- Intro
- Signal processing basics (DFT, aliasing, leakage)
- FRF estimation (H1/H2/coherence)
- Sensor/shaker technology

Course notes : chapter 4 : time and frequency domains, signal processing basics

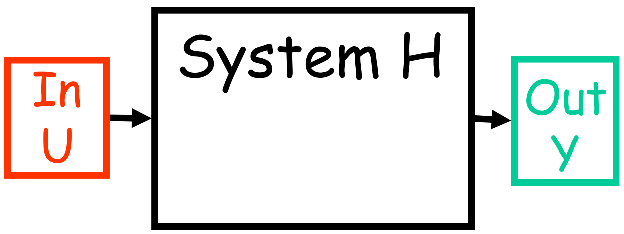
SD Tools VIBRATION SOFTWARE & CONSULTING

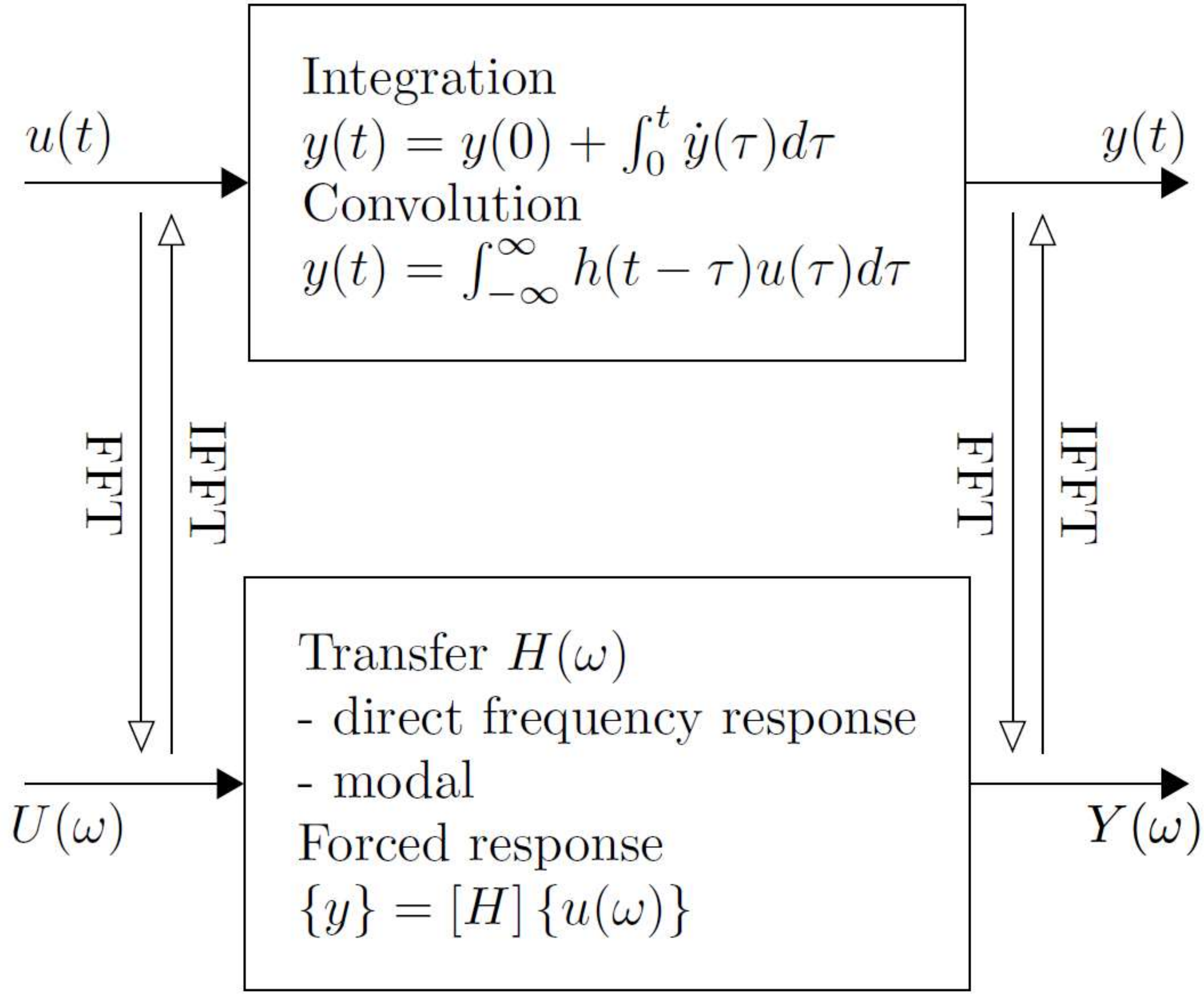
Arts et Métiers Institute of Technology

