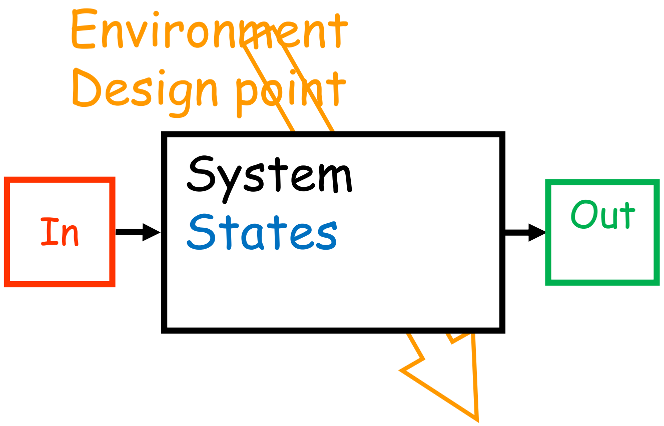
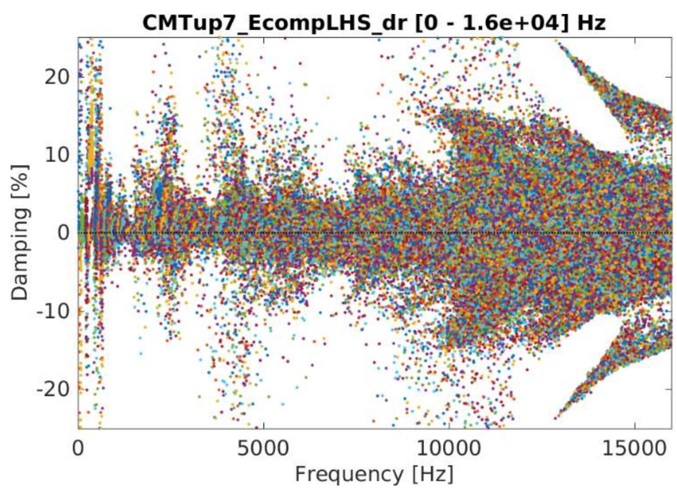
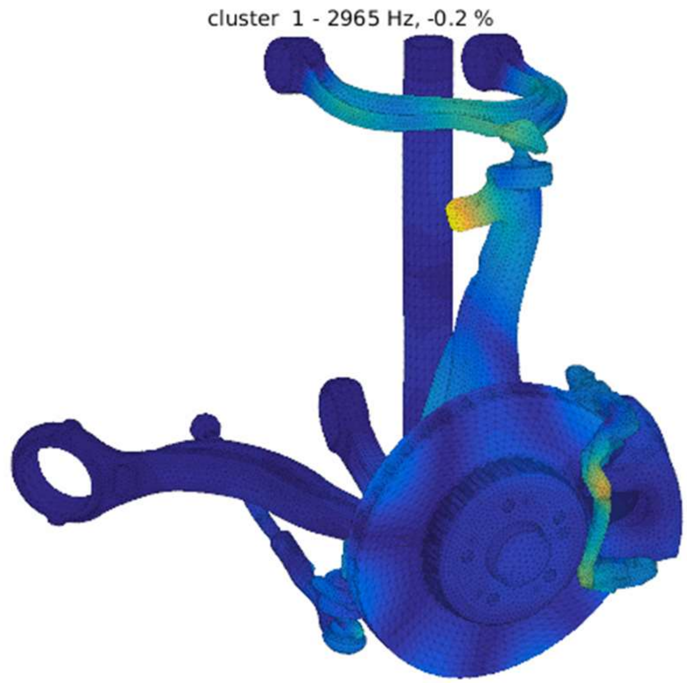
Parametric problems







