Identification (notes chapters 6-7, slides Identification_EMA.pdf)

1. Identification demo
2. Inverse problems : model forms, data (s 7.1)
3. Model forms for identification, discussion of residual terms
4. Frequency domain least-squares solution, implicit NL
5. Evaluation of results (ch 8)

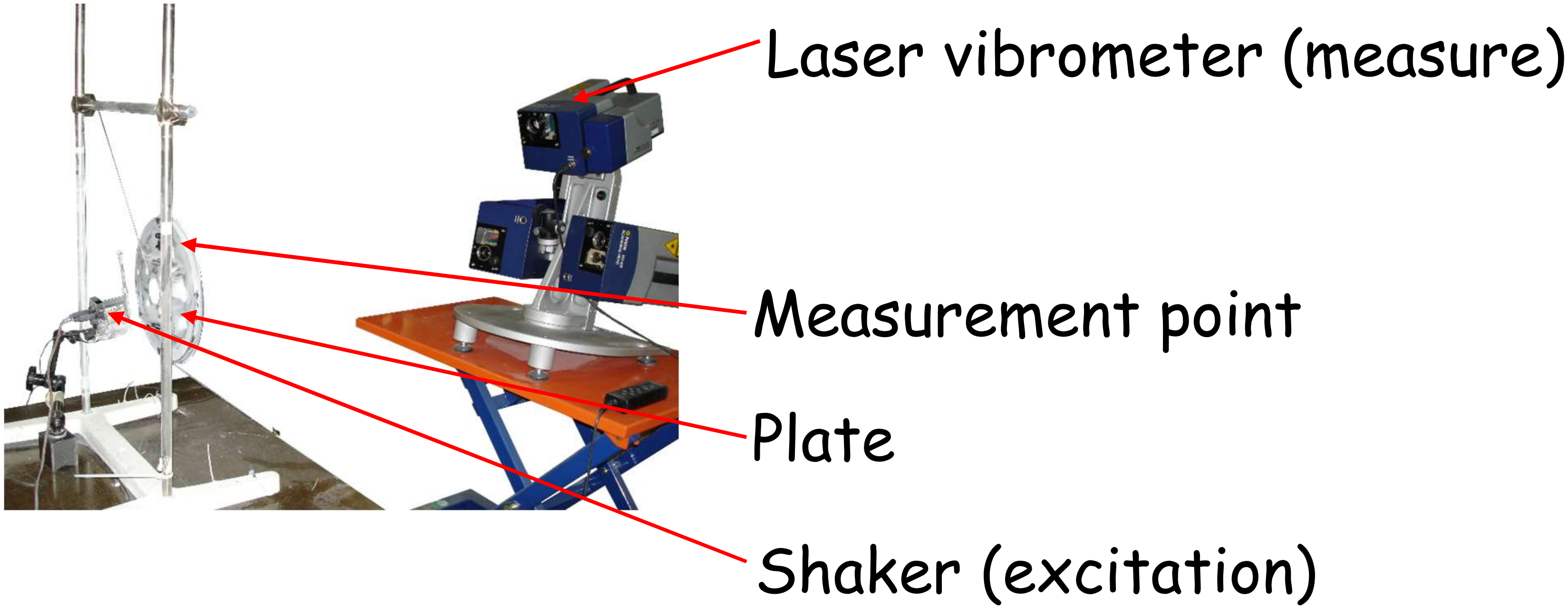
SD Tools
VIBRATION SOFTWARE & CONSULTING

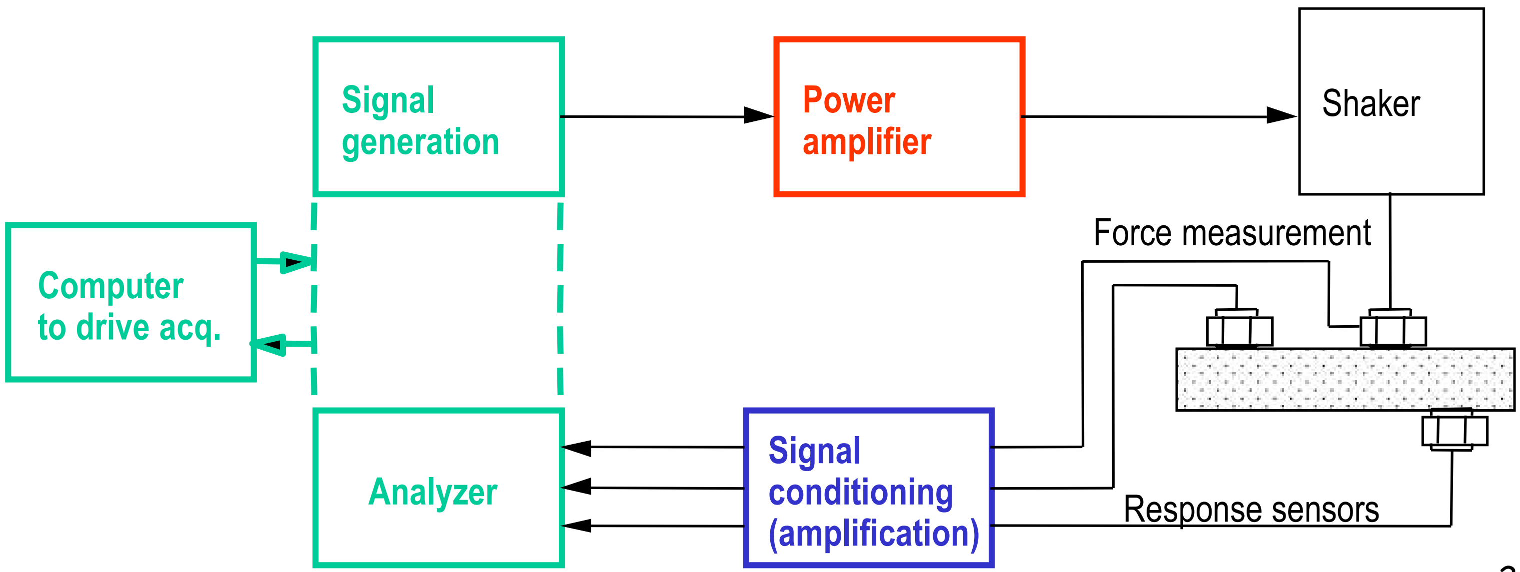
Arts et Métiers Institute of Technology

MS2SC http://savoir.ensam.eu/moodle/course/view.php?id=1874

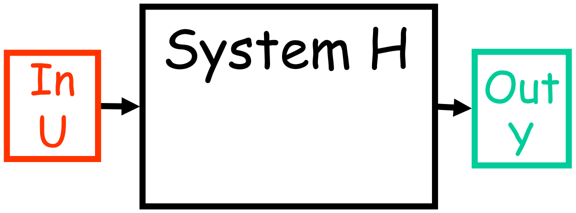
PROVIR http://savoir.ensam.eu/moodle/course/view.php?id=490

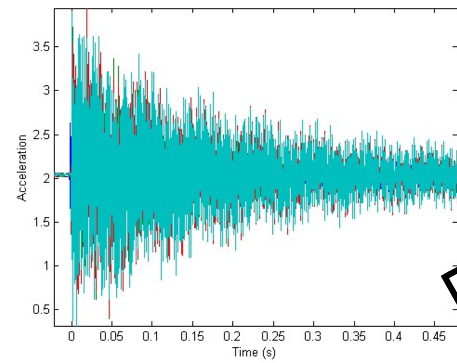
Experimental modal analysis : data





Assumption LTI system = transfers exist





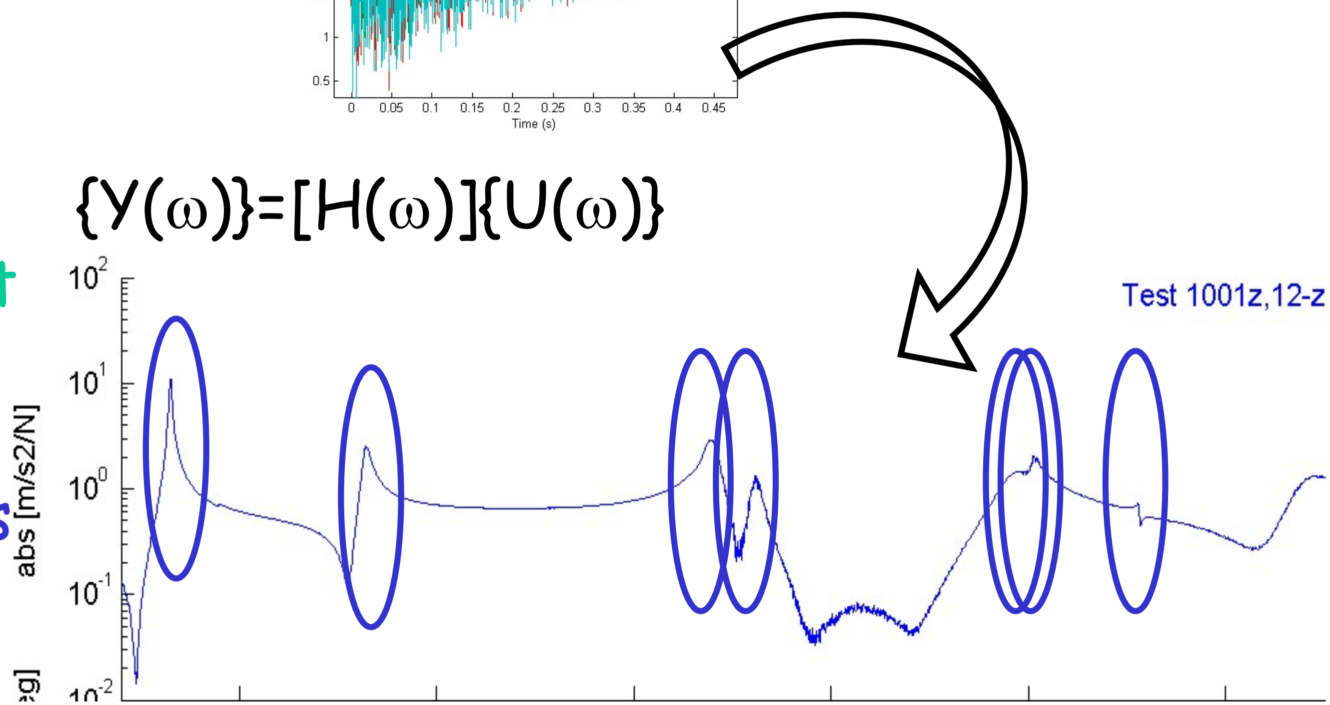
Transfers estimated from time response

ONE input

ONE output

$$\{Y(\omega)\}=[H(\omega)]\{U(\omega)\}$$

MANY resonances



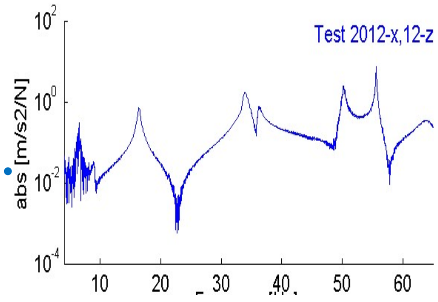
Some standard texts for the vibration community

[1] D. J. Ewins, Modal Testing: Theory and Practice. John Wiley and Sons, Inc., New York, NY, 1984.
[2] W. Heylen et P. Sas, Modal analysis theory and testing. Katholieke Universteit Leuven, Departement Werktuigkunde, 2006
[3] K. G. McConnell, Vibration Testing. Theory and Practice. Wiley Interscience, New-York, 1995

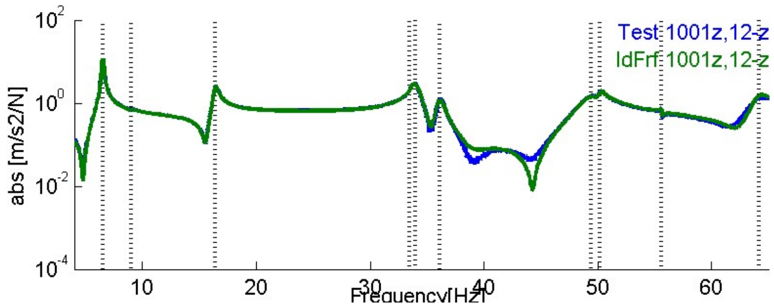
Experimental modal analysis: an inverse problem

See also chapter 5 (signal processing & non-parametric ID) & Signal_Test.pdf

Data
- Transfer
- Hankel mat.

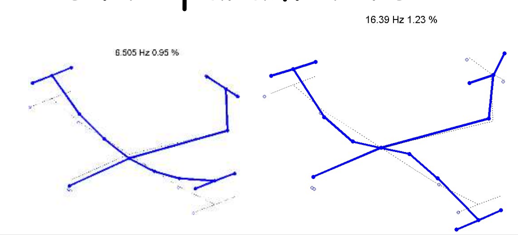


Objective H_{test}-H_{id}



Optimization

Result : parameters



Family of models

SVD $\sum_i \left\{\begin{matrix}U_i\\ \downarrow\\ sensors\end{matrix}\right\}\{a_i(t) \;\to\; time\}$

State space $\{\dot{x}\} = [A]\{x\} + [B]\{u\}$

Pole residue $\sum \frac{[R_j]_{NS\times NA}}{s-\lambda_j}$

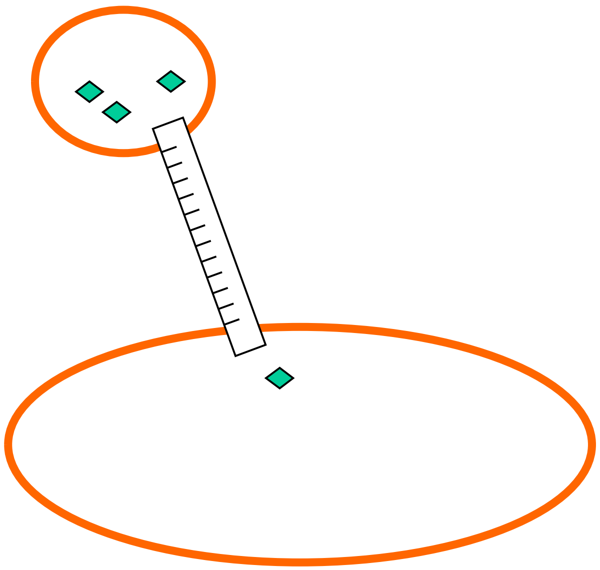
Rational fraction, modal model, second order …

