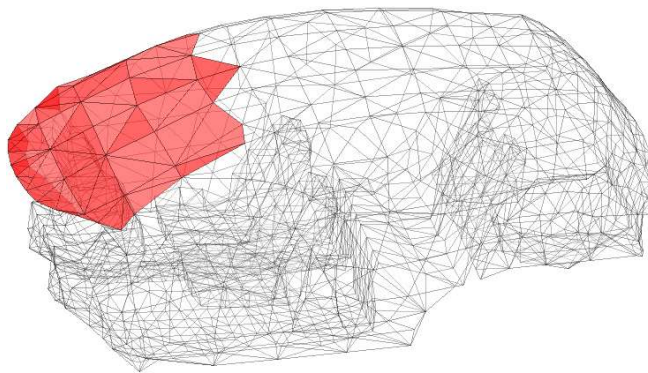


$$\begin{bmatrix} K & -C \\ 0 & F \end{bmatrix} \begin{Bmatrix} U \\ p \end{Bmatrix} - \omega^2 \begin{bmatrix} M & 0 \\ C^T & K_p \end{bmatrix} \begin{Bmatrix} U \\ p \end{Bmatrix} = \begin{Bmatrix} F^{ext} \\ 0 \end{Bmatrix}$$

**Fluid**



**Solid**



# Limiting case : continuity

Solve with zero relative interface motion

$$\{y_{1Int} - y_{2Int}\} = [c_1 \quad -c_2]_{Ng \times (N_1 + N_2)} \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = 0$$

- Classical : eliminate constraint (T kernel)  $[T]$  with  $[c_{Int}] [T] = 0$   
 $\{q\} = [T] \{q_R\}$   
 $[T^T Z T] \{q_R\} = [T^T b] \{u\}$
- Lagrange multiplier solution
- Penalize (approximate energy)

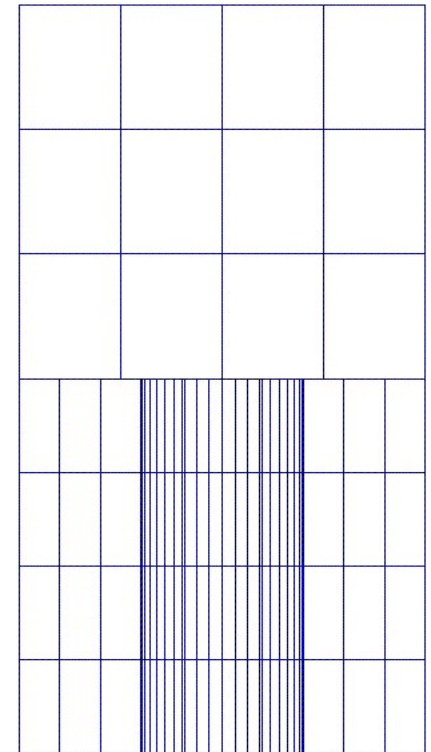
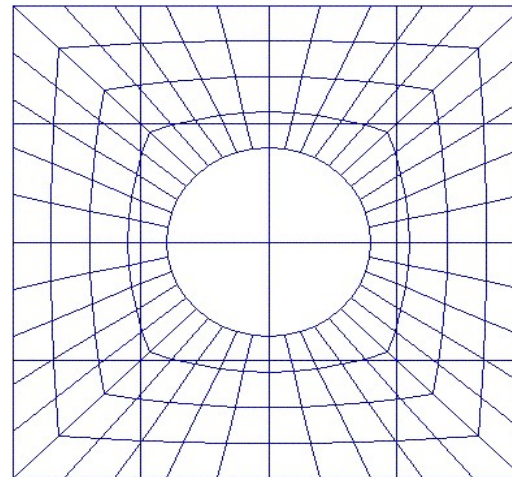
$$\begin{bmatrix} Z(s) & c_{Int}^T \\ c_{Int} & 0 \end{bmatrix} \begin{Bmatrix} q \\ \lambda \end{Bmatrix} = \begin{Bmatrix} F \\ 0 \end{Bmatrix}$$

$$\left( \begin{bmatrix} Z_1 & 0 \\ 0 & Z_2 \end{bmatrix} + \begin{bmatrix} c_1^T \\ -c_2^T \end{bmatrix} \begin{bmatrix} I \\ \epsilon \end{bmatrix} [c_1 \quad -c_2] \right) \begin{Bmatrix} q_1 \\ q_2 \end{Bmatrix} = [b] \{u(s)\}$$

- Other approach : continuity enforced over volume (Ben Dhia, Arlequin)