MS2SC http://savoir.ensam.eu/moodle/course/view.php?id=1874

PROVIR http://savoir.ensam.eu/moodle/course/view.php?id=490

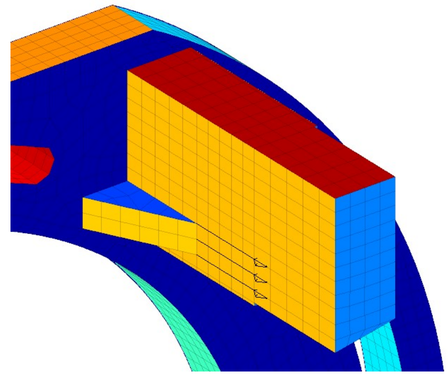
Geometry parametrization / morphing

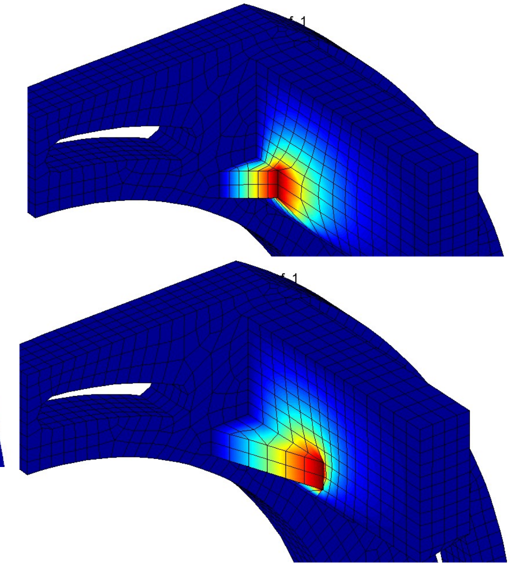
- Shape optimization / morphing

$$P(x) = \sum_i \{p_i(x)\} q_{imaster}$$

fe_shapeoptim BuildFromSel

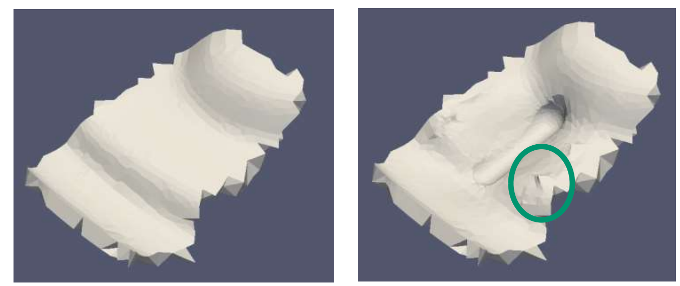
1) fix bottom face, 2) prescribe edge motion
3) deform edges (straight)
4) deform faces (flat)
5) deform interior (good elements ?)

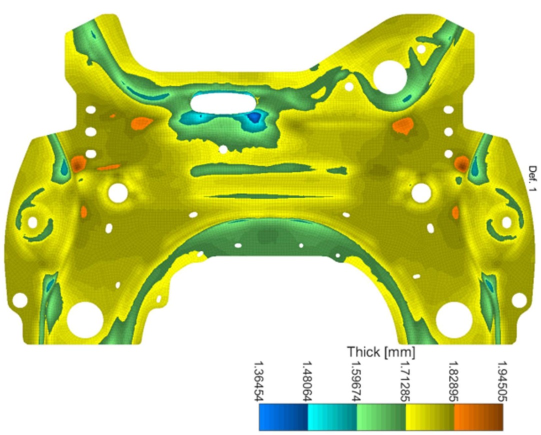


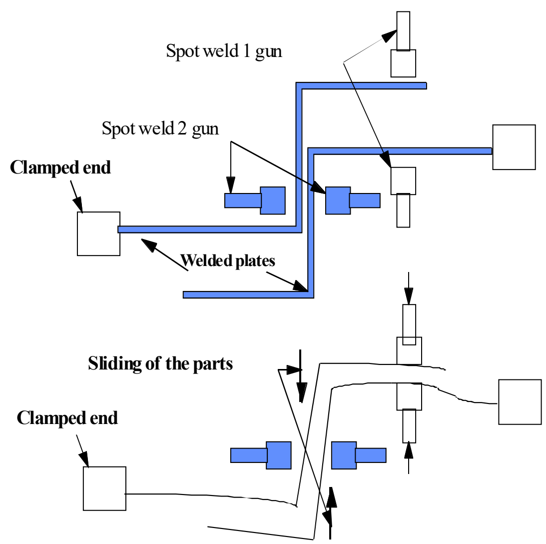


- Process simulation + field projection

fe_shapeoptim Interp





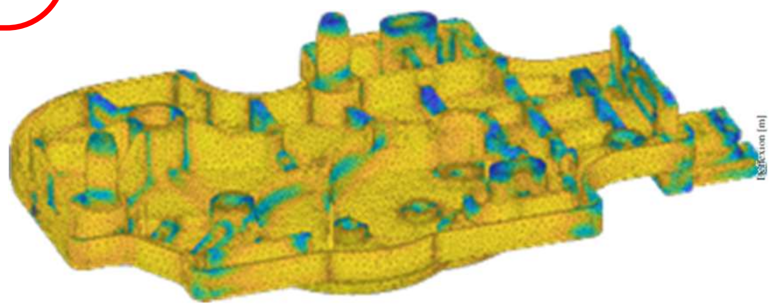


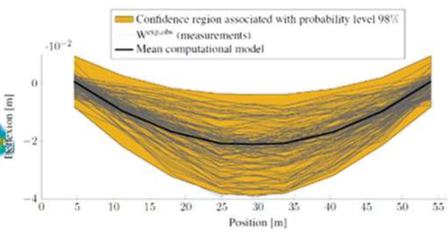
[1] de Paula, Rejdych, Chancelier, Vermot, Balmes, « On the influence of geometry updating on modal correlation of brake components. », in *Vibrations, Chocs & Bruit*, 2012.
[2] G. Vermot Des Roches, E. Balmes, et S. Nacivet, « Error localization and updating of junction properties for an engine cradle model », in ISMA, Leuven, Belgium, 2016, p. ID 372.
[3] E. Blain, « Etudes expérimentales et numériques de la dispersion vibratoire d'assemblages soudés par points », Ph.D. thesis, Ecole Centrale de Paris, 2000.