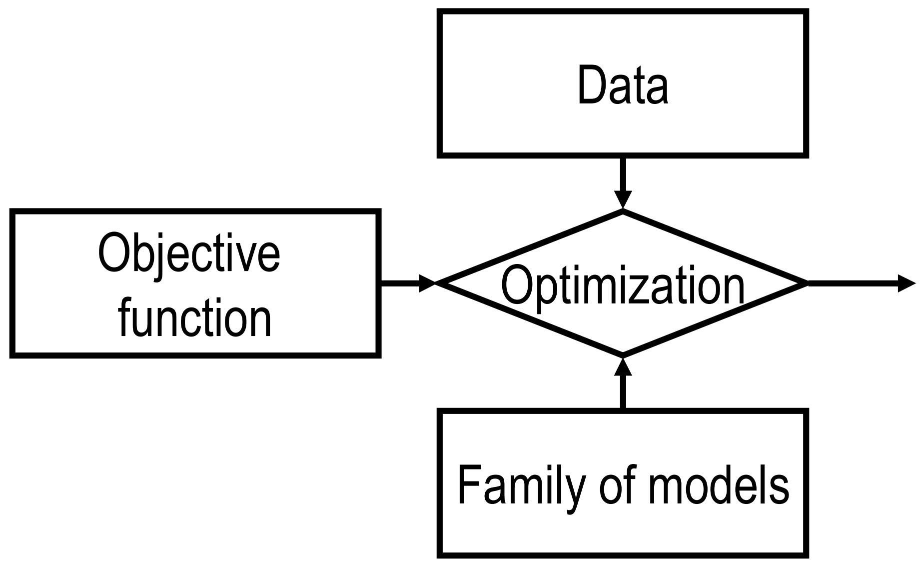
LTI model forms

- See MATLAB control or scipy.signal.lti
- ss state-space $$\{\dot{x}\} = [A]_{N\times N}\{x\} + [B]\{u\}_{NA\times 1}$$ $$\{y\}_{NS\times 1} = [c]\{x\} + D\{u\}$$
- Mechanical SS $$\left\{\begin{array}{c}\dot{p}\\ \ddot{p}\end{array}\right\} = \left[\begin{array}{cc}0 & I\\ -\left[\backslash\omega_j^2\backslash\right] & -\Gamma\end{array}\right]\left\{\begin{array}{c}p\\ \dot{p}\end{array}\right\} + \left[\begin{array}{c}0\\ \phi_j^T b\end{array}\right]\{u(t)\}$$ $$\{y(t)\} = \left[\begin{array}{cc}c\phi_j & 0\end{array}\right]\left\{\begin{array}{c}p\\ \dot{p}\end{array}\right\}$$
- tf $H = \frac{\sum_i [b_i]_{NS\times NA} s^i}{\sum_j a_j s^j}$ $$\left\{\begin{array}{c}\dot{q}\\ \ddot{q}\end{array}\right\} = \left[\begin{array}{cc}0 & I\\ -M^{-1}K & -M^{-1}C\end{array}\right]\left\{\begin{array}{c}q\\ \dot{q}\end{array}\right\} + \left[\begin{array}{c}0\\ M^{-1}b\end{array}\right]\{u(t)\}$$ $$\{y(t)\} = \left[\begin{array}{cc}c & 0\end{array}\right]\left\{\begin{array}{c}q\\ \dot{q}\end{array}\right\}$$
- zpk (zero pole gain) $H = k\frac{\prod_{k<N}(s-[z_j]_{NS\times NA})}{\prod_{j=1}^{N}(s-p_j)}$
- residue (partial fraction expansion = modal form)

$$[H(s)]_{NS\times NA} = \sum_{j=1}^{N/2}\left(\frac{[R_j]_{NS\times NA}}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}\right)$$

Inverse problem





This is done on the blackboard

Objective function

- Least squares
  - Objective : minimize distance
  - Variants : Log-least squares, output-error, …
- Singular value decomposition
  - Objective : constrained rank approximation

Circle fit / -3 dB

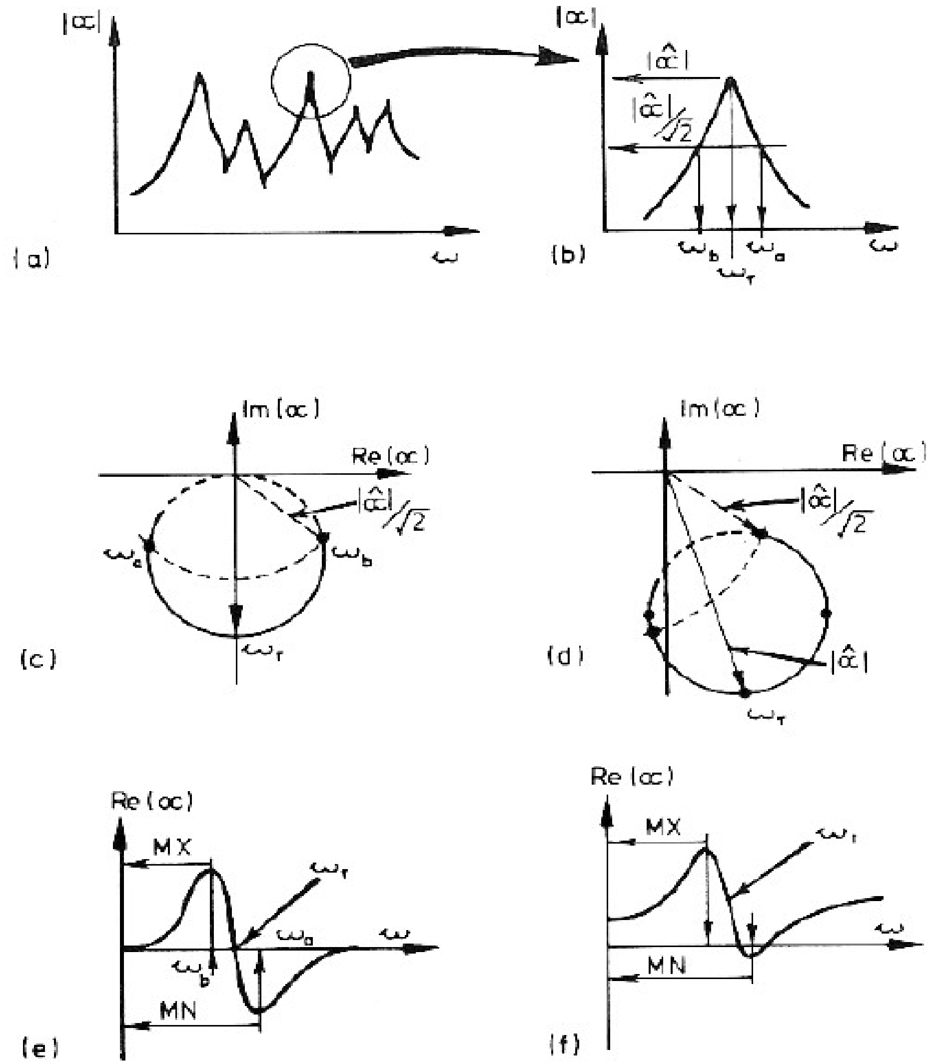
- Assumed model form

$$[H(s)] \approx \frac{A_j}{s^2 + 2\zeta_j \omega_j s + \omega_j^2}$$

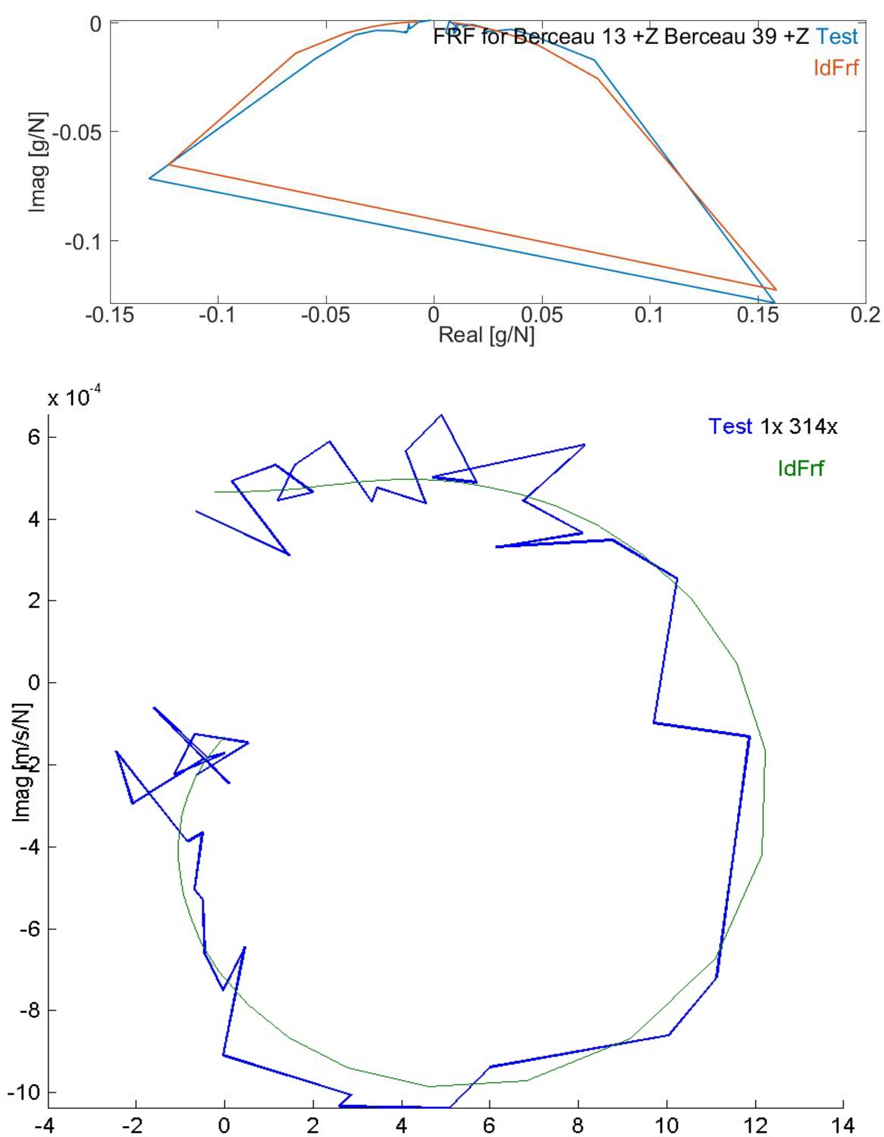
- Geometrical properties of Nyquist give ω,ζ