MS2SC http://savoir.ensam.eu/moodle/course/view.php?id=1874

PROVIR http://savoir.ensam.eu/moodle/course/view.php?id=490

Why do we need frequency responses

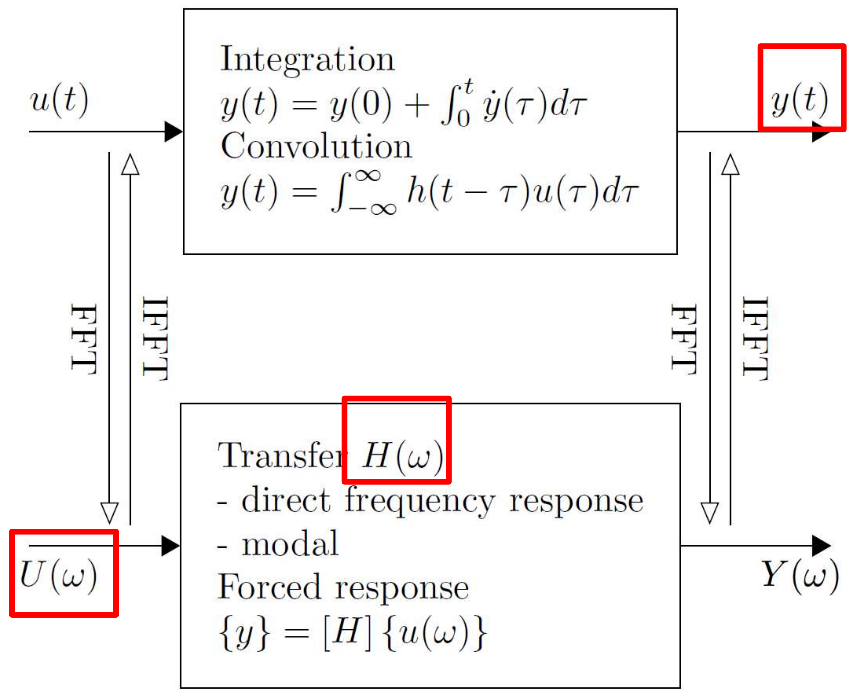


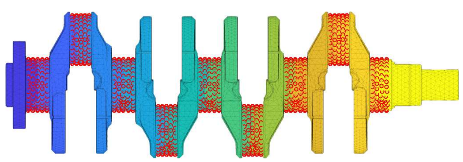


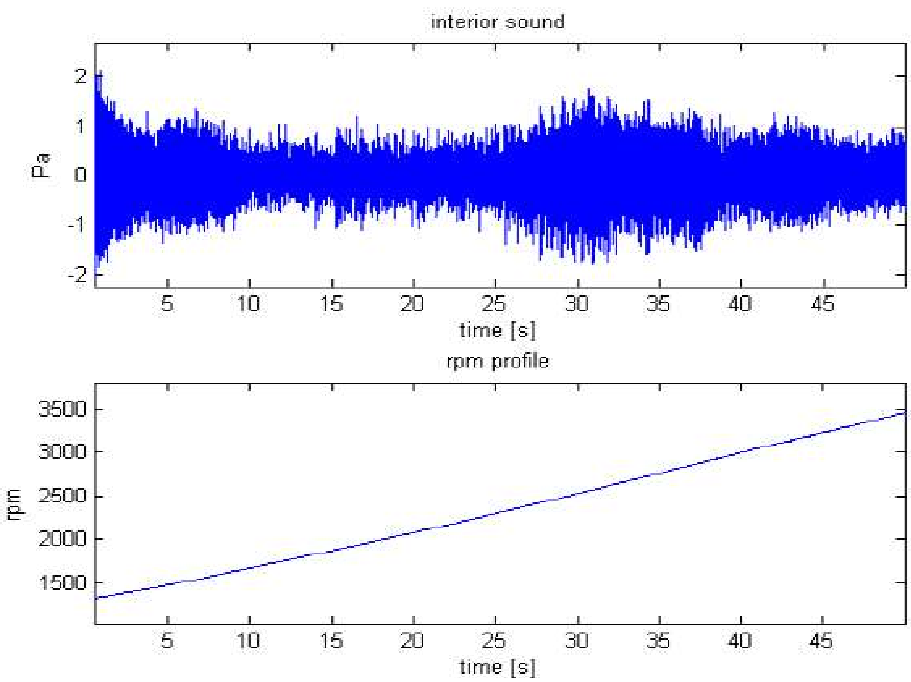
- Scenario 1 : run-up
- Scenario 2 : modal testing