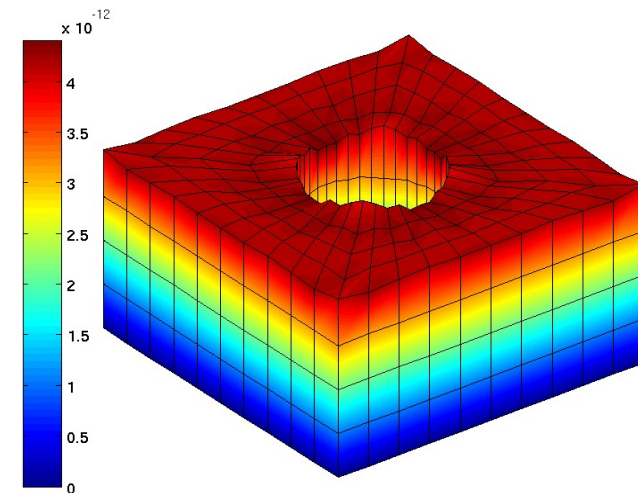
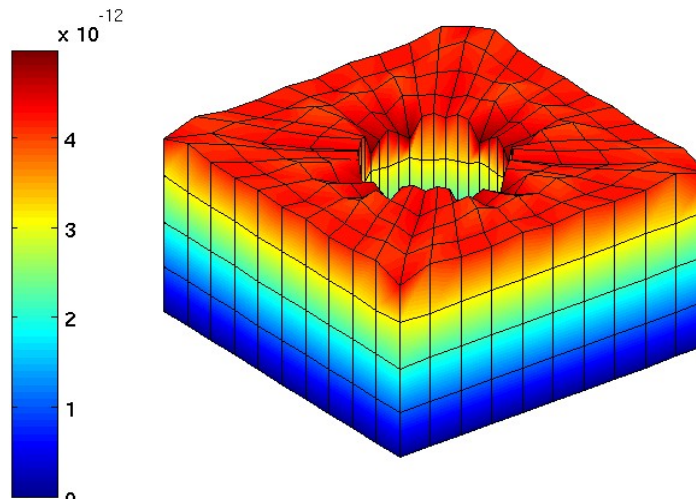
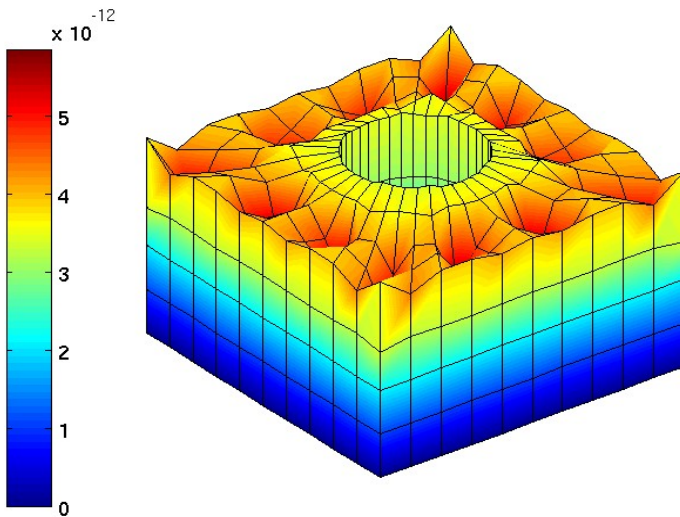
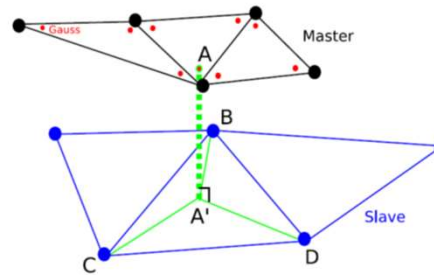
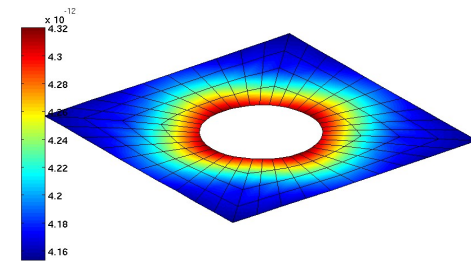
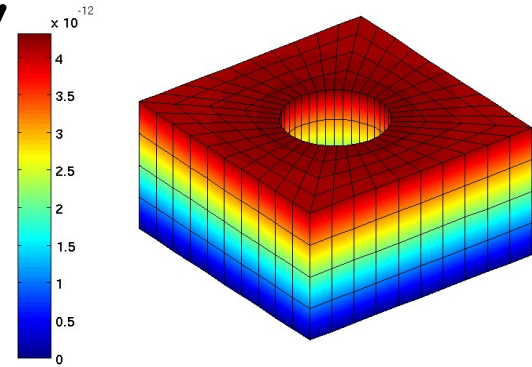
# Incompatible meshes

- Occur regularly
  - Result of **automated meshing** (conform mesh generation can be very difficult)
  - Contact problems
- Test case : compression of 2 cubes
  - Cube over drilled cube
  - Coarse upper cube
  - Refined lower cube
  - Master upper cube



# Incompatible mesh issues

- Solution depends of interpolation strategy
  - Number of **contact points** matched
  - Number of **slave elements** matched
- **Poor results** when using coarse mesh as master



# Discontinuity : numerical implementation

- Construction of a third interface

- Domain **intersection**
- Nodes of **both surfaces**
- Delaunay triangulation

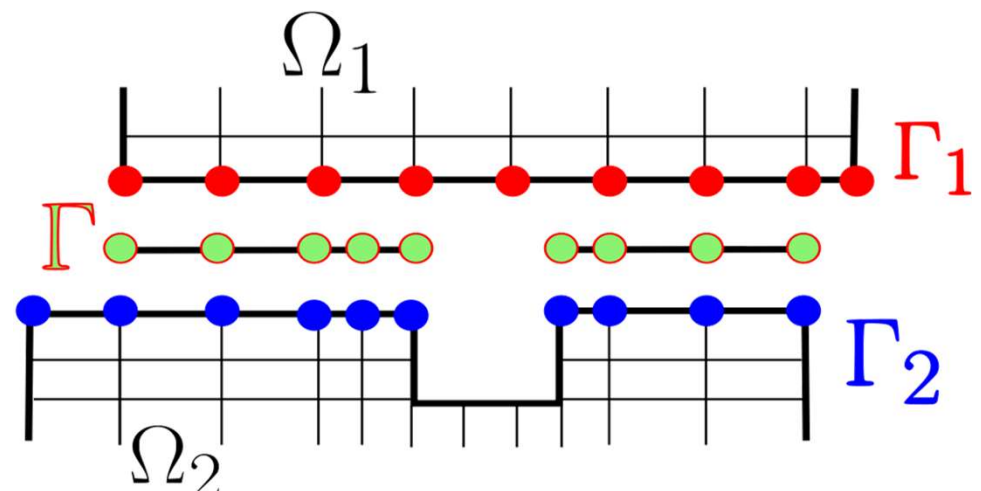
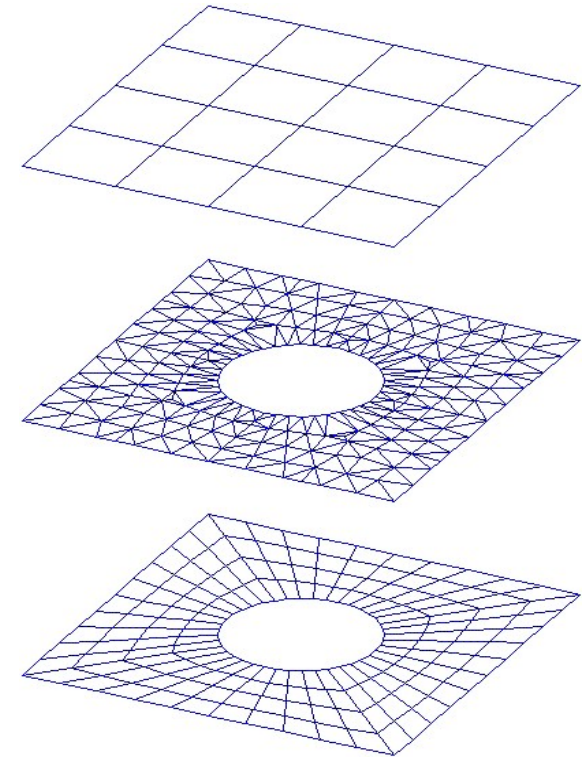
- Gap observation at **nodes or Gauss points**  $\Gamma$