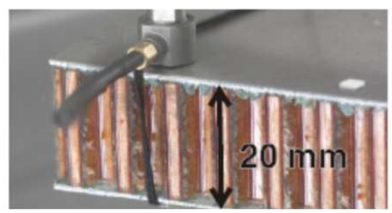
Direct problems : material parameters

- Uniform
- Field
- Equivalent
  (at certain scales)

- Geometry
- Material parameters
- Junction representation
- Equivalent parameters







- Basic parametrization tool : dependence on constitutive parameters C_{ij}

$$K = \int_{\Omega} \{\epsilon\}^T [C_{ij}] \{\epsilon\} = \sum_g B^T [C_{ij}] B \, w_g = \sum_{ij} C_{ij} \left[\sum_g B^T [C_{ij}^u] B w_g \right] = \sum_{ij} C_{ij} [K_{ij}^u]$$

Constitutive law parameterization

- Example : bar stiffness $K = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$ proportional to E
- Classical problem shell thickness parameterized with

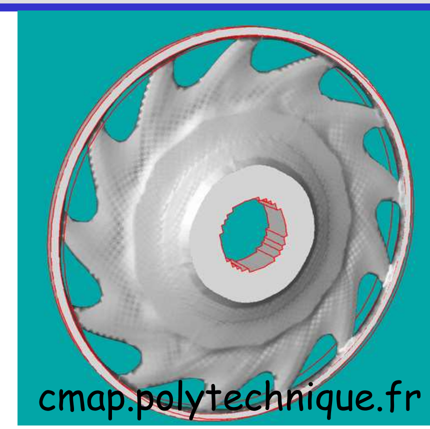
$$\beta_1 = t, \beta_2 = t^3, \beta_3 = t^2$$

$$\begin{Bmatrix} N_{xx} \\ N_{yy} \\ N_{xy} \\ M_{xx} \\ M_{yy} \\ M_{xy} \end{Bmatrix} = \begin{bmatrix} A & B \\ B^T & D \end{bmatrix} \begin{Bmatrix} \epsilon_{xx} \\ \epsilon_{yy} \\ \epsilon_{xy} \\ \kappa_{xx} \\ \kappa_{yy} \\ \kappa_{xy} \end{Bmatrix}$$

- Current practice : weighted element sum

$$[M(p)] = \sum_{j=1}^{NE} \alpha_k(p)\,[M_k^e] \qquad [K(p)] = \sum_{j=1}^{NE} \beta_k(p)\,[K_k^e]$$