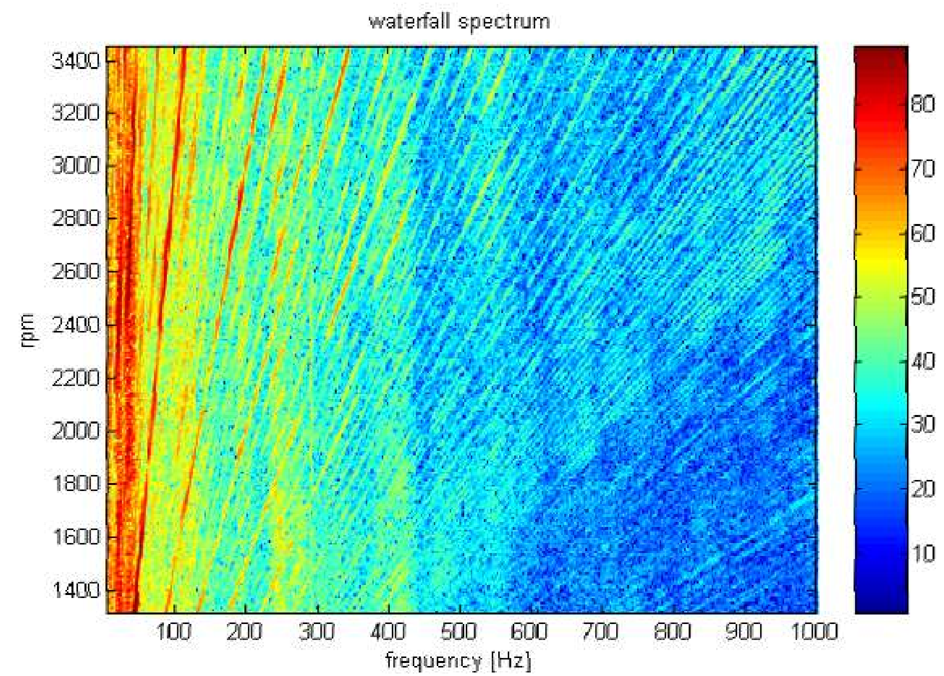
Frequencies in a run-up

- Periodic excitation :
  signal with harmonics $u(t) = \sum_n u_n(n\omega t)$
- Constant system $H(\omega)$
  or time varying & NL
- Output contains
  both harmonic & constant contributions

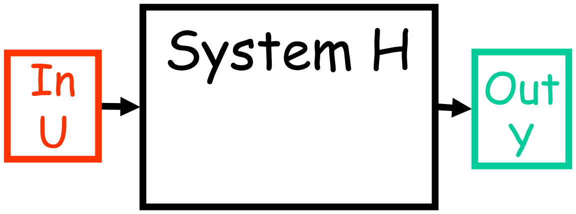


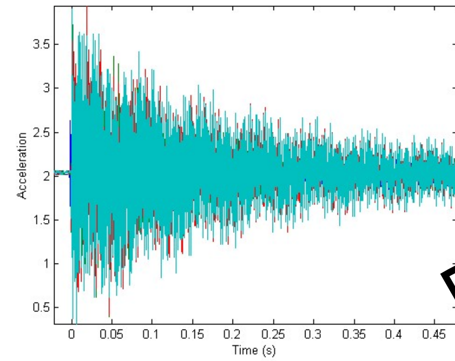






Assumption LTI system = transfers exist





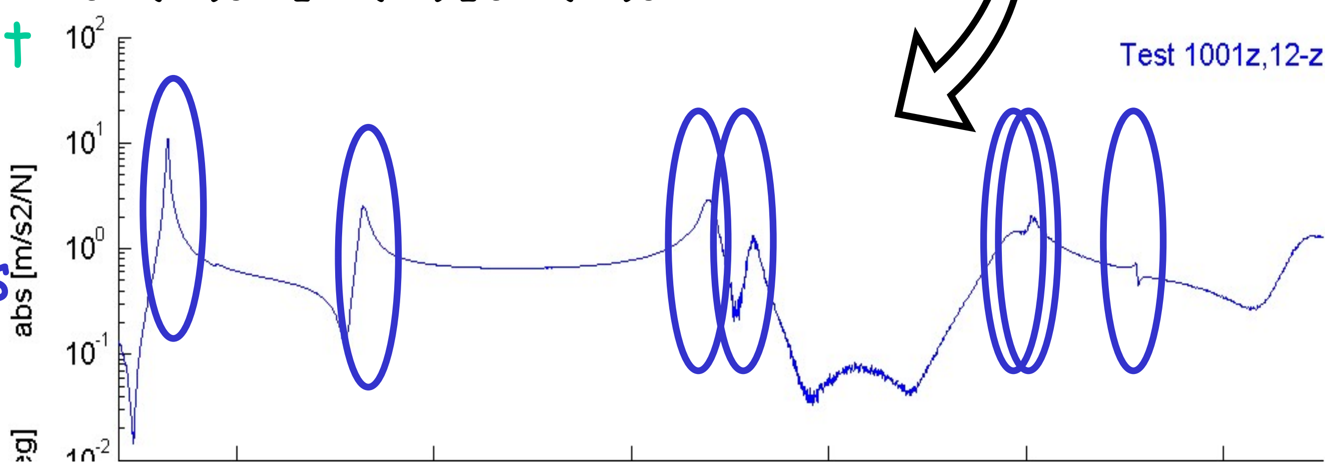
Transfers estimated from time response

ONE input

ONE output

MANY resonances

{Y(ω)}=[H(ω)]{U(ω)}



Bode plot : visualization of transfer function

Some standard texts for the vibration community

[1] D. J. Ewins, Modal Testing: Theory and Practice. John Wiley and Sons, Inc., New York, NY, 1984 & 2009 (2nd edition)