- Projection for  $\Gamma_1$  and  $\Gamma_2$
- Cross product operator

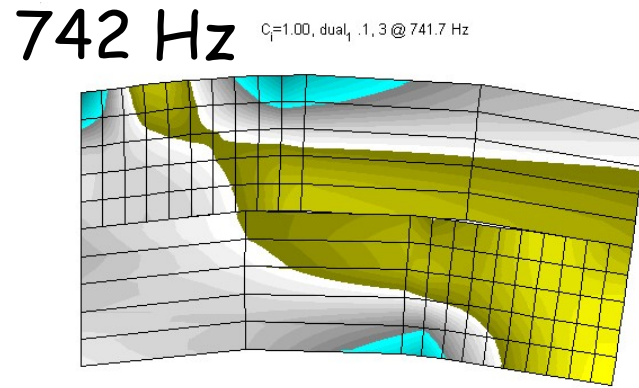
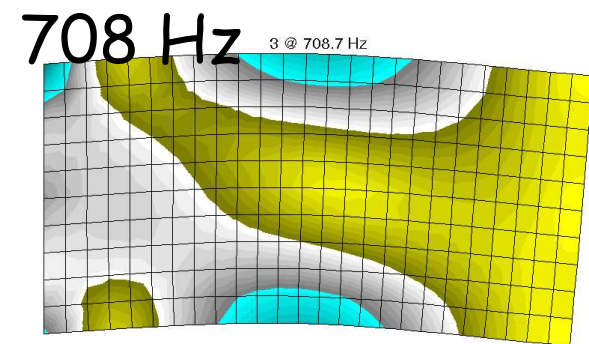
$$[A] = [C_{NOR}] [C_{NOR}]^T$$

- $[A]$  ill conditioned if

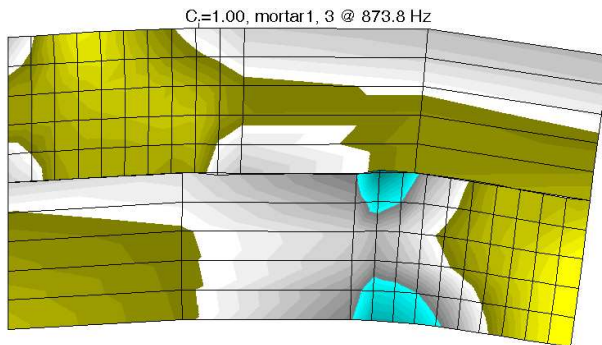
- **Under integration**
- Master points not matched



# Incompatibility and locking

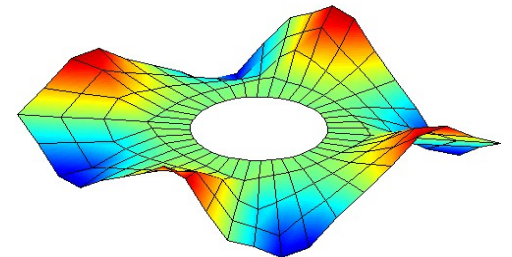
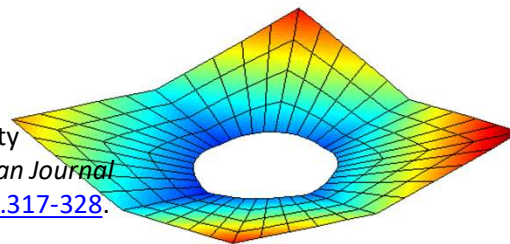
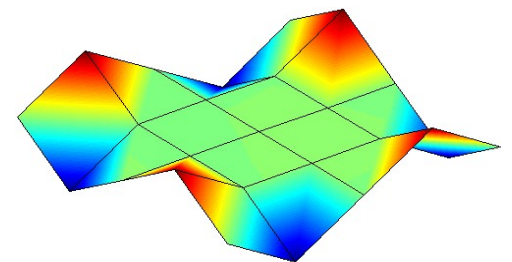
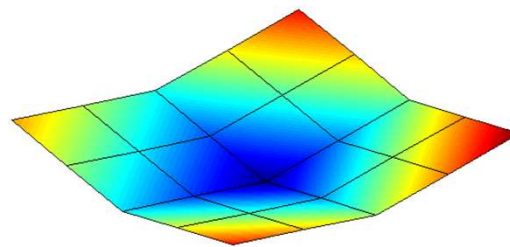


873 Hz=locking



- Strong continuity = locking
- Weak sense for continuity needed [1]

[Skip to Vector sets and bases](#)