$$\zeta_j \approx \frac{\omega_b - \omega_a}{2\omega_n}$$

- Poor accounting of other contributions



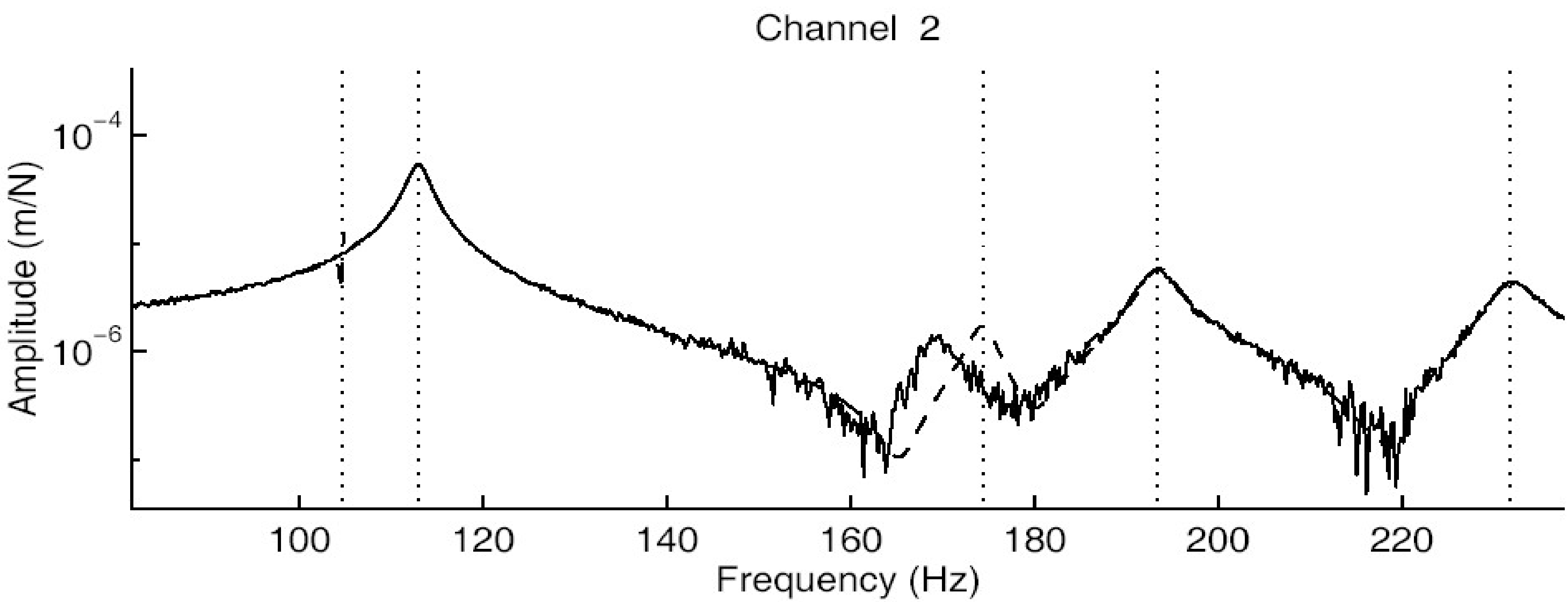
-3dB failures



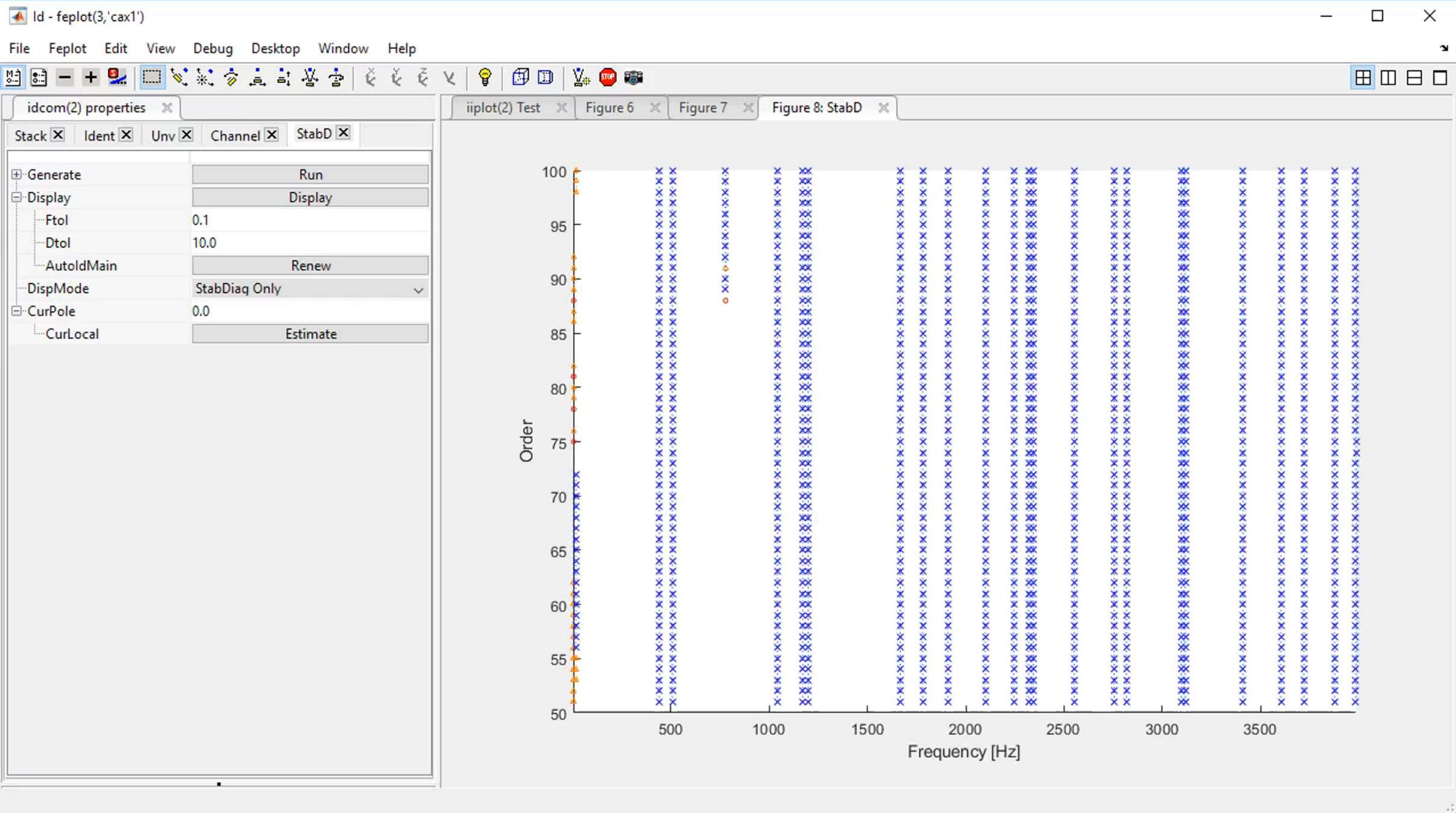
This is done on the blackboard

Example : FD polynomial

- This is done on the blackboard
- Section 7.4.1 (frequency domain ARX)



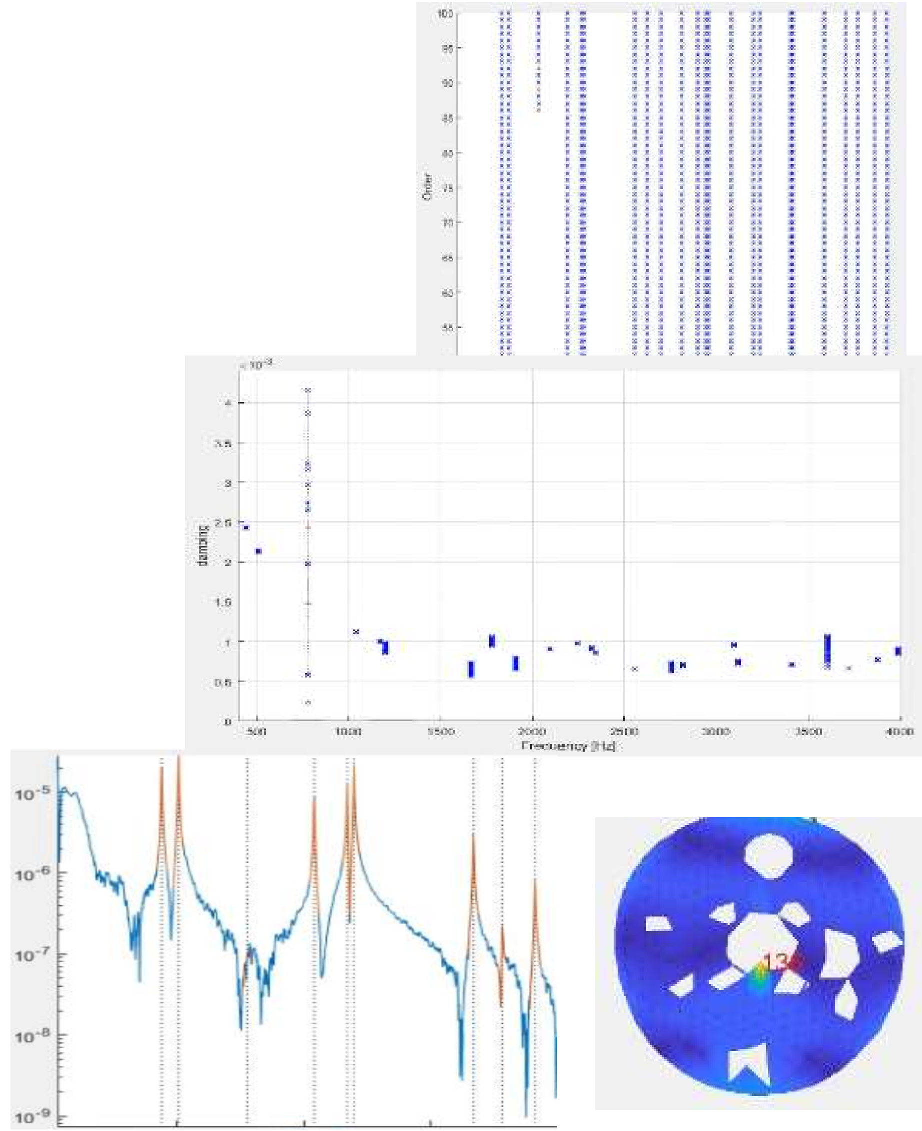
Classical linear system Id



https://www.youtube.com/channel/UCy5PHLw70NqfDDvg24tXJWg

Identification phases

- Initialization pick from :
  - Single pole estimate [1]
  - Stabilization diagram [2]
- Estimate by band (why ? [1,3])
- How can problems be detected ? [3-4]
- Re-optimize poles (why ?)



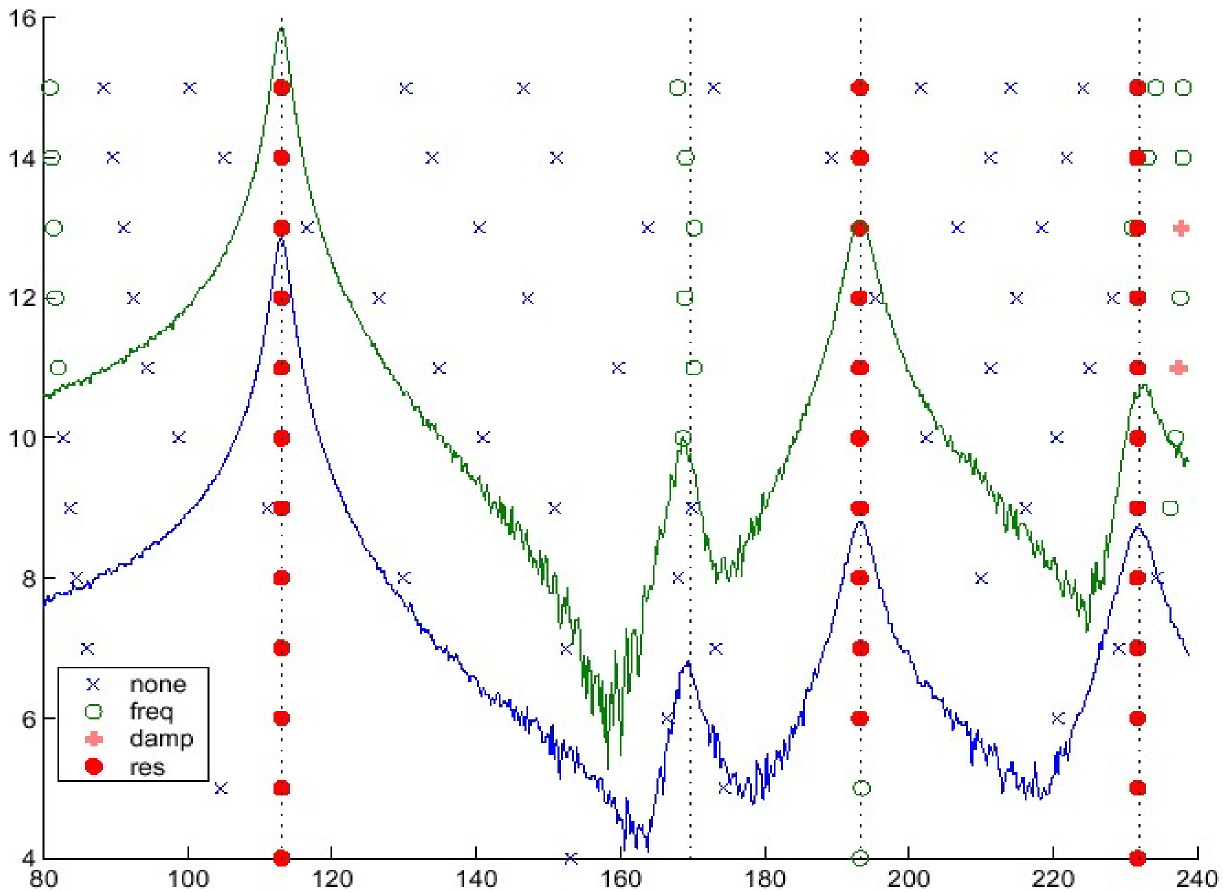
[1] E. Balmes, « Frequency domain identification of structural dynamics using the pole/residue parametrization », IMAC 1996

[2] P. Verboven, « Frequency-domain system identification for modal analysis », Ph.D. thesis, 2002.

[3] G. Martin, « Méthodes de corrélation calcul/essai pour l'analyse du crissement », Ph.D. CIFRE SDTools, Arts et Metiers ParisTech, Paris, 2017