[2] W. Heylen et P. Sas, Modal analysis theory and testing. Katholieke Universteit Leuven, Departement Werktuigkunde, 2006

[3] K. G. McConnell, Vibration Testing. Theory and Practice. Wiley Interscience, New-York, 1995

Contents

- Signal processing basics
  - Continuous & discrete for different usage
  - Base problems : aliasing, leakage
- FRF estimation
- Sensor/shaker technology

Fourier transform : discrete & continuous

- Continuous $Y(\omega) = \int_{-\infty}^{+\infty} y(t) e^{-j\omega t} dt \text{ and } y(t) = \int_{-\infty}^{+\infty} Y(\omega) e^{j\omega t}$
- Periodic in time : discrete in frequency (series, example: harmonic balance methods)

$$Y(k\Delta f) = \frac{1}{T} \int_0^T y(t) e^{-j2\pi\Delta f t} dt \text{ with } y(t) = \sum_{k=-\infty}^{+\infty} Y(k\Delta f) e^{j2\pi k \Delta f t}$$

- Discrete in time : periodic in frequency (sample application periodic FEM)

$$Y(f) = \sum_{n=-\infty}^{+\infty} y(n\Delta t) e^{-j\omega n \Delta t} \text{ and } y(n\Delta t) = \frac{1}{\omega_s} \int_{-\omega_s/2}^{\omega_s/2} Y(\omega) e^{j\omega n \Delta t} d\omega$$

- Finite length discrete time : finite frequency (measurements) (DFT using fft library)

$$Y(k\Delta f) = \frac{1}{N} \sum_{n=0}^{N-1} y(n\Delta t) e^{j2\pi nk/N} \text{ and } y(n\Delta t) = \sum_{k=0}^{N-1} Y(k\Delta f) e^{j2\pi nk/N}$$

Time and frequency

- Discrete transform only depends values ($e^{\frac{j\pi nk}{N}}$ regular on unit circle)

Time and frequencies are

- regularly spaced
- found by

k =[1:N]

t_k =[0:N-1]Δt
=([0:N-1]/N)T

f_k =[0:N-1]/(NΔt)
=[0:N-1]/(T)

- Typical objectives are
  - Bandwidth
  - Frequency resolution

Impose	Consequence	Influence de N
Δt	$F_{Max} = \frac{1}{2\Delta t}$	$T = N\Delta t$ $\Delta f = \frac{1}{N\Delta t}$
F_{Max}	$\Delta t = \frac{1}{2F_{Max}}$	$T = N\Delta t$ $\Delta f = \frac{1}{N\Delta t}$
Δf	$T = \frac{1}{\Delta f}$	$\Delta t = \frac{T}{N}$ $F_{Max} = \frac{N}{2}\Delta f$
T	$\Delta f = \frac{1}{T}$	$\Delta t = \frac{T}{N}$ $F_{Max} = \frac{N}{2}\Delta f$

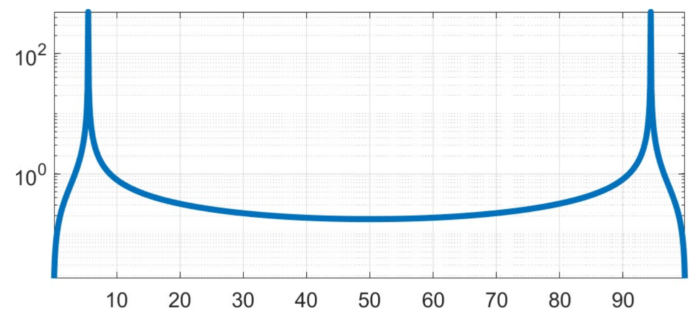
DFT Properties

Linearity	$Y_1 + Y_2 = \mathcal{F}(y_1 + y_2)$ and $\alpha Y_1 = \mathcal{F}(\alpha y_1)$
Base time change	$\mathcal{F}(y(at)) = Y(f/a)/\lvert a \rvert$
Time delay	$\mathcal{F}(y(t - t_0)) = Y(f) e^{-j2\pi f t_0}$
Time derivative	$\mathcal{F}(\dot{y}) = i\omega \mathcal{F}(y)$
Convolution	$A(\omega)B(\omega) = \mathcal{F}\left[\int_{-\infty}^{+\infty} a(\tau) b(t - \tau)\right]$
Energy (Parceval)	$E = \int_{-\infty}^{+\infty} \lvert g(t) \rvert^2 \, dt = \int_{-\infty}^{+\infty} \lvert G(f) \rvert^2 \, df$