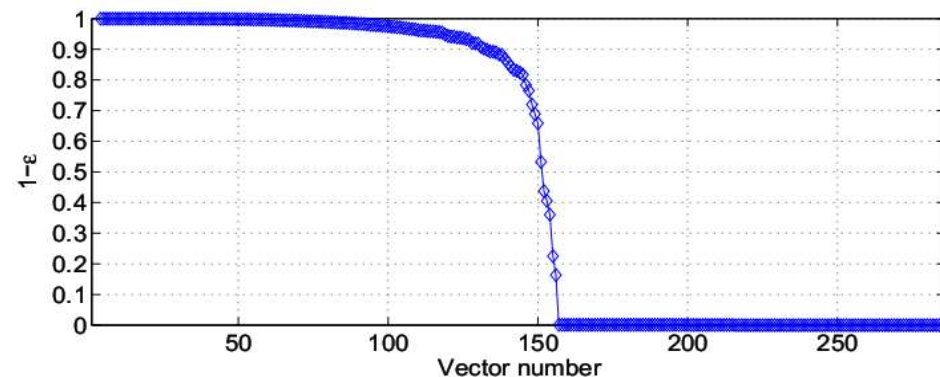
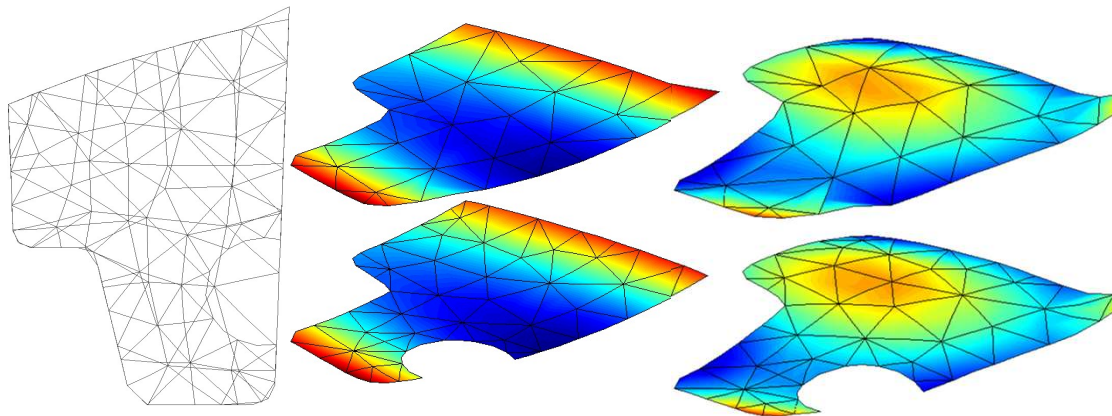
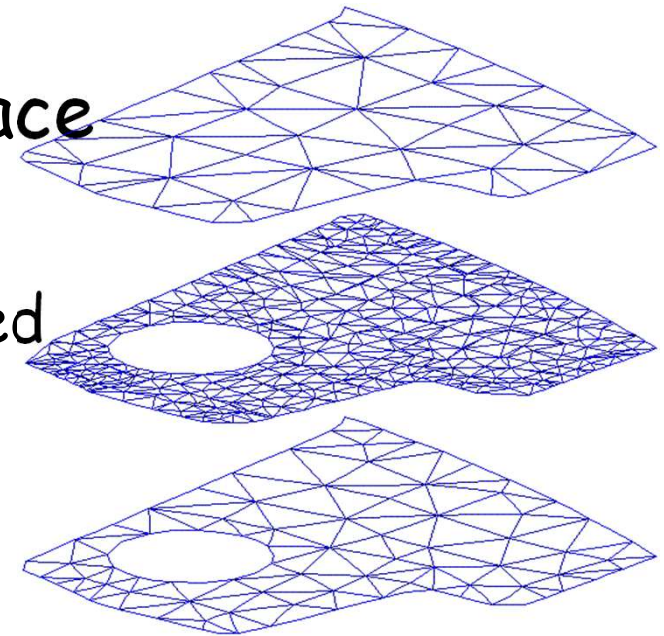
[1] G. Vermot Des Roches, E. Balmes, H. Ben Dhia, and R. Lemaire, "Compatibility measure and penalized contact resolution for incompatible interfaces," *European Journal Of Computational Mechanics*, vol. 19, pp. 317–329, 2010, doi: [10.3166/ejcm.19.317-328](https://doi.org/10.3166/ejcm.19.317-328).

# Quality measurement (1- $\varepsilon$ )-compatibility

- Measure the **norm difference** between the basis vectors of  $\Gamma_1$  and their projection on  $\Gamma_2$ 
$$C_2^1(\{q_1\}) = \frac{\|\pi_2^1 \{q_1\}\|}{\|\{q_1\}\|}$$
- Realize this leads to an eigenvalue problem
$$C_2^1(\{q_1\})^2 = \frac{\{q_1\}^T [A_{21}]^T [A_{22}]^{-1} [A_{21}] \{q_1\}}{\{q_1\}^T [A_{11}] \{q_1\}}$$
- Use of an inner product with **mechanical meaning** (pressure load with surface stiffness density)

# Illustration on a brake model

- Master/Slave strategy not obvious
- Mesh refinement differences
- Application to the pad/caliper interface
- Compatibility issues
  - **Spurious movements** for partially matched contact elements
  - Movement over **drilled parts**