[4] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

Stabilization diagrams

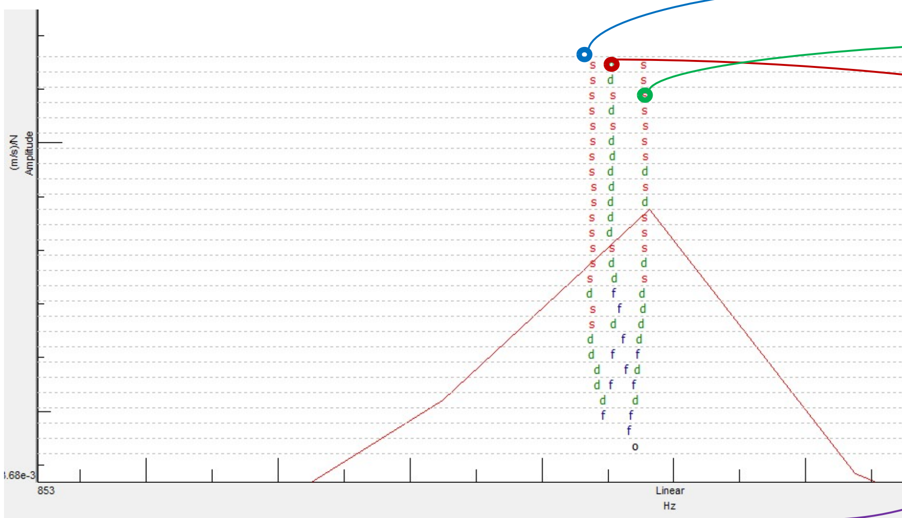


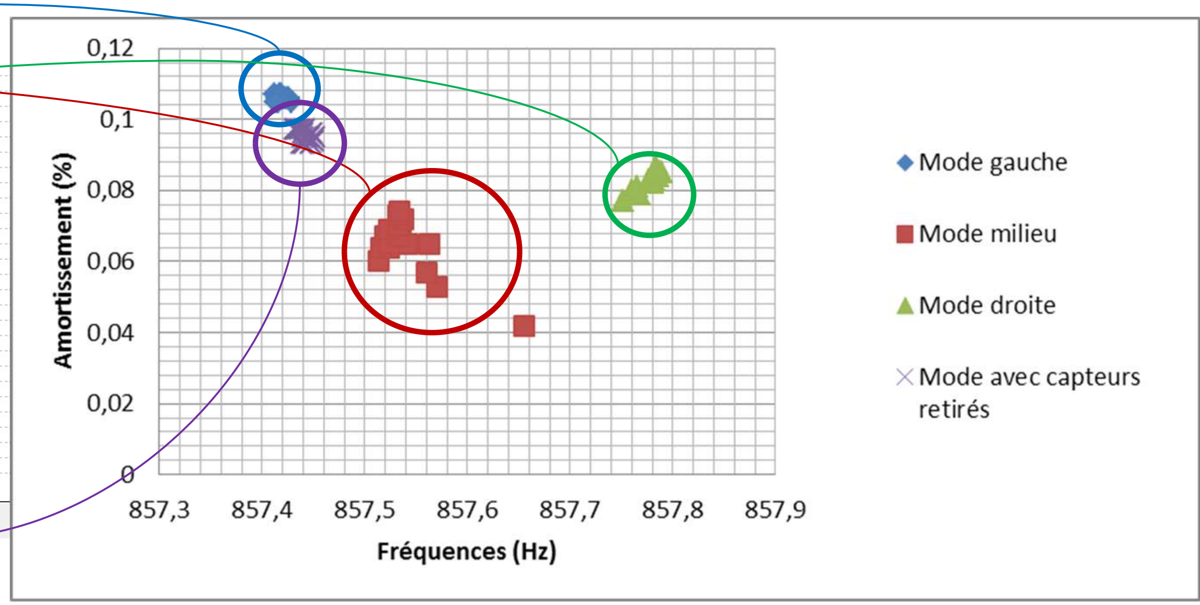
$$\Delta\omega_j/\omega_j < 1\%$$

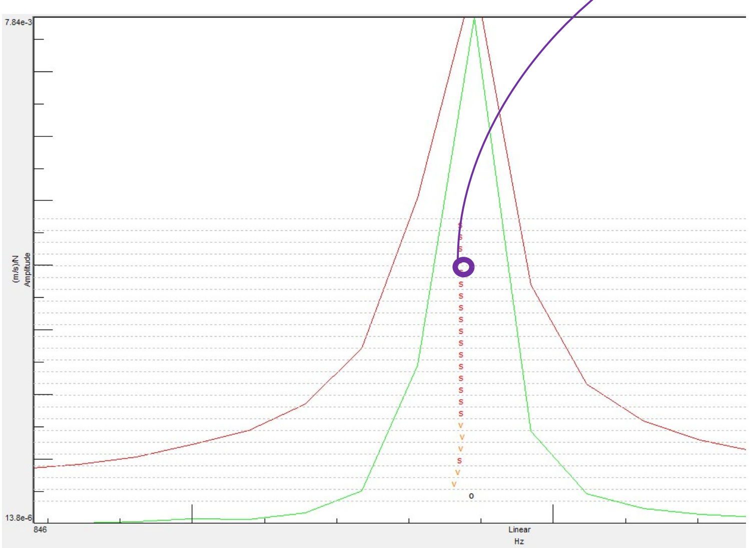
$$\Delta\zeta_j/\zeta_j < 10\%$$

$$\Delta R_j/R_j < 10\%$$

Stabilization diagrams & bias

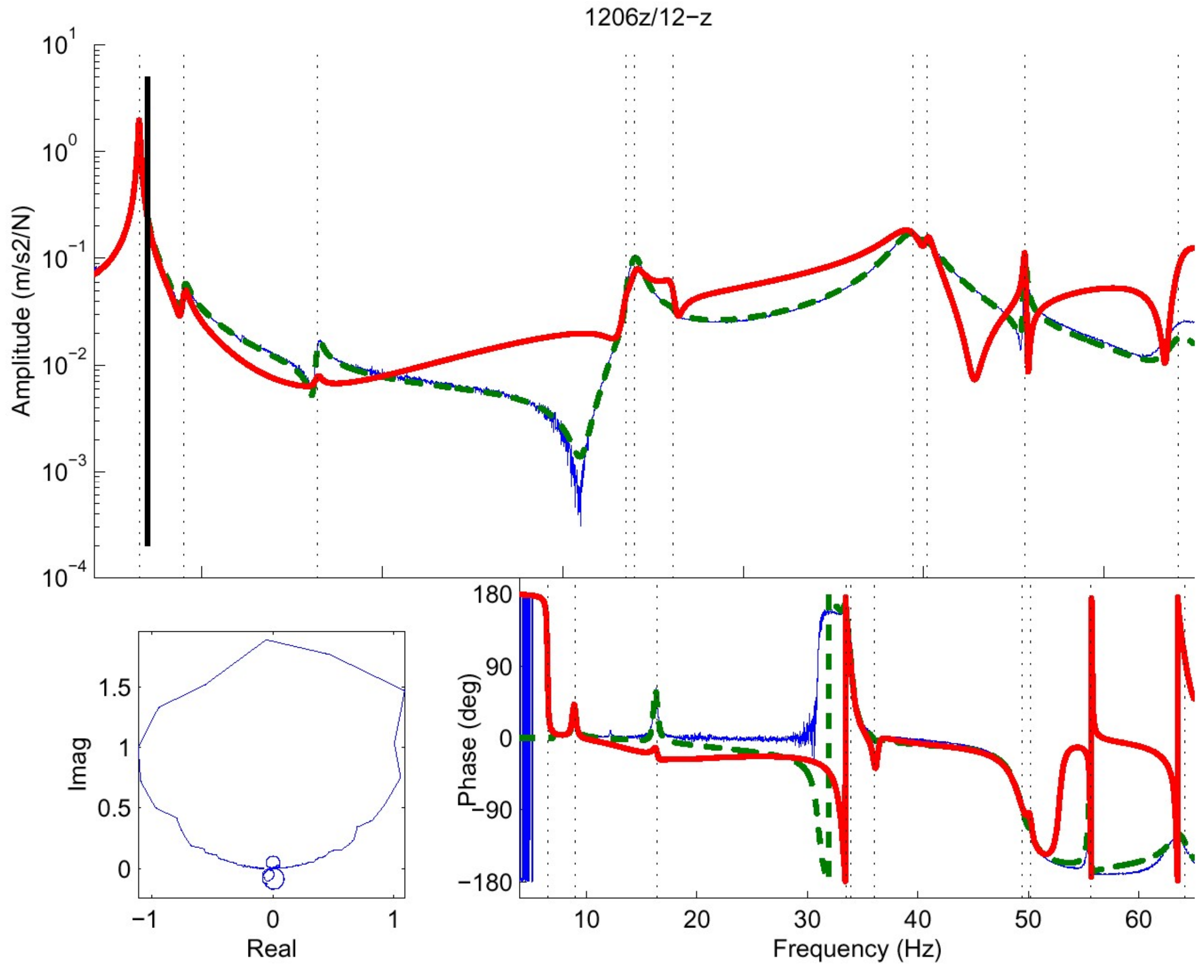






- Traditional representation with order does not show dispersion
- Removing wrong sensors improves result
  noise adds bias + *dispersion?*

Noise induced bias -> work by band

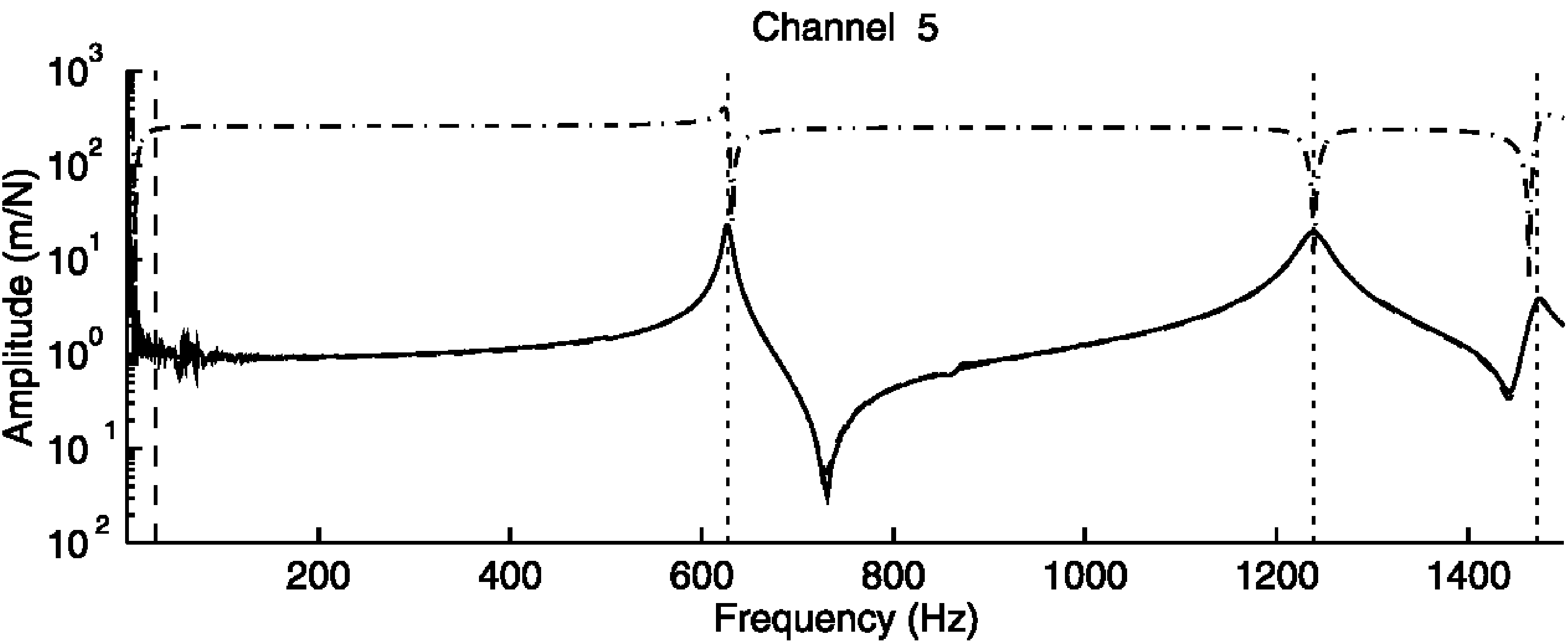


Inconsistence between data and family of models induces bias / variability

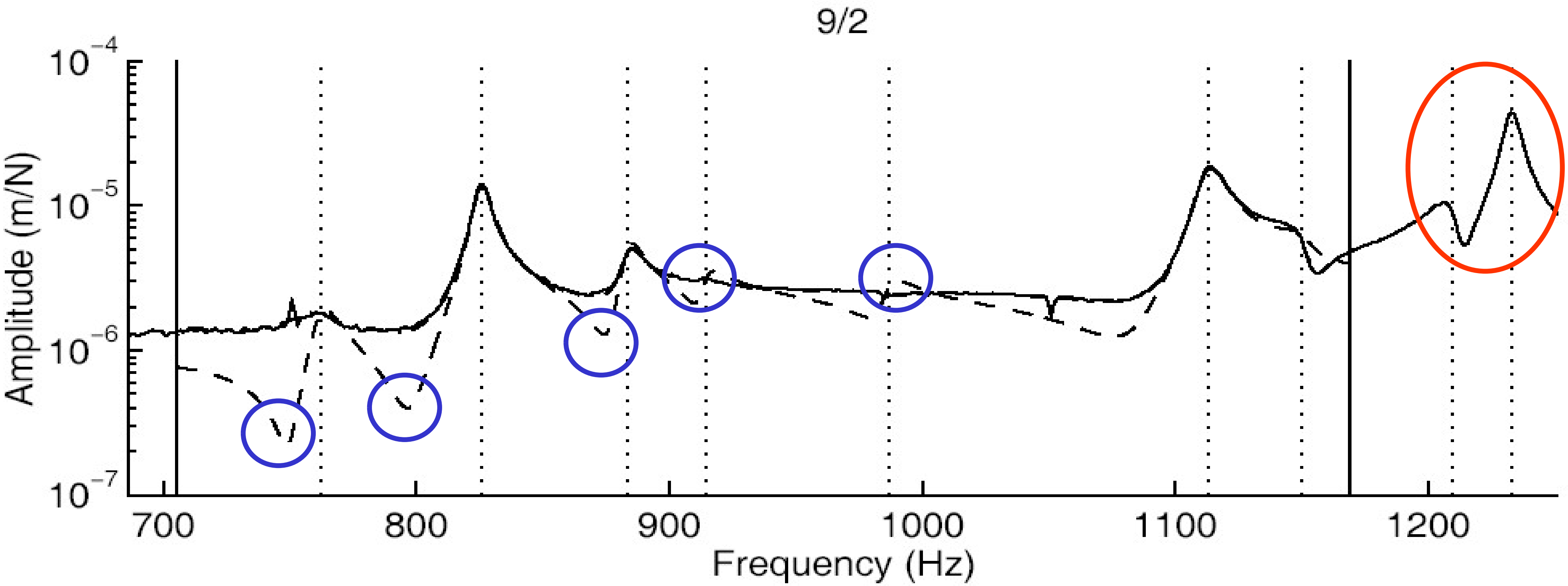
[1] M. Böswald, « Analysis of the bias in modal parameters obtained with frequency-domain rational fraction polynomial estimators », Proceedings of ISMA 2016

[2] El-kafafy, De Troyer, Peeters, Guillaume, « Fast maximum-likelihood identification of modal parameters with uncertainty intervals: A modal model-based formulation », MSSP, 2013

Low frequency noise bias



Effects of residual terms



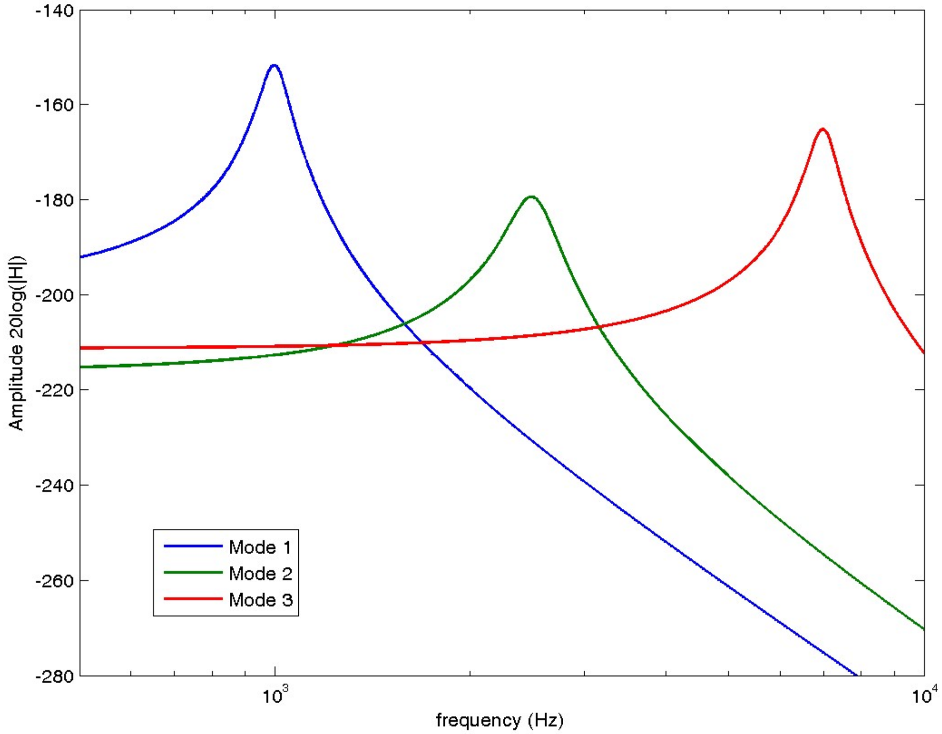
Here : ignoring two nearby modes induces bias

In general : account for flexibility and inertia effects (lower/upper residual)