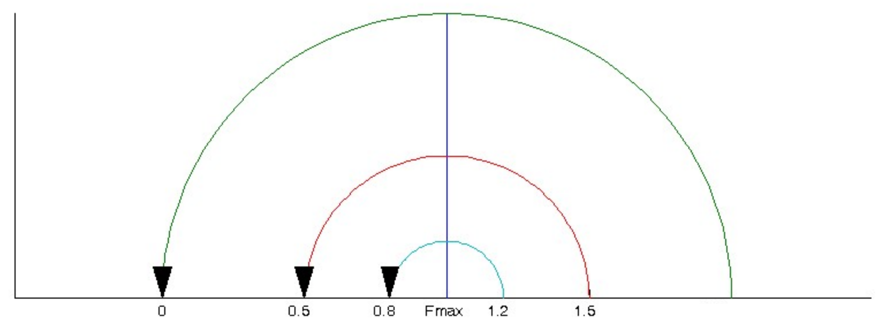
Continuous vs. DFT problem 1 : aliasing

```
t=linspace(0,10,1000);
u=sin(5.5*2*pi*t); U=fft(u); U(1)=0;
f=(0:length(t)-1)/diff(t(1:2))/length(t);
h=semilogy(f,abs(U));axis('tight');
```

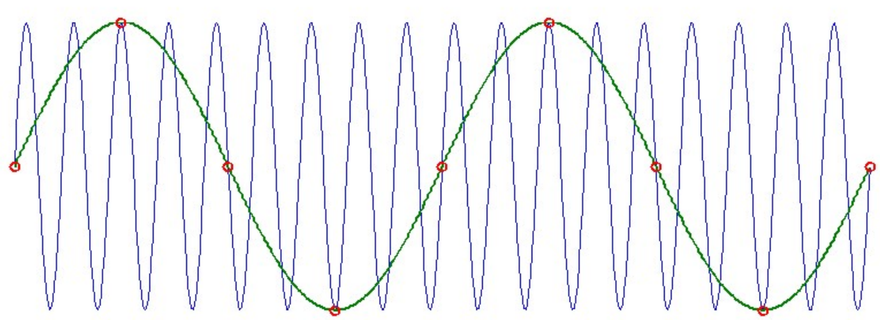
Cannot distinguish first and second half of spectrum