# Classical reduction bases + variants

**CMS = coupling + reduction**

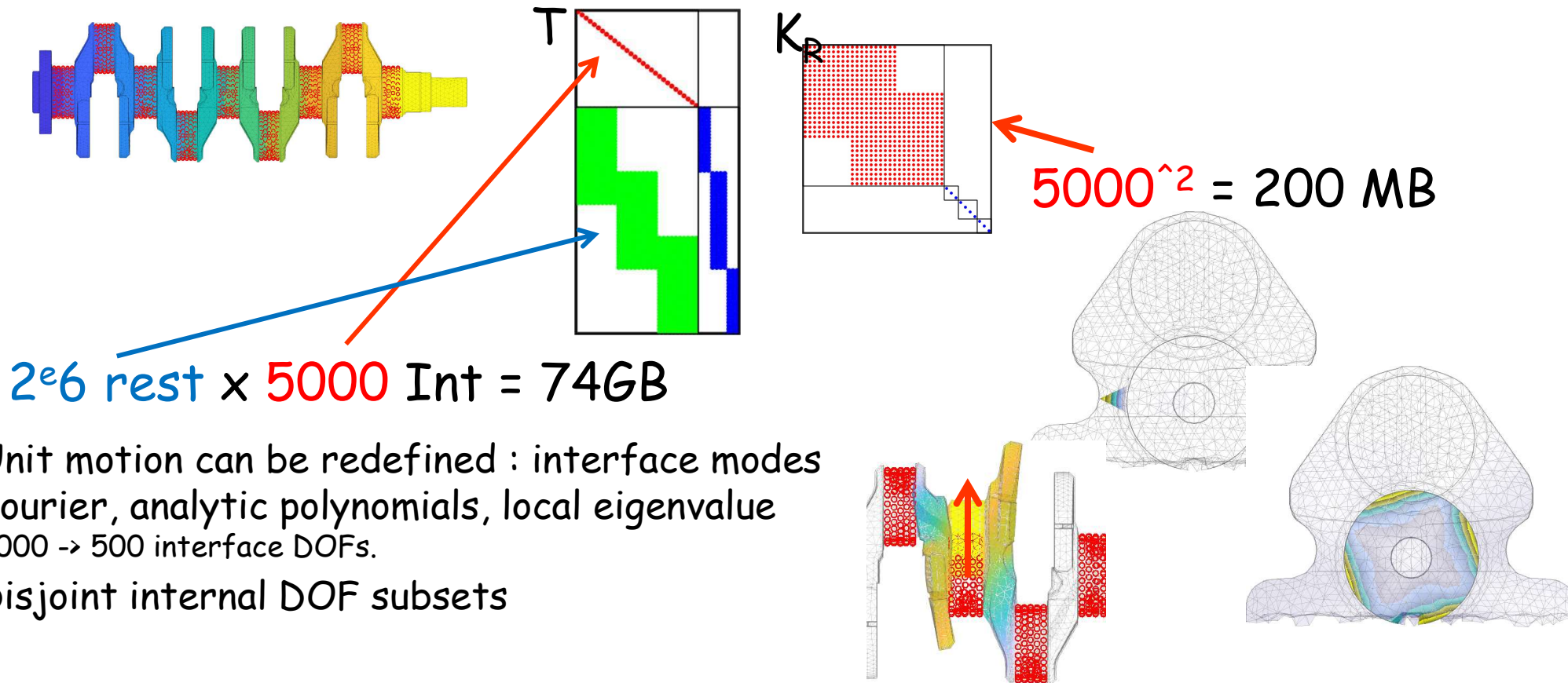
- Static condensation + fixed interface modes = Craig-Bampton
- Free modes + attachment modes (static correction)
- ... + residual vectors for parametric changes

Discuss now :

- ... + interface modes
- CMT : Trace of assembled modes
- ... + component modes
- ODS, POD, Snapshot POD, ... (see [Avanded\\_Modal\\_Periodic.pdf](#))

# Interface reduction / model size / sparsity

- Craig-Bampton often sub-performant because of interfaces



## Separate requirements for learning shapes :

bandwidth, inputs external & parameter  
truncation, sparsity

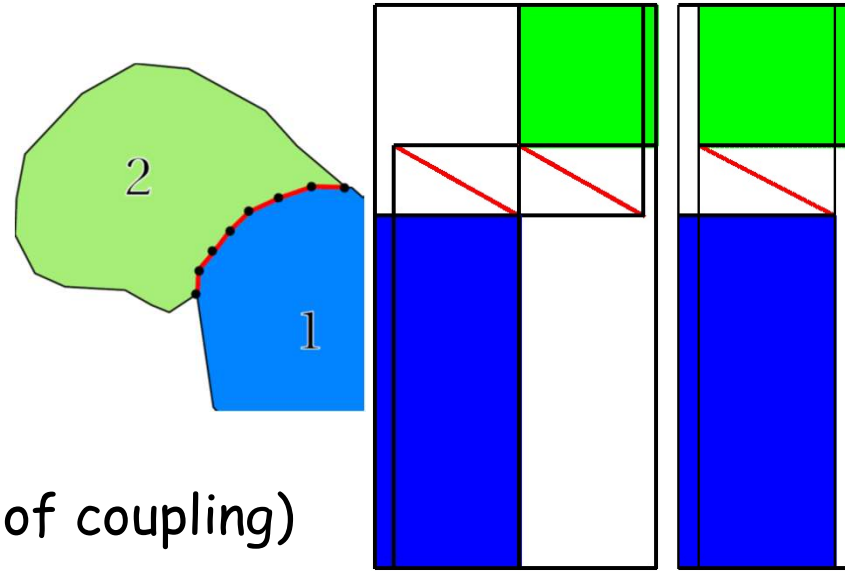
# Interfaces for coupling

## Classical CMS : continuity coupling

- Reduced independently
- All interface motion (or interface modes)
- Assembly by continuity

### Difficulties

- Mesh incompatibility
- Large interfaces
- Strong coupling (reduction requires knowledge of coupling)

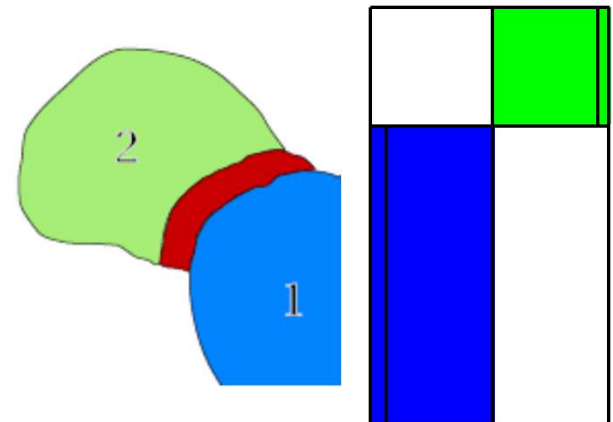
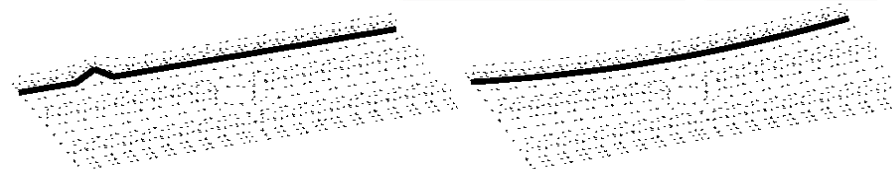


## Disjoint components : energy coupling

- Assembly by computation of interface energy (example Arlequin)