Identification model

- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$



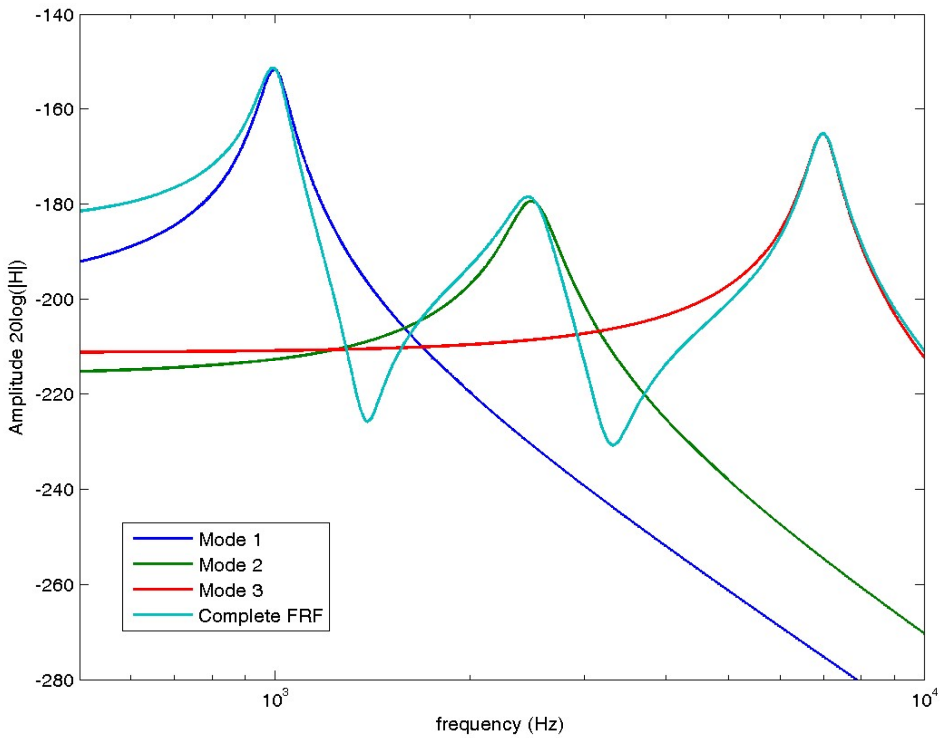
$\boxed{[H_1(s)]}$ $\boxed{[H_2(s)]}$ $\boxed{[H_3(s)]}$

Identification model

- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$

$\boxed{[H_1(s)]}$ $\boxed{[H_2(s)]}$ $\boxed{[H_3(s)]}$



$$[H_{model}(s)] = \boxed{[H_1(s)]} + \boxed{[H_2(s)]} + \boxed{[H_3(s)]}$$

Identification model

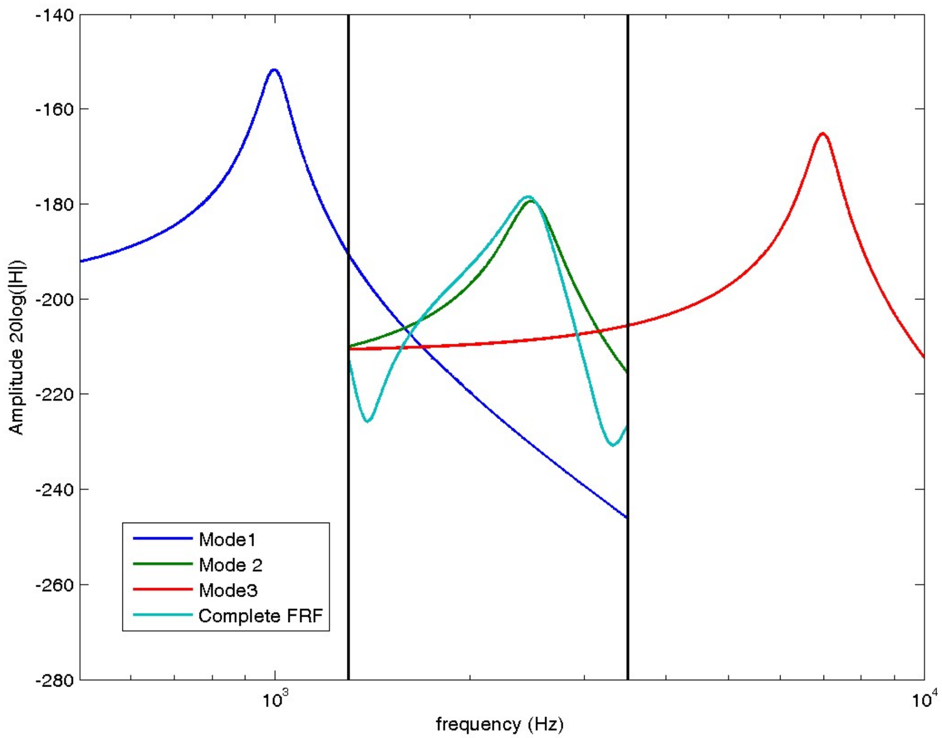
- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$

Bandwidth

Low residual $\quad$ in $\quad$ High residual

$\boxed{\cancel{[H_1(s)]}} \quad \boxed{[H_2(s)]} \quad \boxed{\cancel{[H_3(s)]}}$

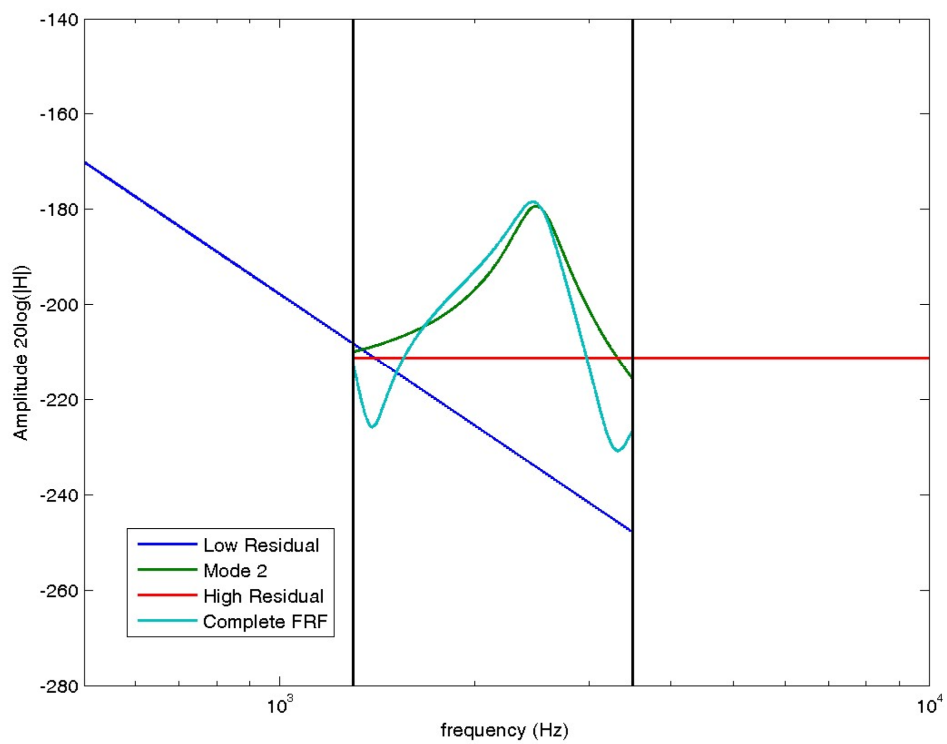


$$[H_{model}(s)] = \boxed{\cancel{[H_1(s)]}} + \boxed{[H_2(s)]} + \boxed{\cancel{[H_3(s)]}}$$

Identification model

- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$



Bandwidth

Low residual: $\boxed{\cancel{[H_1(s)]}} \rightarrow \boxed{\dfrac{[E]}{s^2}}$

in: $\boxed{[H_2(s)]}$

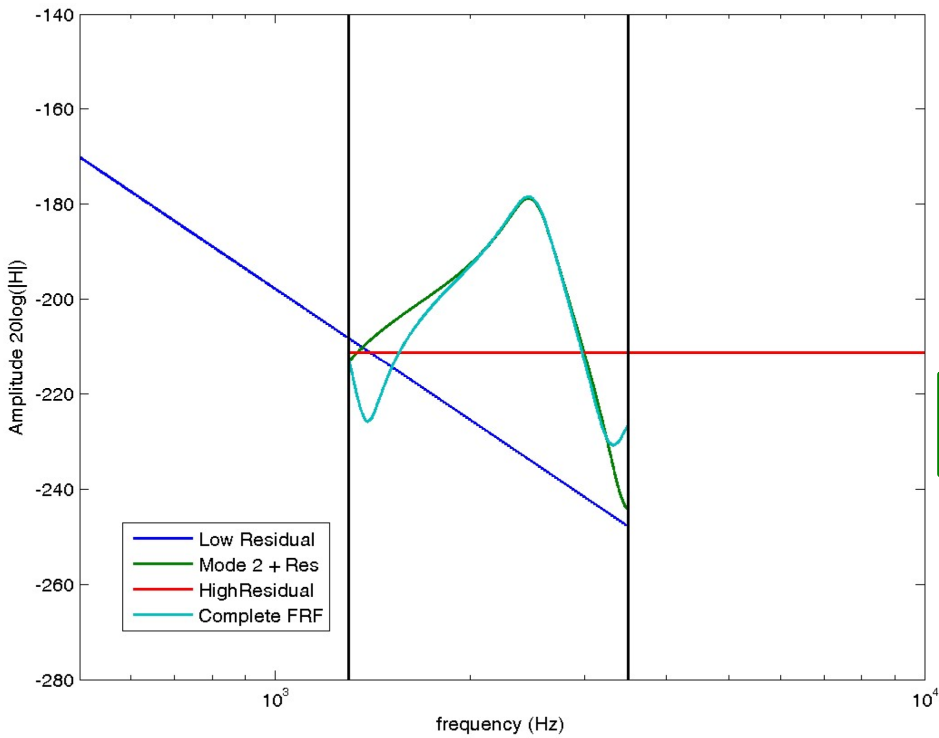
High residual: $\boxed{\cancel{[H_3(s)]}} \rightarrow \boxed{[F]}$

$$[H_{model}(s)] = \boxed{\cancel{[H_1(s)]}} + \boxed{[H_2(s)]} + \boxed{\cancel{[H_3(s)]}}$$

Identification model

- Pole/Residue with high and low residual terms

$$[H_j(s)] = \frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j}$$



Bandwidth

Low residual | in | High residual

$\boxed{\cancel{[H_1(s)]}}$ $\rightarrow$ $\boxed{\frac{[E]}{s^2}}$

$\boxed{[H_2(s)]}$

$\boxed{\cancel{[H_3(s)]}}$ $\rightarrow$ $\boxed{[F]}$

$$\boxed{[H_{model}(s)]} = \boxed{\frac{[E]}{s^2}} + [H_2(s)] + \boxed{[F]}$$

Identification strategies

$$\min_{\lambda_j} \min_{R_j,E,F} \left| [\alpha(s)]_{test} - \sum_{j \in \texttt{identified}} \left(\frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j} \right) + [E(s)] \right|^2$$

Freq. domain linear least squares with residuals
almost always used
Unknowns λ_j (2*Np), R_j (Ns*Na*2*Np), E,F (Ns*Na*2)

Determination of poles : main differentiator between methods

Two main strategies

- select in over-specified set
- build gradually

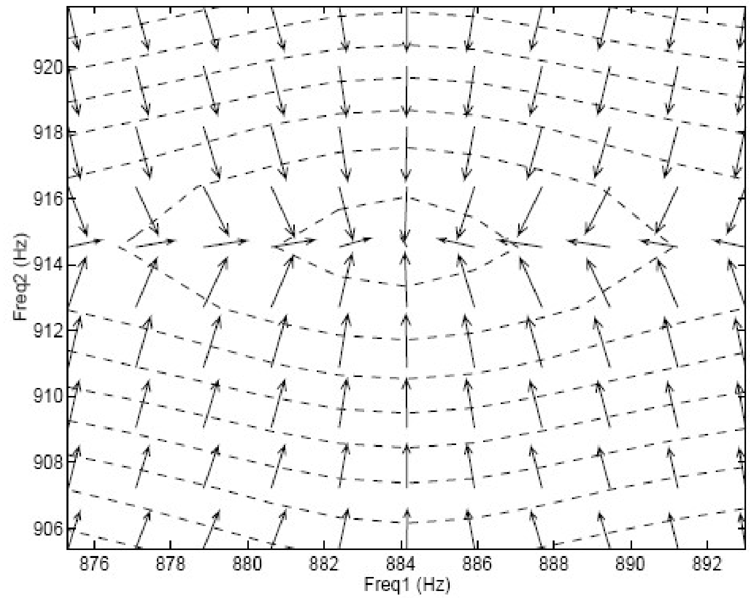
Frequency domain output error

$$\min_{\lambda_j} \min_{R_j,E,F} \left| [\alpha(s)]_{test} - \sum_{j \in \texttt{identified}} \left(\frac{[R_j]}{s-\lambda_j} + \frac{[\bar{R}_j]}{s-\bar{\lambda}_j} \right) + [E(s)] \right|^2$$

- R_j,E,F implicit functions of λ_j
- $\frac{\partial J}{\partial \lambda_j} = \frac{\partial R^T R}{\partial \lambda_j} = 2R^T \frac{\partial R}{\partial \lambda_j}$ can be computed analytically
- around a pole gradient not very affected by error on other poles

SDT implements

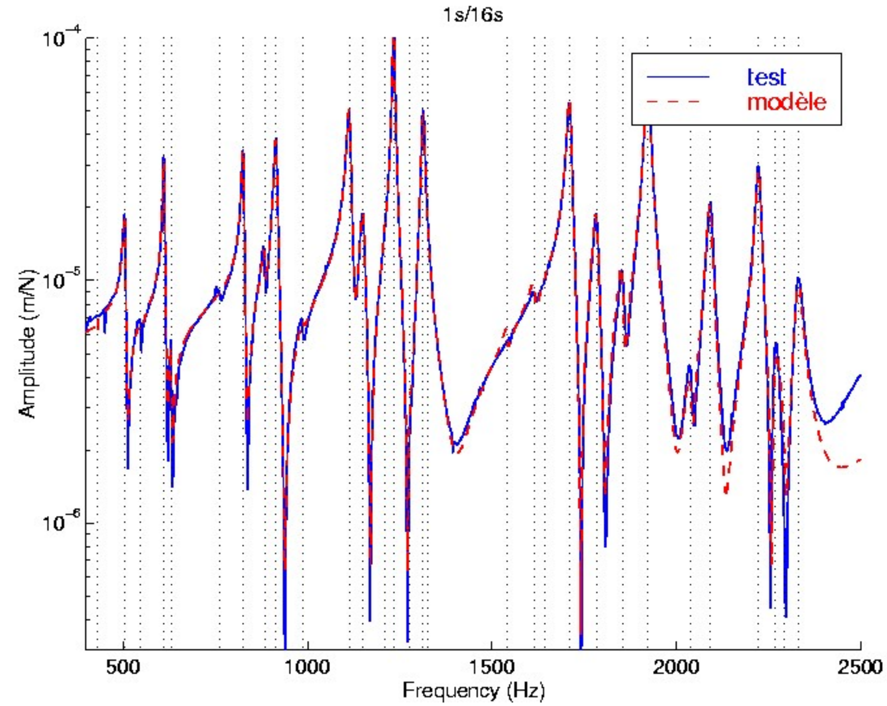
- Fast optimization
- All λ_j changed at each step using gradient only