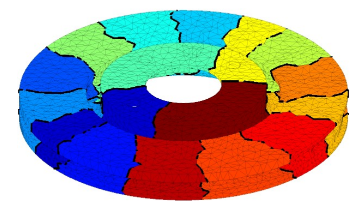
Element/model weights

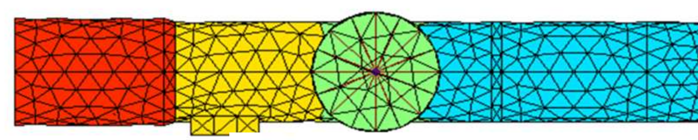


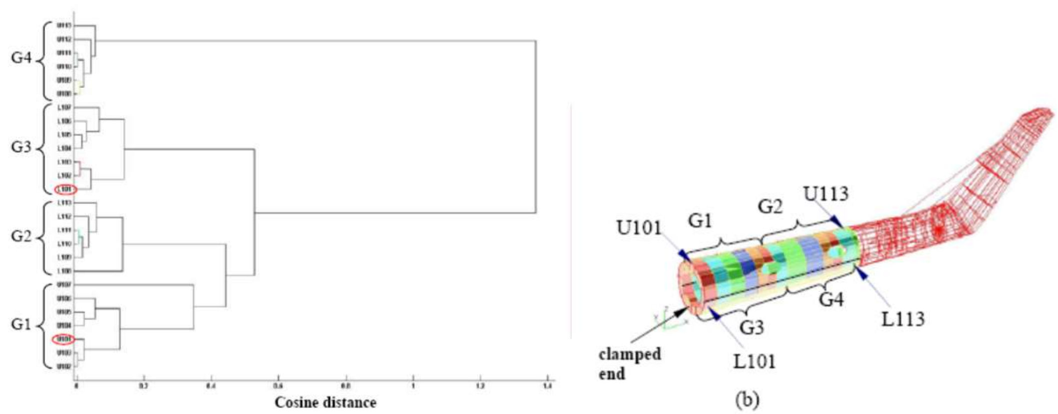
$$K(p) = \sum_e \alpha_e(p)[K^e]$$

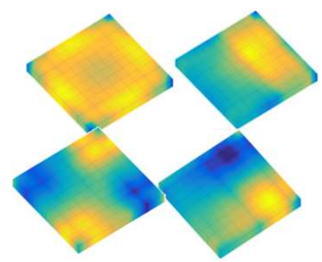
Weighted element matrices = standard

- Element-wise (topology optimization)
- Field/groupwise
  - parameter groups …
  - solution of eigenvalue problem (polynomial chaos, Ghanem, Soize, …)
  - Clustering (for example k-means a usual data-mining algorithm)









Clustering of Parameter Sensitivities: Examples from a Helicopter Airframe Model Updating Exercise. Shahverdi, Mottershead & All

SDT implementations : upcom / zCoef / stressCut

- Element wise $K(p) = \sum_e \alpha_k^e [K^e]$ www.sdtools.com/help/upcom.html

$$\text{mind} = \begin{bmatrix} M_s & M_e & K_s & K_e & \alpha_m & \alpha_k \\ \vdots & & & & & \\ elt & & & & & \end{bmatrix}$$

- Group wise $K(p) = \sum_g \alpha_g [K^g]$ www.sdtools.com/help/zCoef.html

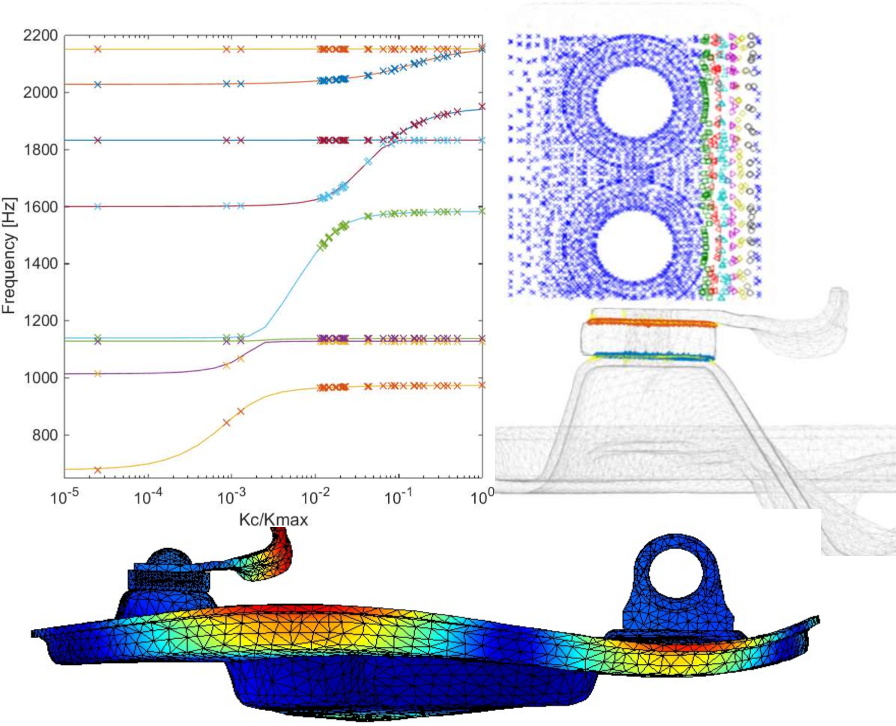
```
zCoef={'Klab','mCoef','zCoef0','zCoefFcn';
       'M'     1        0        '-w.^2';
       'Ke'    0        1        1+i*fe_def('DefEta',[]);
       'Kv'    0        1        'par(1)'};
```

- Disassembly www.sdtools.com/help/corstress.html

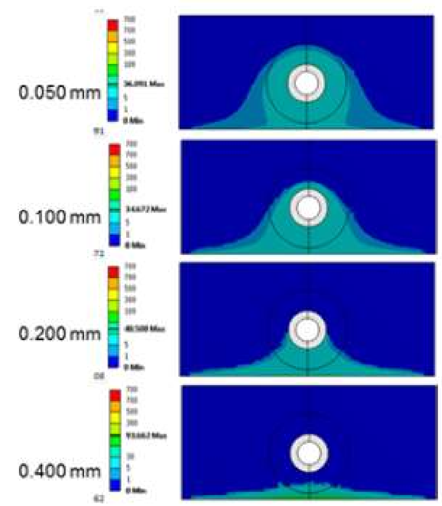
$$K(p) = [b][C_{constit}][c] = \begin{bmatrix} \ddots & & \\ & w_g J_g N_i^g & \\ & & \ddots \end{bmatrix}^T \begin{bmatrix} \ddots & & \\ & C_{ij}^g & \\ & & \ddots \end{bmatrix} \begin{bmatrix} \ddots & & \\ & N_i^g & \\ & & \ddots \end{bmatrix}_{(Ng \times Nstrain) \times N}$$