### Difficulties

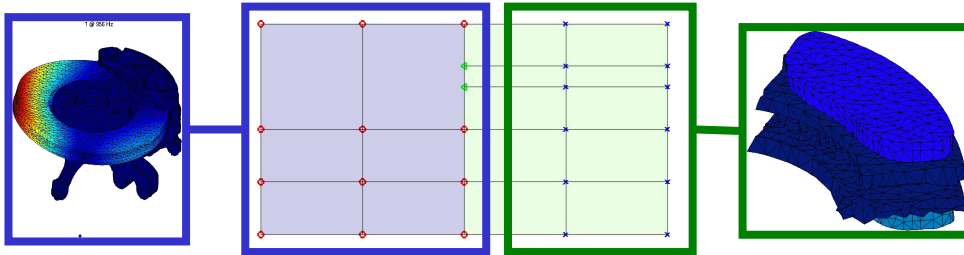
- Use better bases than independent reduction



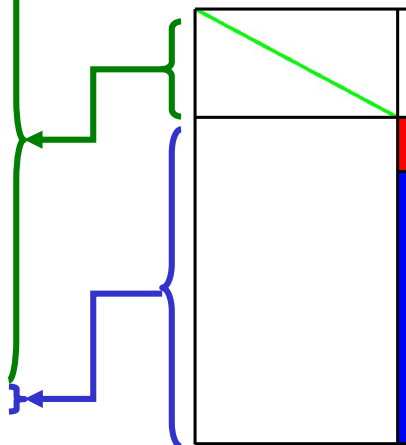
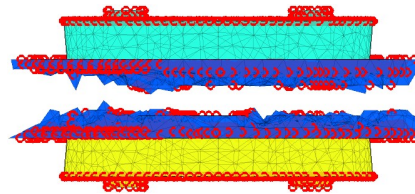
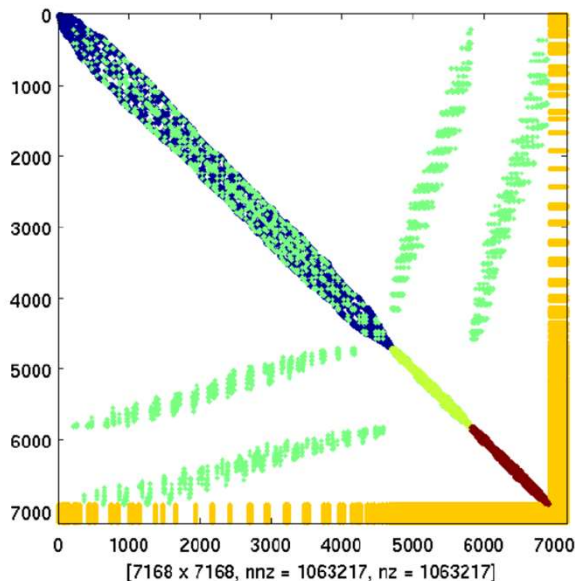
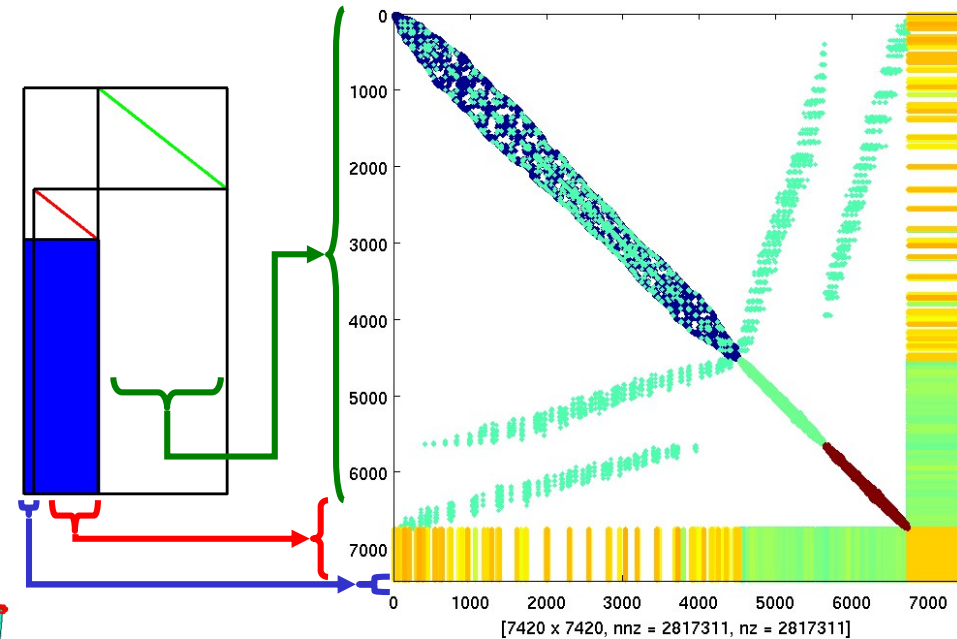
# Revised notion of interface

## Classical CMS (Craig-Bampton)

- **System** is brake without **contact area**



- Reduction : modes of system and interface loads
- **Many interface DOFs** needed heavily populated matrix