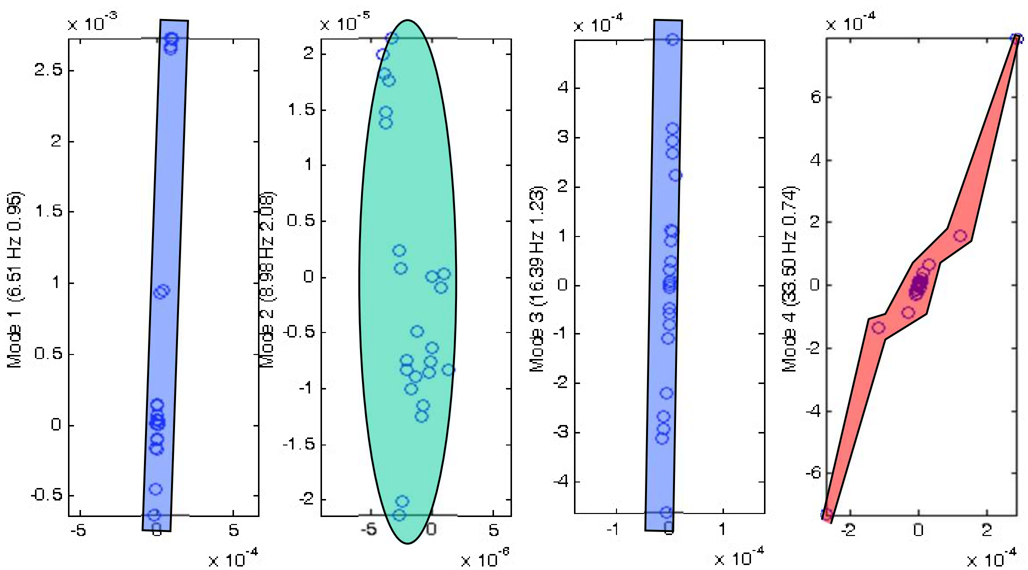


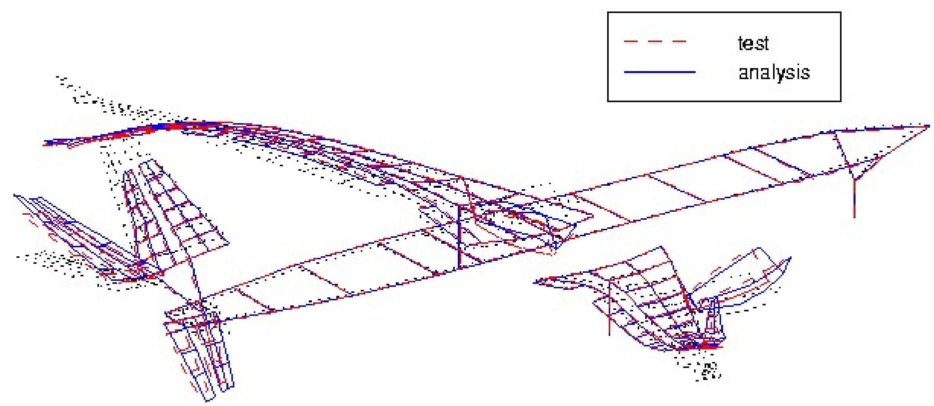
Balmes, PhD MIT 93 & http://www.sdtools.com/pdf/IMAC96id.pdf

Validations of results

- FRF Superposition
- Modeshape visualization
- Residue properties

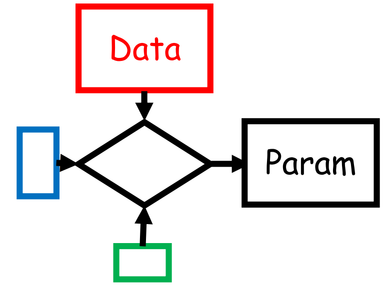




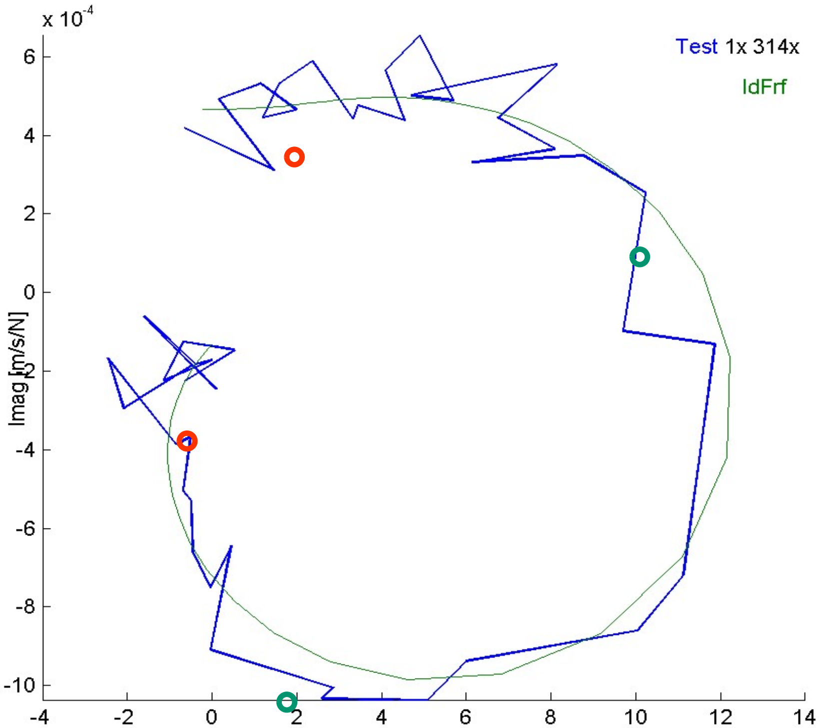


Quality : an error criterion

Is the model well identified?



- Superpose measured and identified FRF for each sensor c, around each mode j
- Compute error on Nyquist



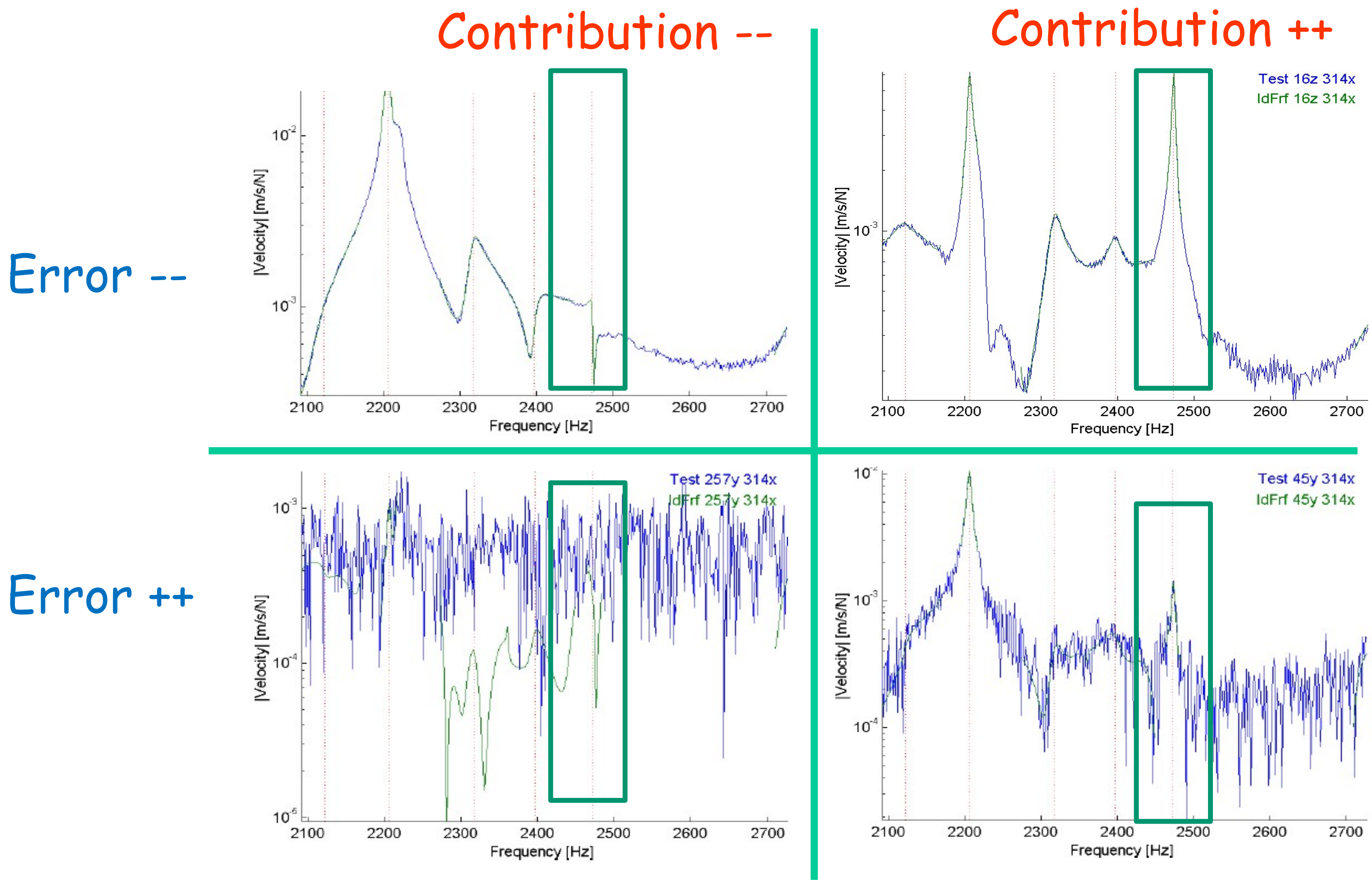
$$e_{jc} = \frac{\int_{\omega_j(1-\alpha\zeta_j)}^{\omega_j(1+\alpha\zeta_j)} |H_{cTest} - H_{cId}|^2}{\int_{\omega_j(1-\alpha\zeta_j)}^{\omega_j(1+\alpha\zeta_j)} |H_{cTest}|^2}$$

"Around each mode" :

- Half power : +/- $\zeta_j \omega_j$
- Visible peak : +/- $5\zeta_j \omega_j$

[1] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

Test/identification error



For each sensor, each mode may have strong error/noise & low contribution

Test error per mode/sensor

[1] G. Martin, E. Balmes, et T. Chancelier, « Characterization of identification errors and uses in localization of poor modal correlation », MSSP 2017.

Evaluate quality of identification

Id - iiplot(2) IdFrf

File Edit View Iiplot Insert Debug Desktop Window Help

idcom(2) properties

Stack Ident Unv Channel StabD Qual

Modes

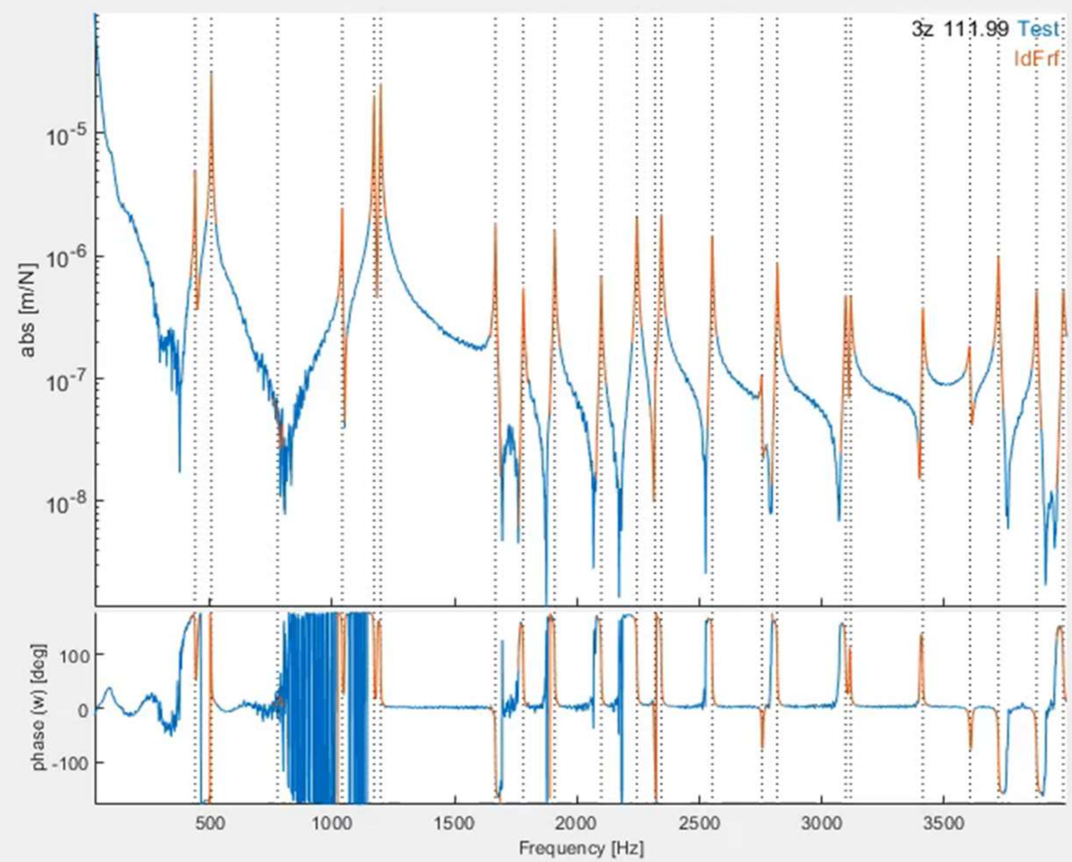
Mode#	Freq[Hz]	Damp[%]	Error[%]	Contrib[...	MPC[%]	NOSmax...
1	441,5	0,24	7	88	90	3
2	507,4	0,21	10	82	94	1
3	778,1	0,35	40	11	1	14
4	1041,6	0,11	7	87	100	19
5	1171,5	0,10	7	77	100	6
6	1198,3	0,10	8	79	100	51
7	1667,4	0,07	12	81	99	55
8	1780,6	0,11	14	61	98	5
9	1909,1	0,08	12	76	100	53
10	2098,7	0,09	5	86	100	5
11	2244,6	0,10	4	82	100	1
12	2319,0	0,09	4	54	100	30
13	2344,2	0,09	4	88	100	7
14	2552,2	0,07	7	93	100	8