Parametrization of contact/sliding

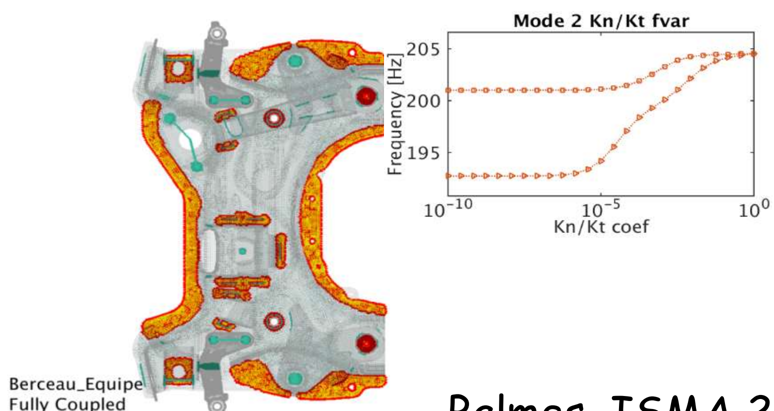
- Variable contact surface, contact, sliding



Chassis Brakes International
Eurobrake 2014



Goth, ISMA 2016



Balmes, ISMA 2016

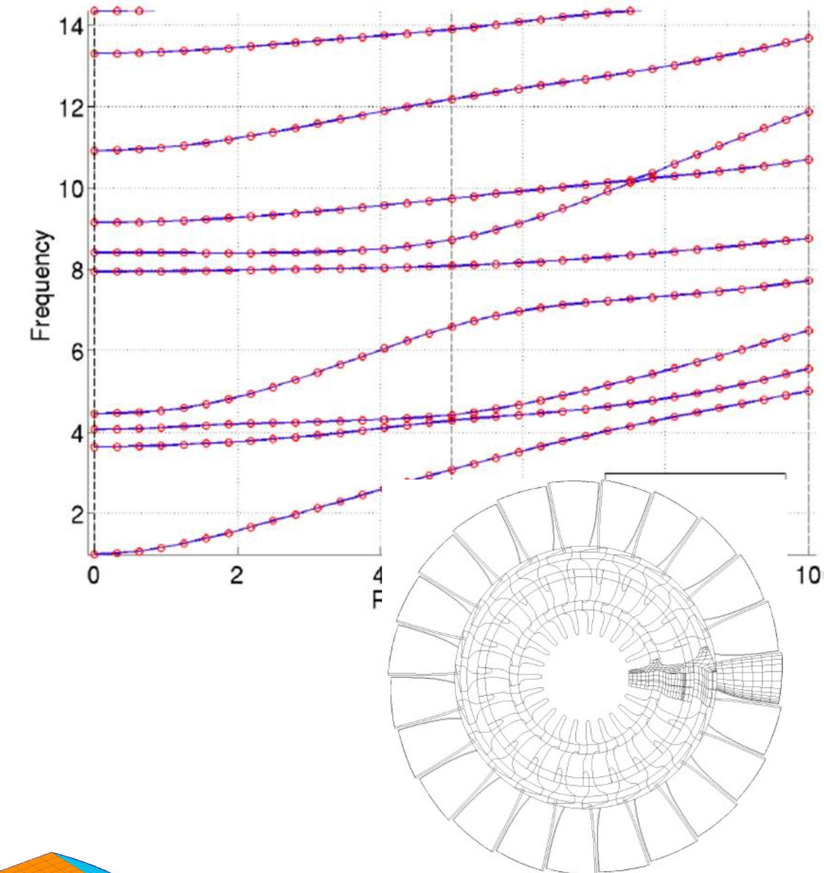
Sensitivity

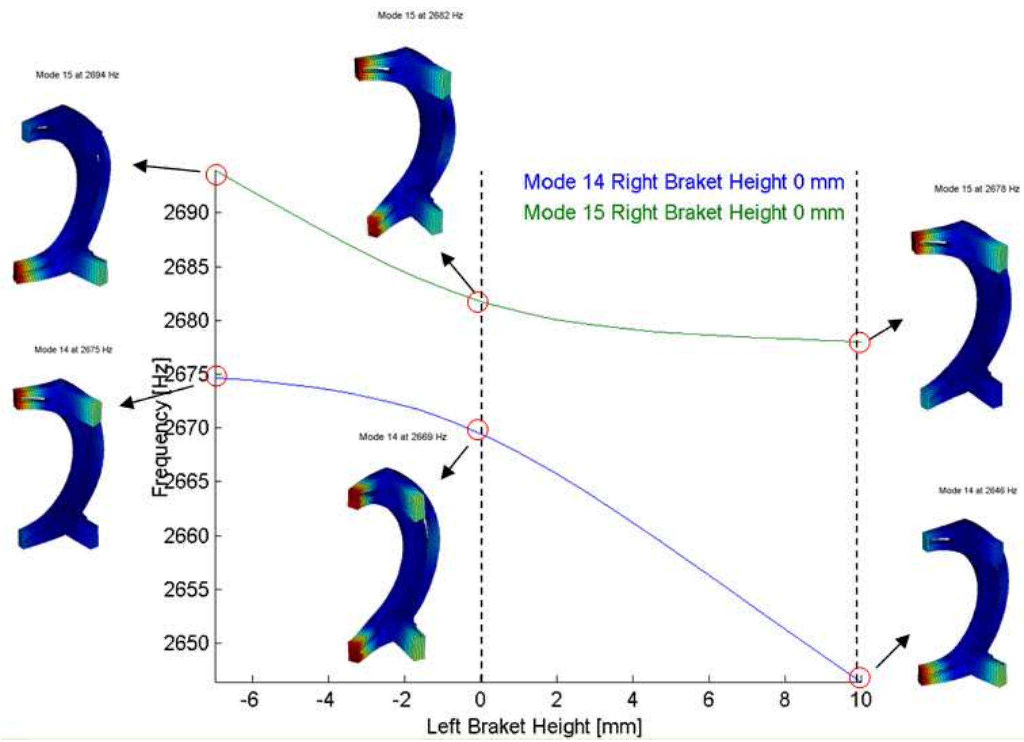
Things done on the blackboard

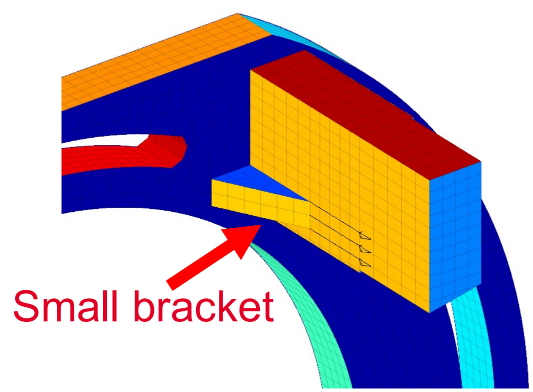
- Static direct and adjunct (poly section 10.2)
- Frequency and shape sensitivity (poly section 10.3)
- Exact shape sensitivity (inverse in presence of null space, same as residual flexibility with RB modes)
- Fox/Kapoor (use of modal coordinates)