## Disjoint component with exact modes

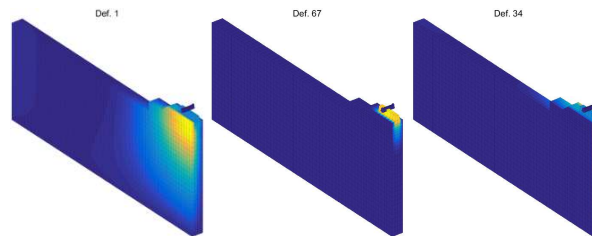
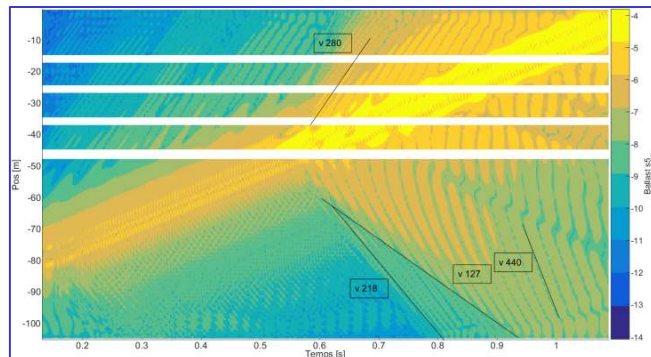
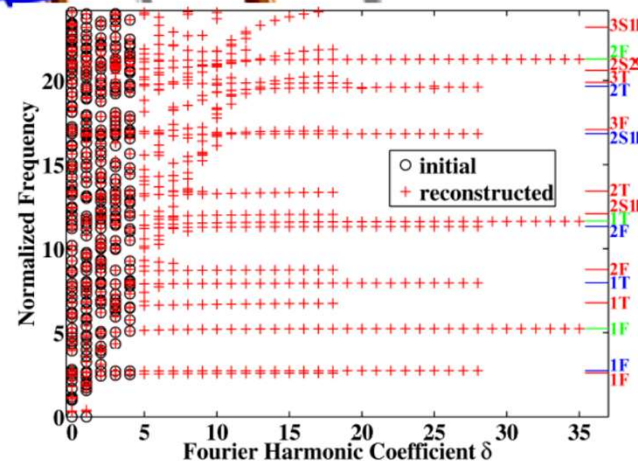
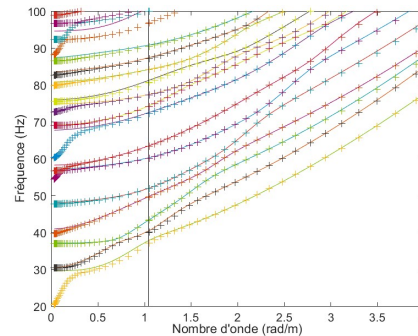
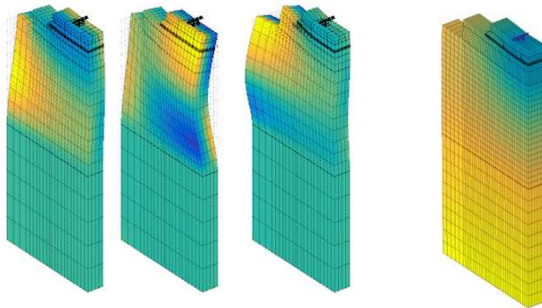
- No reduction of DOFs **internal to contact area**
- Reduction : **trace of full brake modes** on **reduced area** (no need for static response at interface)

# Interface reduction : wave/cyclic

Best interface reduction = learn from full system modes

1. Learn using wave (Floquet)/cyclic solutions
2. Build basis with left/right compatibility
3. Assemble reduced model

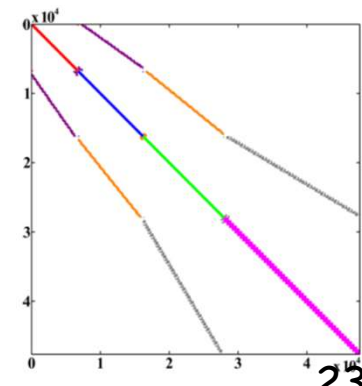
Mode 1 at 3.585 Hz Mode 2 at 6.496 Hz Mode 3 at 10.53 Hz



Arlaud, 2016 <https://pastel.archives-ouvertes.fr/tel-01455077>

Pinault 2020 <https://pastel.archives-ouvertes.fr/tel-03131802>

Sternschuss 2008 <https://tel.archives-ouvertes.fr/tel-00366252>



# Open issues : nominally exact reduced model

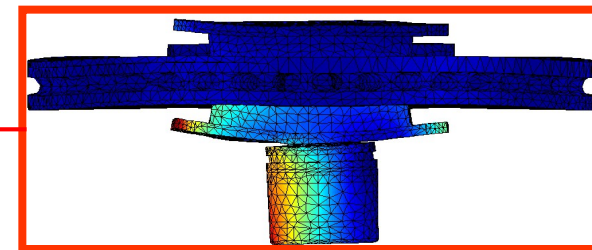
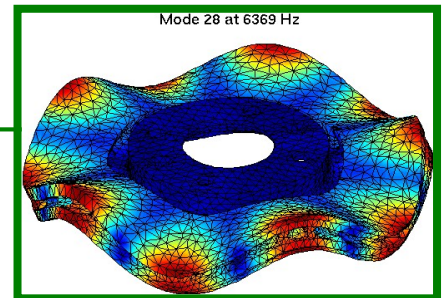
1980 : interest large linear solution

2017 : enhanced coupling

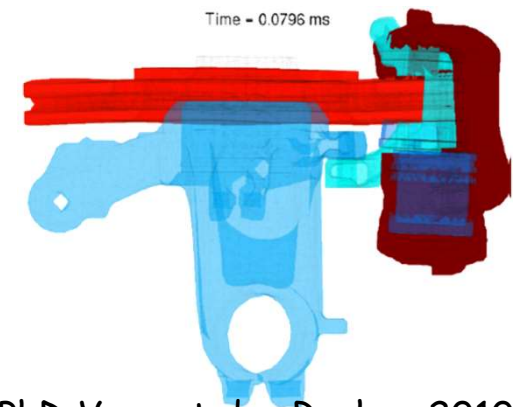
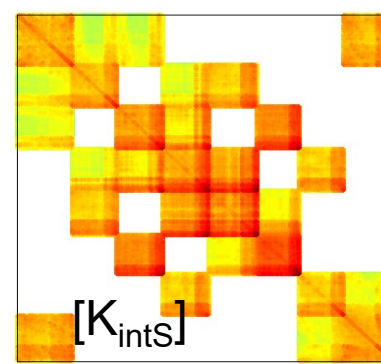
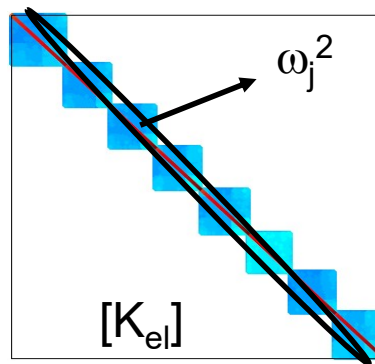
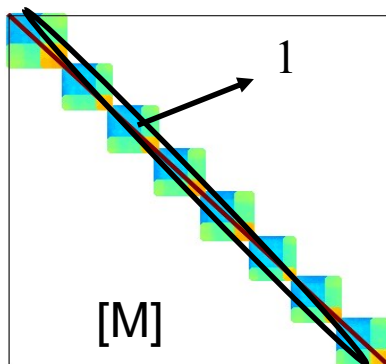
- Component Mode Tuning method
  - free/free real modes (explicit DOFs)
  - trace of the assembled modes on the component

$$[T_{ci}] = \left[ \boxed{[\phi_{ci}]} \quad \boxed{[\Phi_{|ci}]} \right] Orth.$$