I/O Pairs

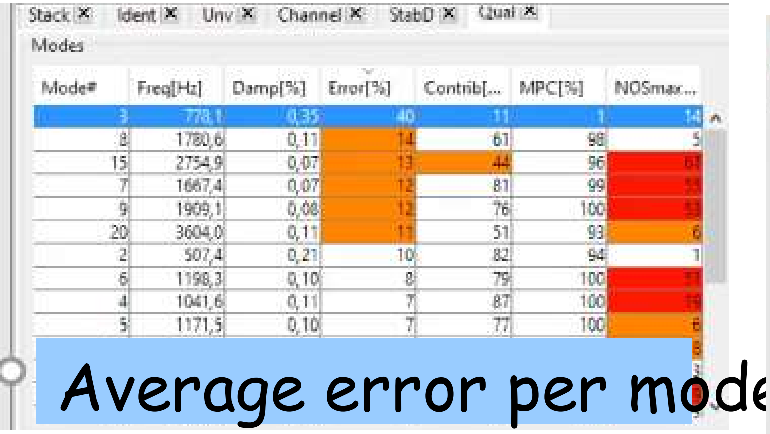
Mode	Out	In	Error	Level	Contrib	NOS
23	1,01	111,99	9,8%	3,5%	85,9%	0,3%
23	2,01	111,99	6,9%	3,9%	73,5%	0,3%
23	3,01	111,99	4,4%	3,7%	70,7%	0,2%
23	4,01	111,99	10,8%	4,4%	52,9%	0,5%
23	5,01	111,99	6,2%	5,6%	54,9%	0,3%
23	6,01	111,99	4,2%	6,6%	59,5%	0,3%
23	7,01	111,99	6,6%	4,2%	57,6%	0,3%
23	8,01	111,99	23,9%	0,9%	0,0%	0,2%
23	9,01	111,99	9,2%	3,5%	73,5%	0,3%
23	10,01	111,99	9,7%	2,6%	52,5%	0,3%
23	11,01	111,99	5,4%	3,4%	55,5%	0,2%
23	12,01	111,99	5,2%	2,4%	29,3%	0,1%
23	13,01	111,99	7,3%	2,3%	15,2%	0,2%
23	14,01	111,99	5,0%	2,8%	27,7%	0,1%

iiplot(2) IdFrf | Figure 1: StabD

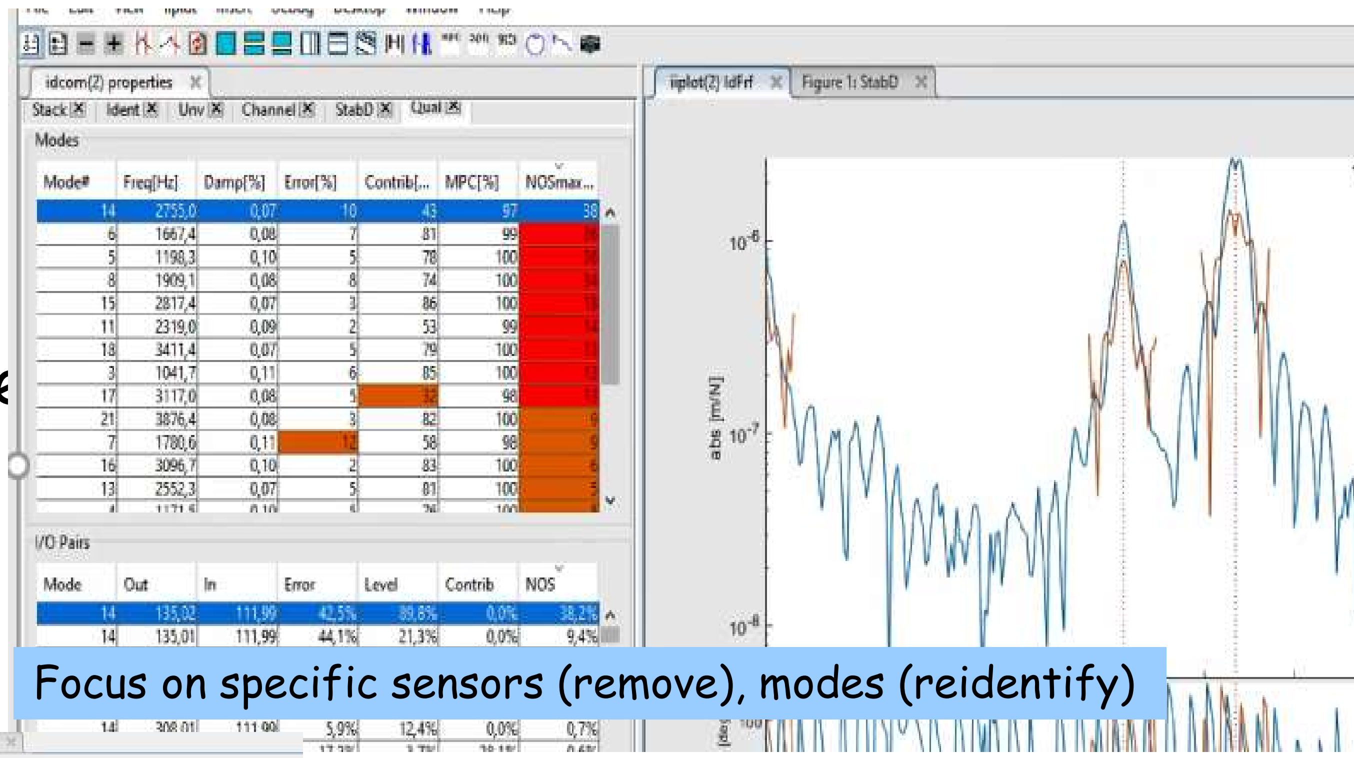
Identified model in IdMain



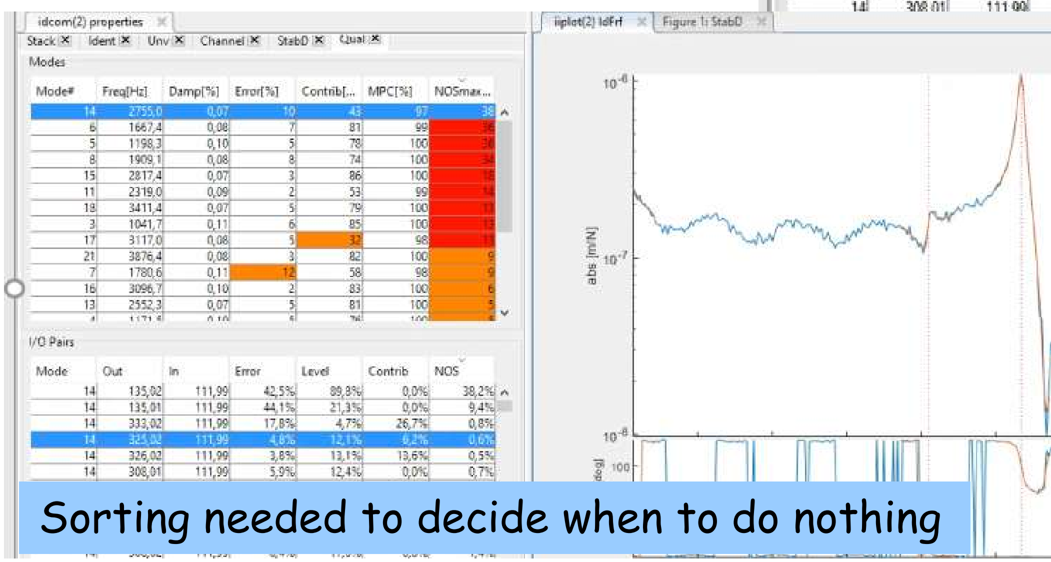
Error analysis usage



Average error per mode

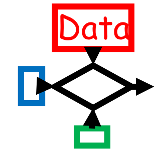


Focus on specific sensors (remove), modes (reidentify)



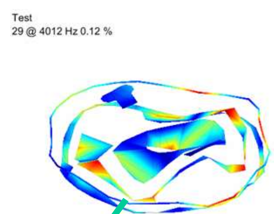
Sorting needed to decide when to do nothing

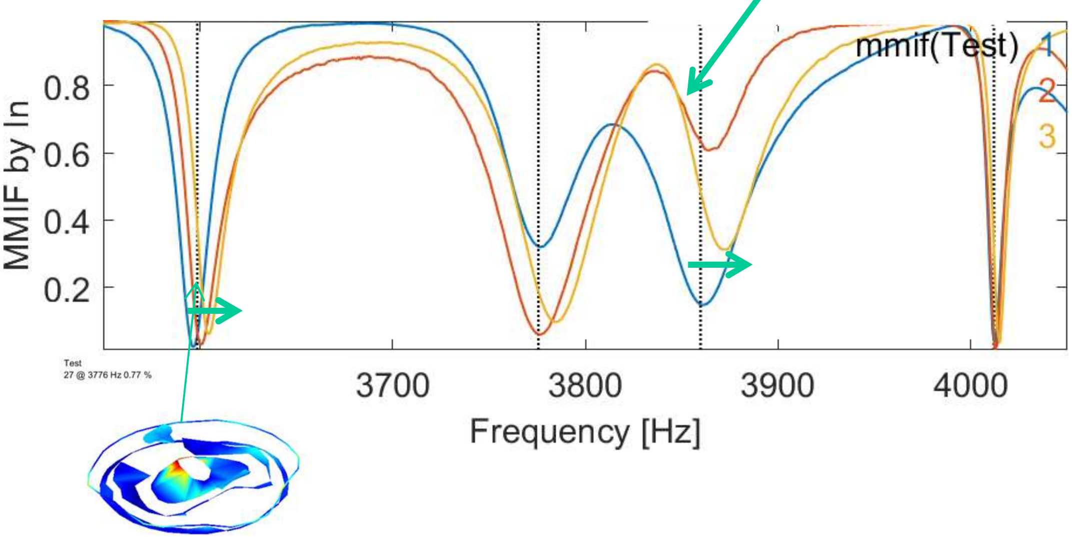
Other errors : NL/LPV ≠ one system

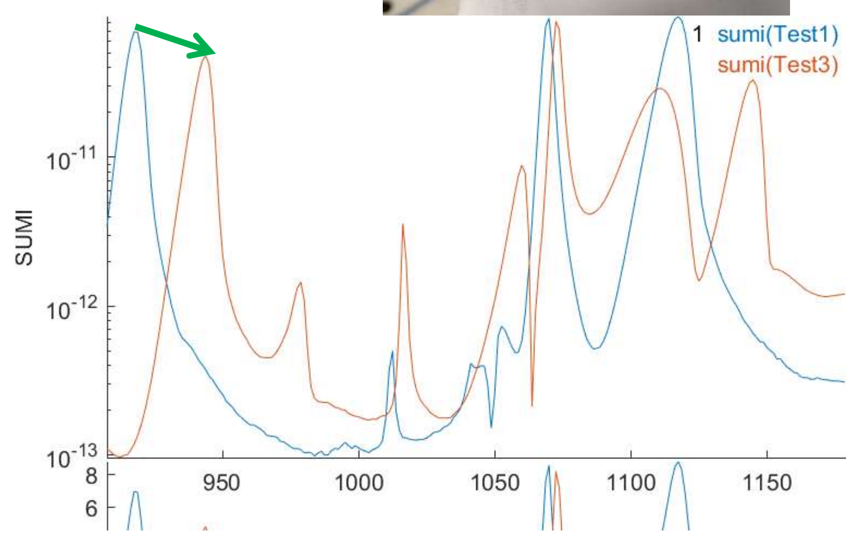


- Non linear system : resonance dependent on input point / accurate positioning

- MMIF or identification per impact location shows significant dispersion
- Multiple identification results are not perfectly coherent

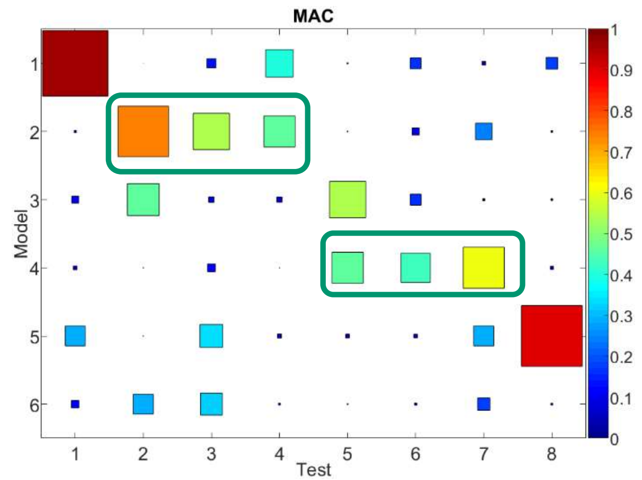




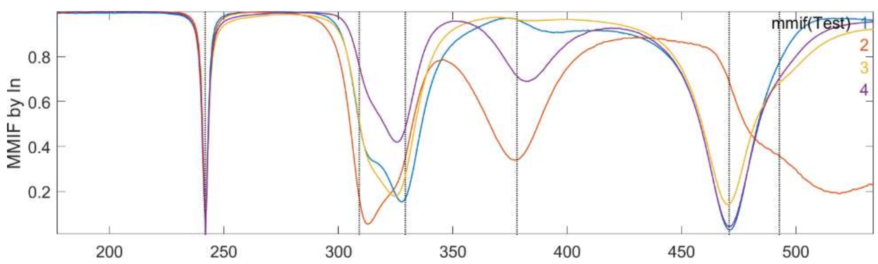


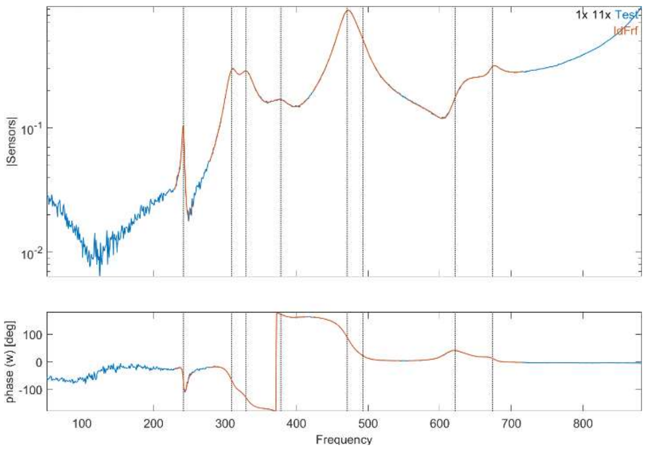
Other errors : local modes

- « Same mode » multiple times



- non structural masses generate global mode duplicates
- « small mass » ≠ « tiny peak » (if this were true proof mass dampers would not work)