Mode crossing

- High sensitivity for close modes associated with mode crossing





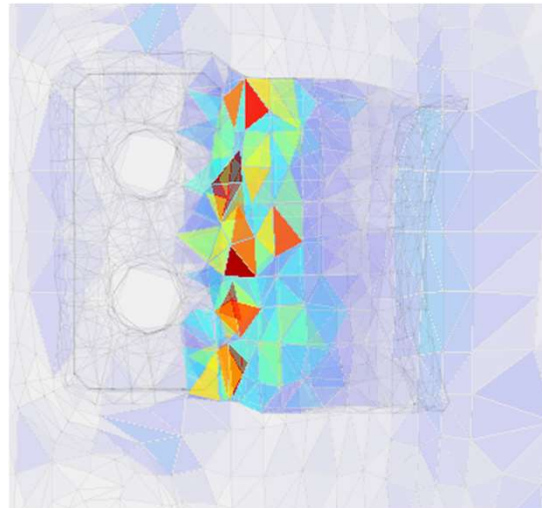


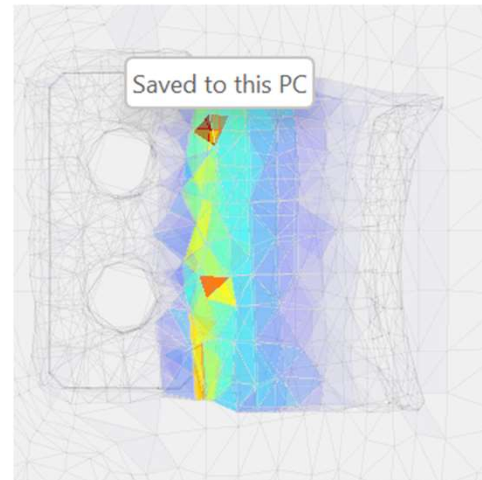
Tricks with energy display

Views by

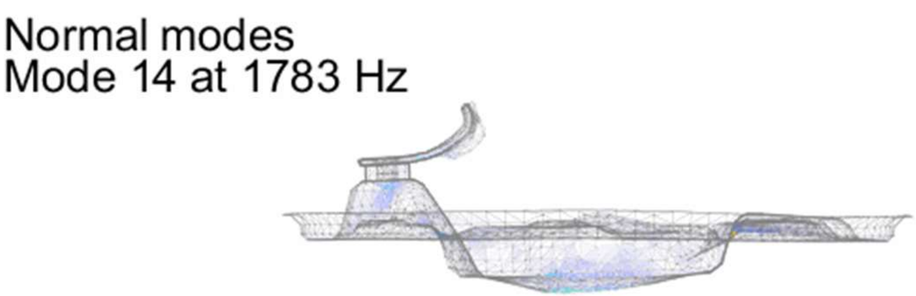
- Element energy
- Energy density
- Energy in group