Shannon : The signal should not have content above $F_{max}=F_{sample}/2$

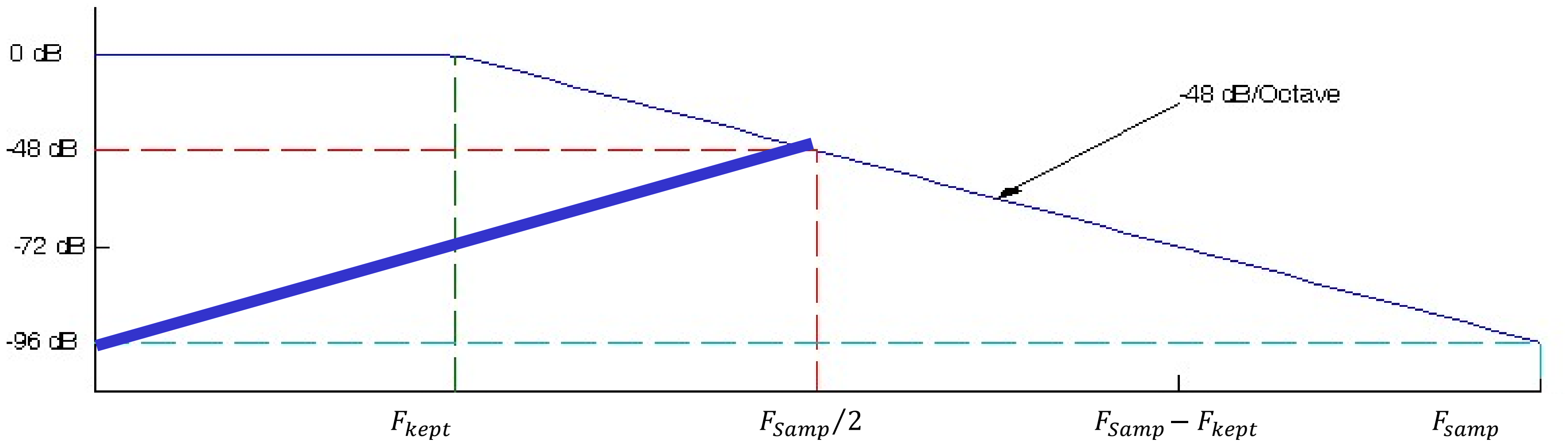


$F_{ech} = 2F_{max}$



Application : lab 1DOF

Anti-aliasing filters



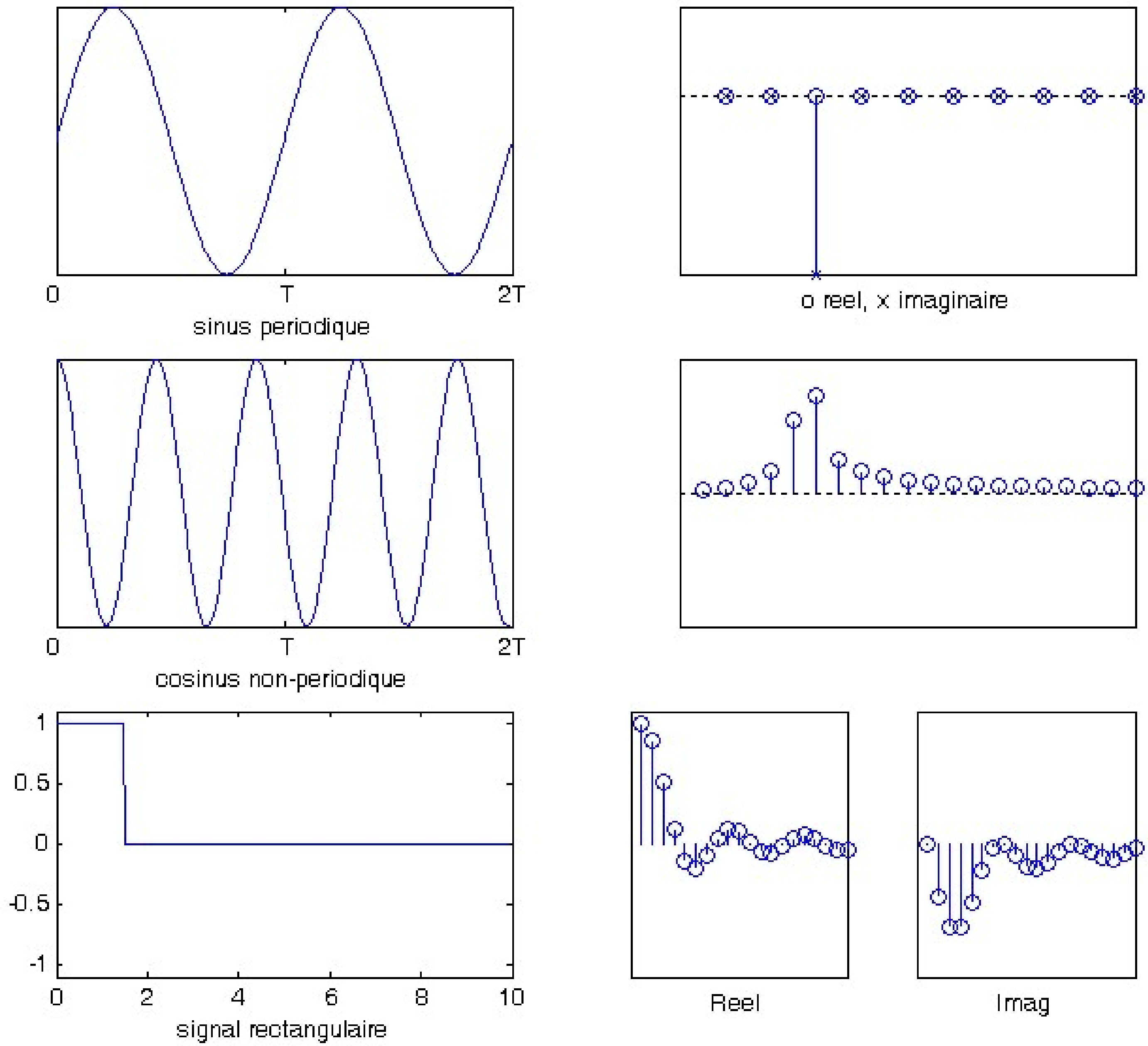
Objective : **aliased signal** reduced by resolution at $F_{kept} = F_{sample}/(2+\alpha)$

Example

- resolution 12 bits=72 dB
- Filter –48 dB/octave
- Reduction $F_{kept} = F_{sample}/4$