- Reduced model is sparse
- Free mode amplitudes are DOFs
- Reduced model has exact nominal modes



Disc  
OuterPad  
Inner Pad  
Anchor  
Caliper  
Piston  
Knuckle  
Hub



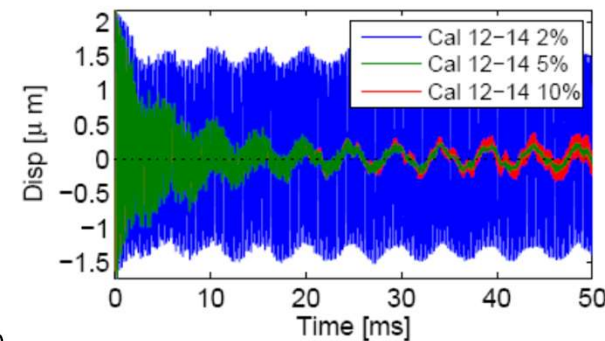
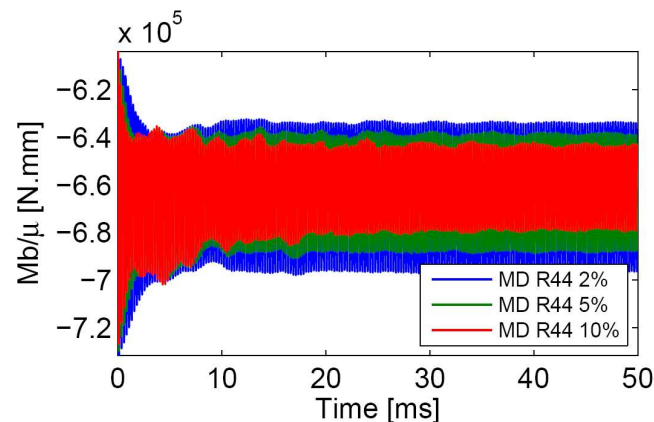
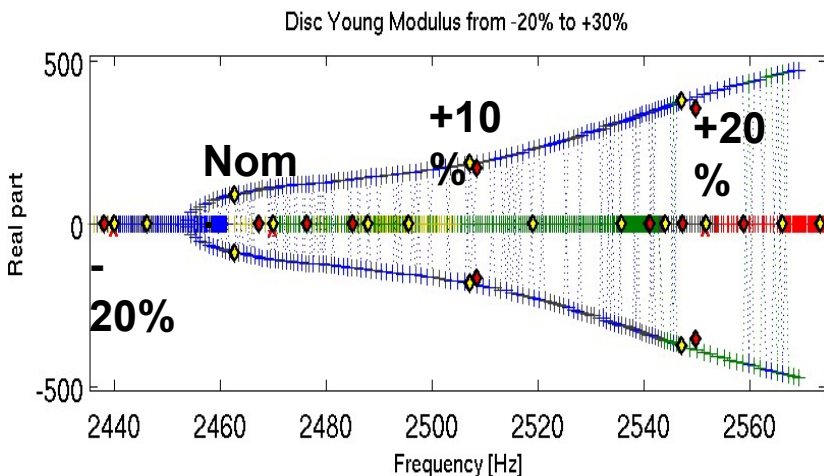
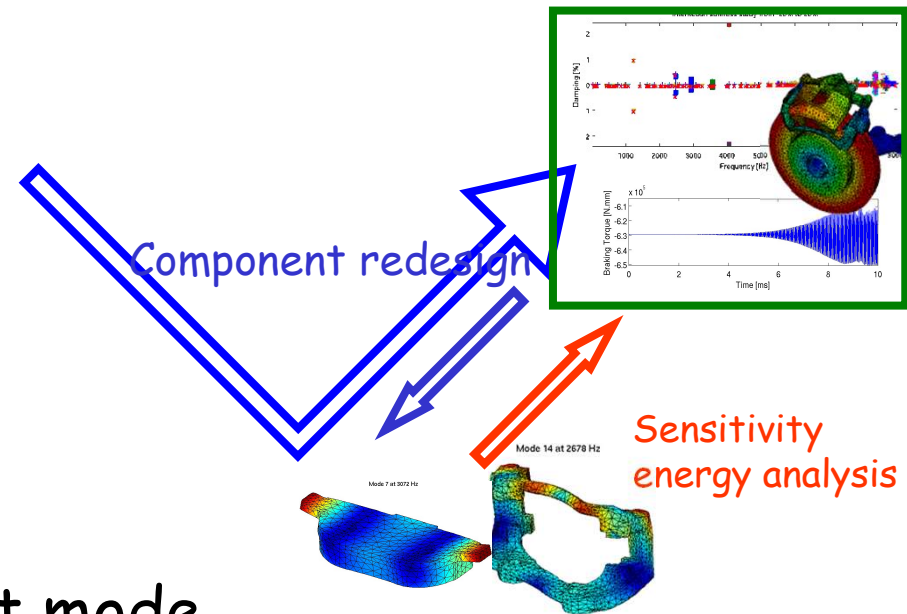
PhD Vermot des Roches 2010  
<https://tel.archives-ouvertes.fr/tel-00589951>

# CMT & design studies

- One reduced model / multiple designs

## Examples

- impact of modulus change
- damping real system or component mode



# Component modes as design parameters

- Component modes can be used as explicit reduced DOFs
- Brake application : which mode of which component should be modified
- Engine application : effect of blade mistuning