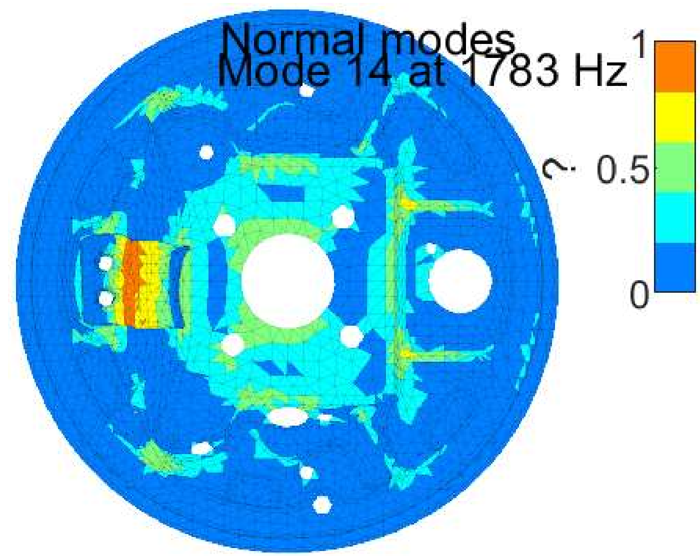
Give different perspectives





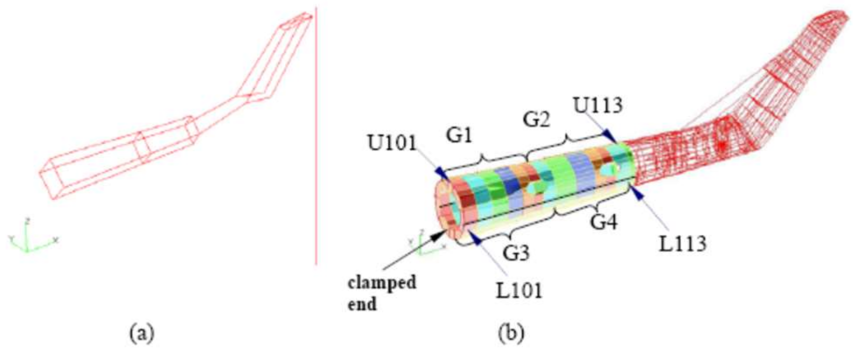
Abaqus output variables : ELSE, ESEDEN

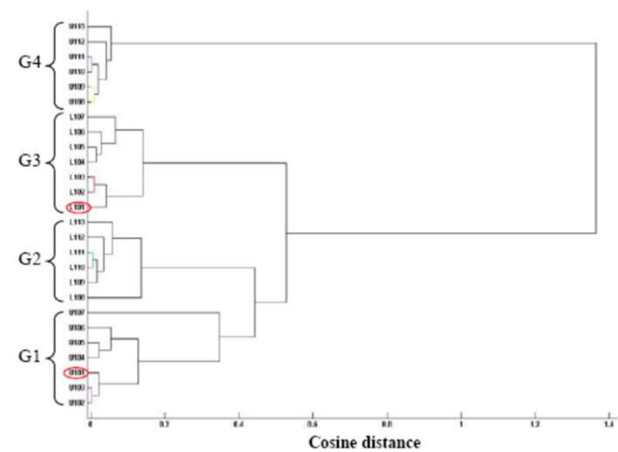




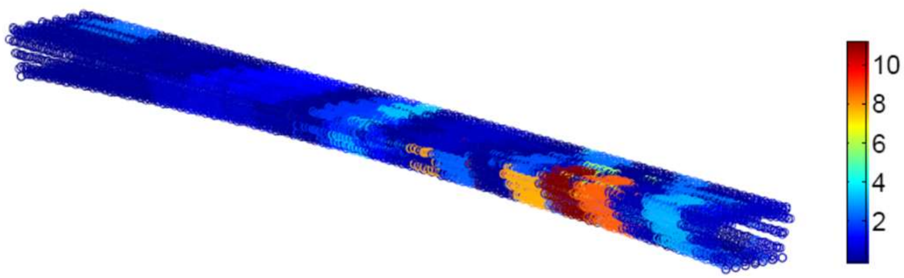
Clustering examples

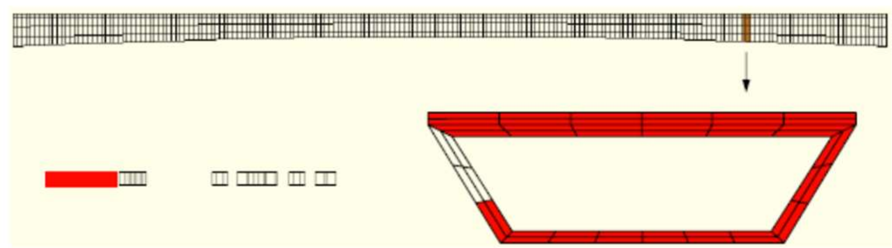
- Helicopter frame





- Bridge deck



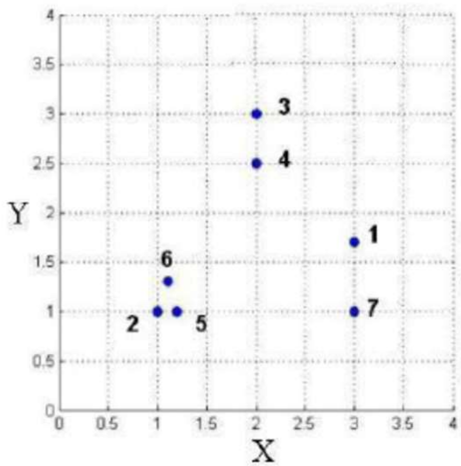


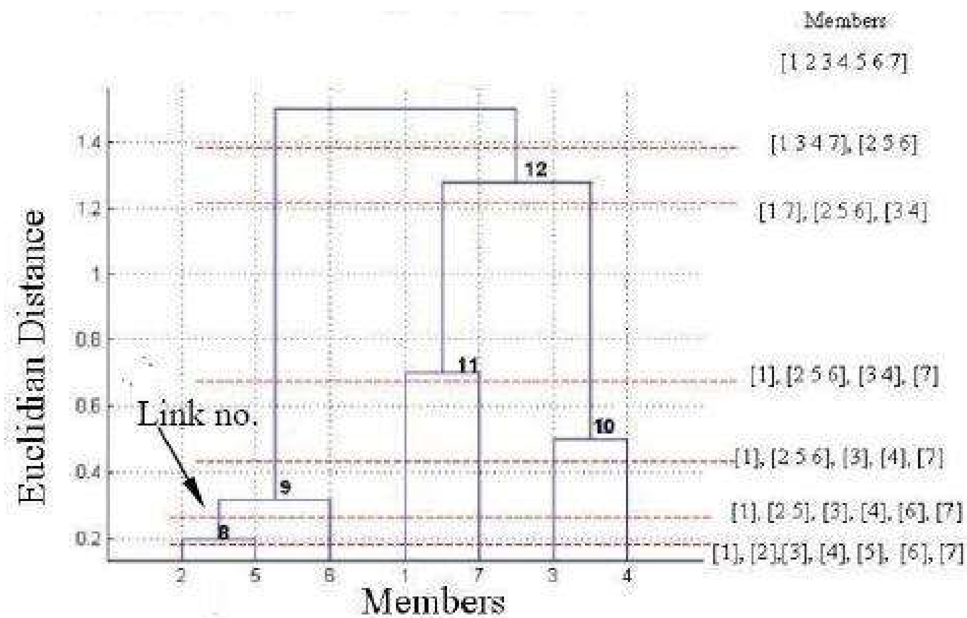
[1] Clustering of Parameter Sensitivities: Examples from a Helicopter Airframe Model Updating Exercise. Shahverdi, Mottershead & All
[2] Statistical Model-Based Damage Localization: a Combined Subspace-Based and Substructuring Approach Balmes, Basseville & All

Sensitivity / clustering

- Clustering techniques (k-means for example) can be used to group elements with similar effects
- Key mathematical notion : cosine distance (subspace in MATLAB)

$$d_{ij} = 1 - \sum_{k=1}^{p} x_{ik}\, x_{jk} \Big/ \left(\sum_{k=1}^{p} x_{ik}^{2} \sum_{k=1}^{p} x_{jk}^{2} \right)^{1/2}$$





Link no.	linked objects		distance
8	2	5	0.2
9	6	8	0.32
10	3	4	0.5
11	1	7	0.7
12	10	11	1.28
	9	12	1.5

Clustering of Parameter Sensitivities: Examples from a Helicopter Airframe Model Updating Exercise. Shahverdi, Mottershead & All

Subspace clustering example

- Clustering
  - 2D complex mode basis
  - Subspace angle tolerance
- Parameter variations per cluster