Modern analyzers mix analog and digital anti-aliasing

Simple signals

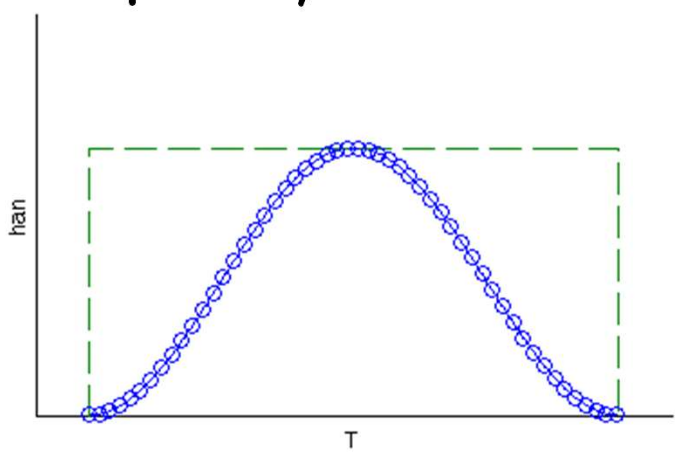


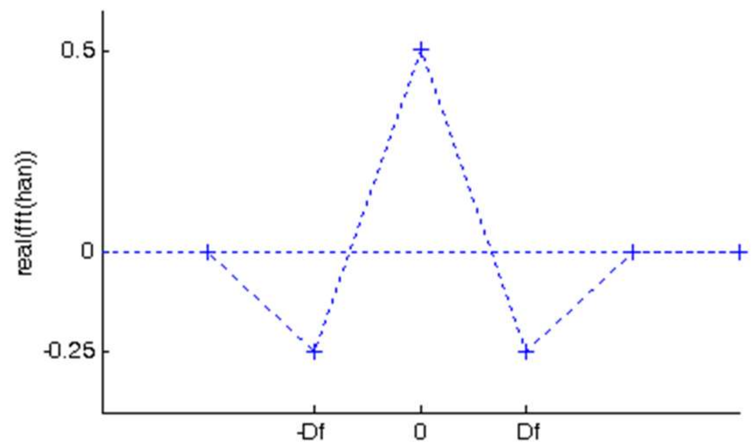
Continuous vs. DFT problem 2 : leakage

DFT = continuous transform of windowed signal

$$y_{Test}(t) = y(t)w(t) \Longleftrightarrow y_{Test}(\omega) = \int_{-\infty}^{+\infty} w(k)y(\omega - k)$$

This is equivalent to a weighted averaging by $w(k)$ in the frequency domain

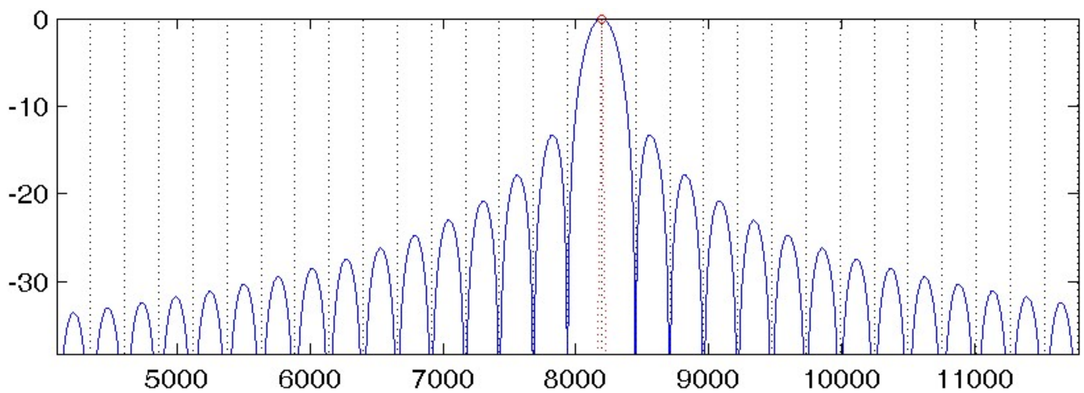


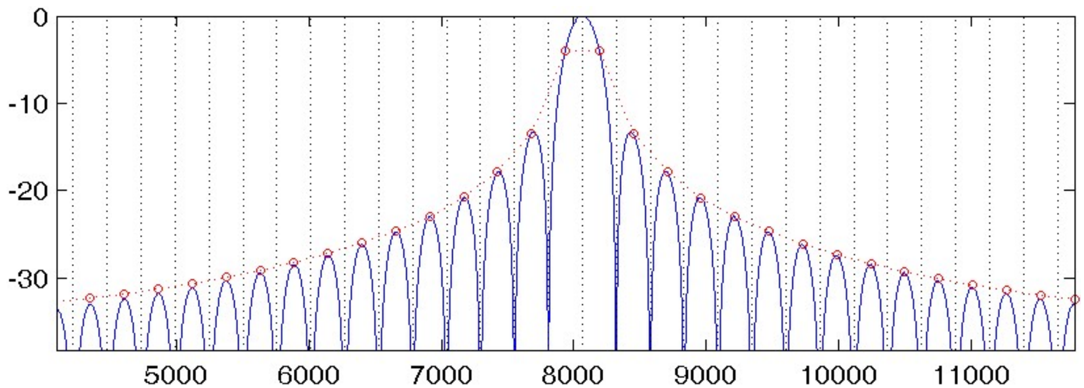


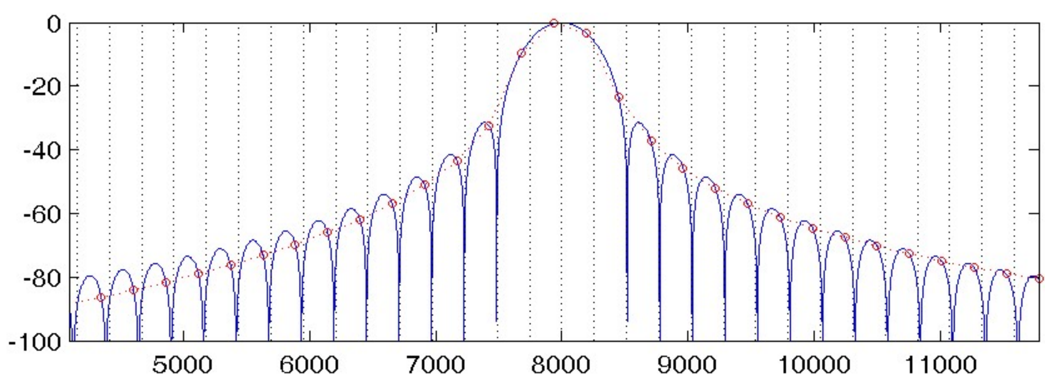
$$w(t) = \left(\sum_{j=0}^{2} a_j \cos\left(\frac{2\pi}{N\Delta t} jt\right)\right)\left(\sum_{k=0}^{N-1} \delta(t - k\Delta t)\right)$$