Subspace methods : IO (ERA)

$$[h(k\Delta t)]_{pq} = \begin{bmatrix} [y(k)] & [y(k+j_1)] & \dots & [y(k+j_{q-1})] \\ [y(k+i_1)] & [y(k+i_1+j_1)] & \dots & [y(k+i_1+j_{q-1})] \\ \vdots & \vdots & \ddots & \vdots \\ [y(k+i_{p-1})] & [y(k+i_{p-1}+j_1)] & \dots & [y(k+i_{p-1}+j_{q-1})] \end{bmatrix}$$

$$[h(k\Delta t)]_{pq} = [C]_p [A]^{k-1} [B]_q = \begin{bmatrix} [C] \\ \vdots \\ [C][A]^{j_{p-1}} \end{bmatrix} [A]^{k-1} \left[[B] \;\dots\; [A]^{j_{q-1}} [B] \right]$$

$$[h(k\Delta t)]_{pq} \approx [U] \left[{}^{\setminus} S_{\setminus} \right] [V]^T$$

$$[A] = \left[\left[{}^{\setminus} S_{\setminus} \right]^{-1/2} [U]^T [h(2\Delta t)]_{pq} [V] \left[{}^{\setminus} S_{\setminus} \right]^{-1/2} \right]$$

$$[B] = \left[{}^{\setminus} S_{\setminus} \right]^{1/2} [V]^T \begin{bmatrix} \left[{}^{\setminus} I_{\setminus} \right] \\ [0] \\ [0] \end{bmatrix}_{qNA \times NA}$$

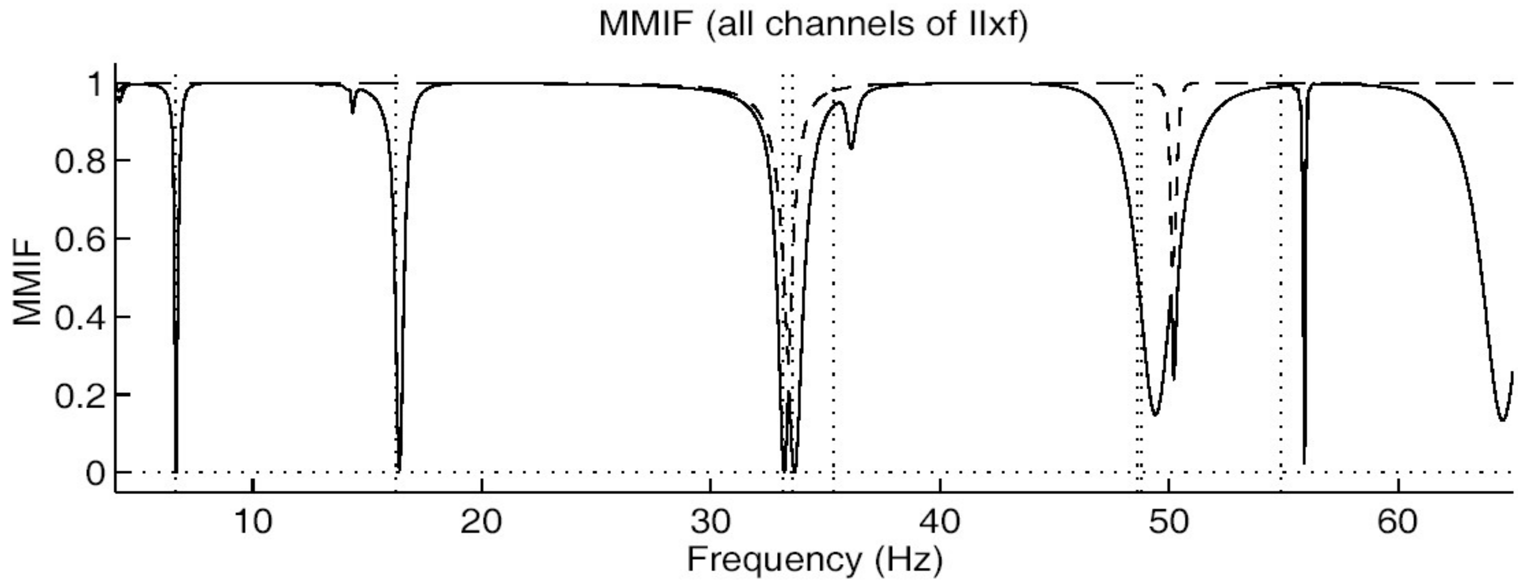
$$[C] = \left[\left[{}^{\setminus} I_{\setminus} \right] [0] \; [0] \right]_{NS \times pNS} [U] \left[{}^{\setminus} S_{\setminus} \right]^{1/2}$$

[1] J. N. Juang and R. S. Pappa, “An Eigensystem Realization Algorithm (ERA) for Modal Parameter Estimation and Model Reduction,” *J. Guidance, Control, and Dynamics*, vol. 8, no. 5, pp. 620–627, 1985.
[2] E. Reynders, “System identification methods for (operational) modal analysis: review and comparison,” Archives of Computational Methods in Engineering, vol. 19, no. 1, pp. 51–124, 2012.

Order selection : MMIF

- Mode indicator functions
- Related to force appropriation (do by hand)
- Appropriation excites a single mode (allow picture taking)

$$MMIF_i = min_{u \perp u_{j<i}} \frac{u^T[Re(H)^T Re(H)]u}{u^T[H^H H]u}$$

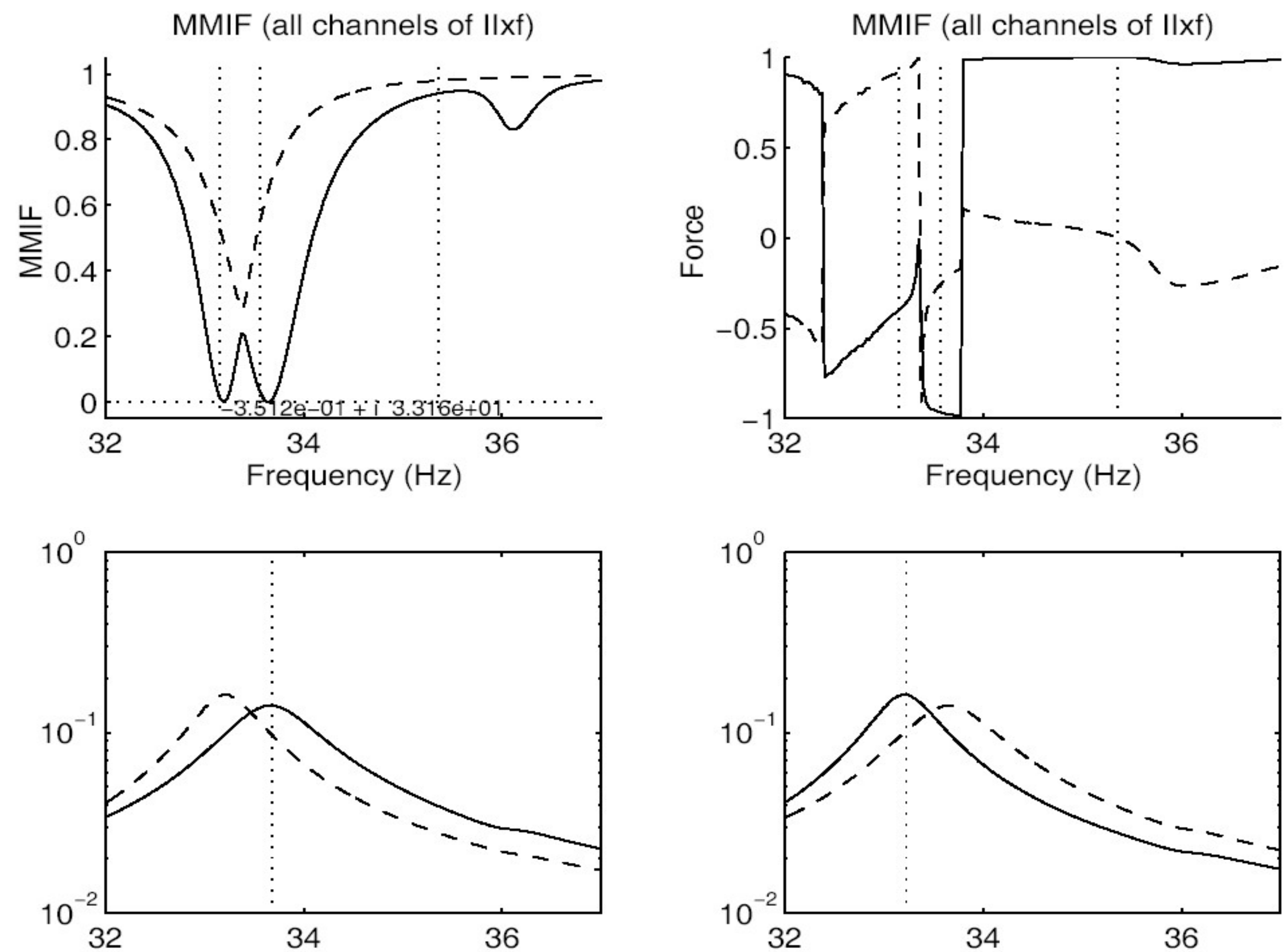


Order selection : MMIF

- Multi exciter allows spatial filter $\{u_i(t)\}_{NA} = \{w_i\}_{NS} a(t)$
- Appropriation : choose $\{w_i\}_{NS}$ to make other modes unobservable
- Keywords

  Modal coordinates

  Modal filter

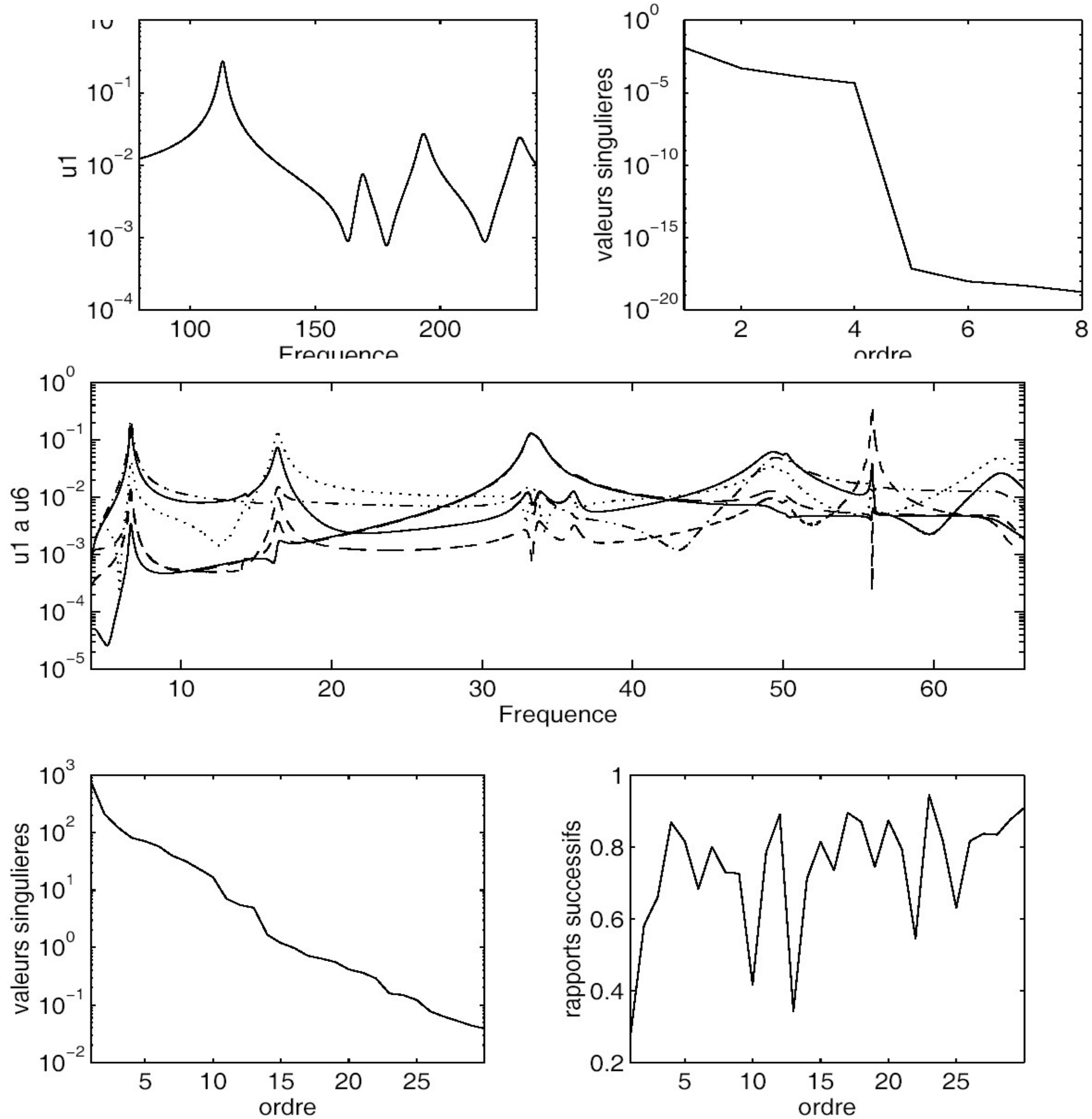


Order selection : SVD

$$[H_{kl}(\omega)]_{NW\times N_{kl}} = \left[\frac{1}{\omega_{1:NW}-\lambda_{1:NM}}\right]_{NW\times NM}[R_{j,kl}]_{NM\times N_{kl}}$$

- Rank of H is limited
- Perfect for simulation
- Problem for real data
- More useful for data volume reduction

P. Guillaume & al. 2005 Improved techniques for outlier detection



Properties of residues

- Minimal

$$\frac{R_j}{s-\lambda_j} = \frac{\{c\psi_j\}\left\{\psi_j^T b\right\} + \{c\psi_{j+1}\}\left\{\psi_{j+1}^T b\right\}}{s-\lambda_j}$$

- Reciprocal

$$\left([c_{\mathtt{col}}]\{\psi_j\}\right)^T = \{\psi_j\}^T [b_{\mathtt{col}}]$$

- Second order model (modal damping)

$$\frac{[R_j]}{s-\lambda_j} + \frac{\left[\bar{R}_j\right]}{s-\bar{\lambda}_j} = \frac{2(s+\zeta_j\omega_j)Re(R_j) + 2\omega_j\sqrt{1-\zeta_j^2}Im(R_j)}{s^2+2\zeta_j\omega_j s+\omega_j^2}$$

$$[R_j] = \frac{T_j}{i\omega_j\sqrt{1-\zeta_j^2}} = [c]\,\frac{\phi_j}{\sqrt{i\omega_j\sqrt{1-\zeta_j^2}}}\frac{\phi_j^T}{\sqrt{i\omega_j\sqrt{1-\zeta_j^2}}}\,[b]$$

- Modal mass (reciprocal, second order)

$$\frac{1}{\mu_j(c_{col})} = c_{col}\phi_j\phi_j^T b_{col} = (c_{col}\phi_j)^2 = T_{jcol} = i\omega_j\sqrt{1-\zeta_j^2}R_{jcol}$$

- Second order non proportional damping