Keywords for leakage free methods : analytic signals, spectrum reassignment

Leakage (df offset)







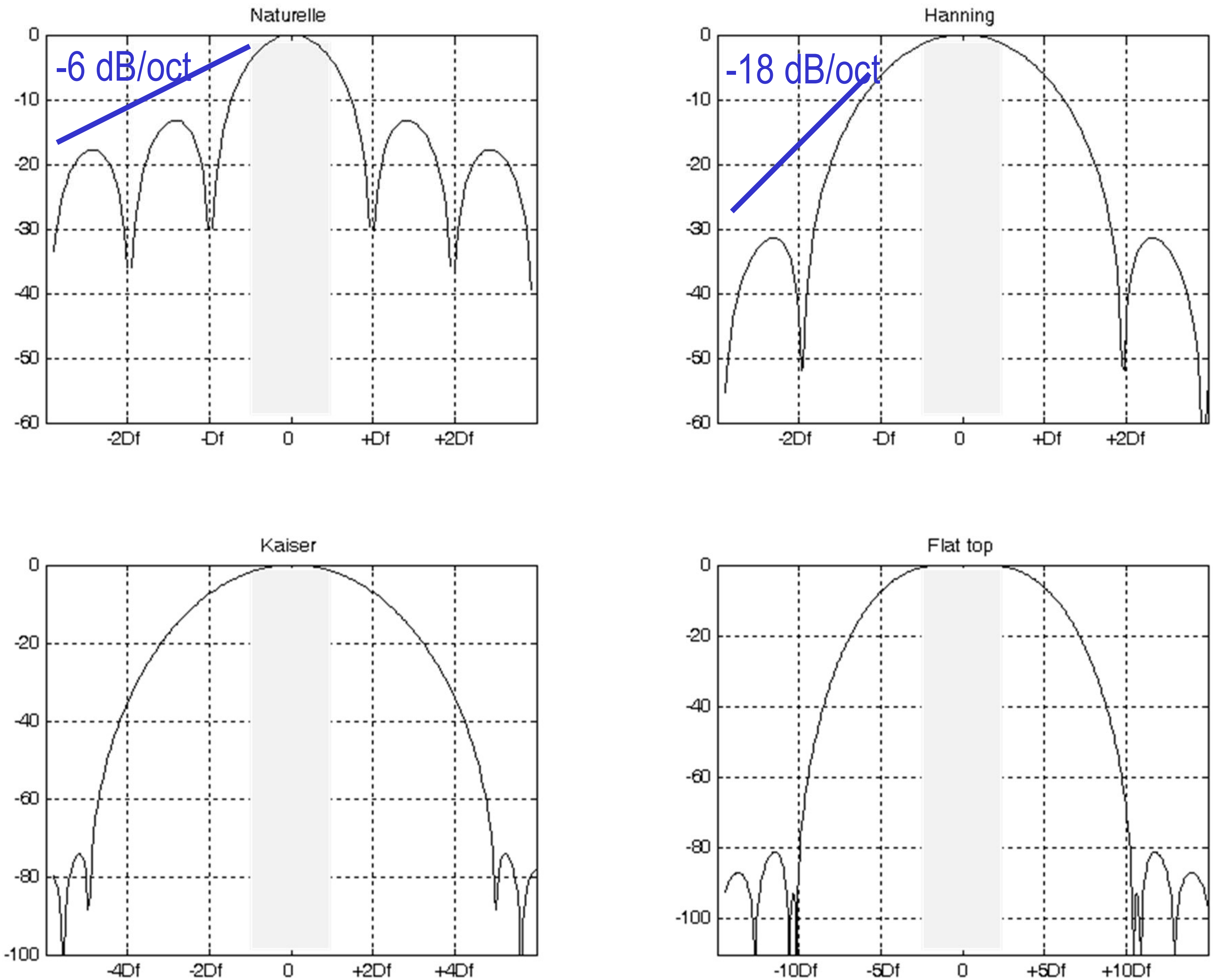
$$y(n\delta\omega) = \int_{-\infty}^{\infty} w(k) Y_{ContFT}(\omega_{in} - k)dk$$

For sine

$$y(n\delta\omega) = w(n\delta\omega + \omega_{in})$$

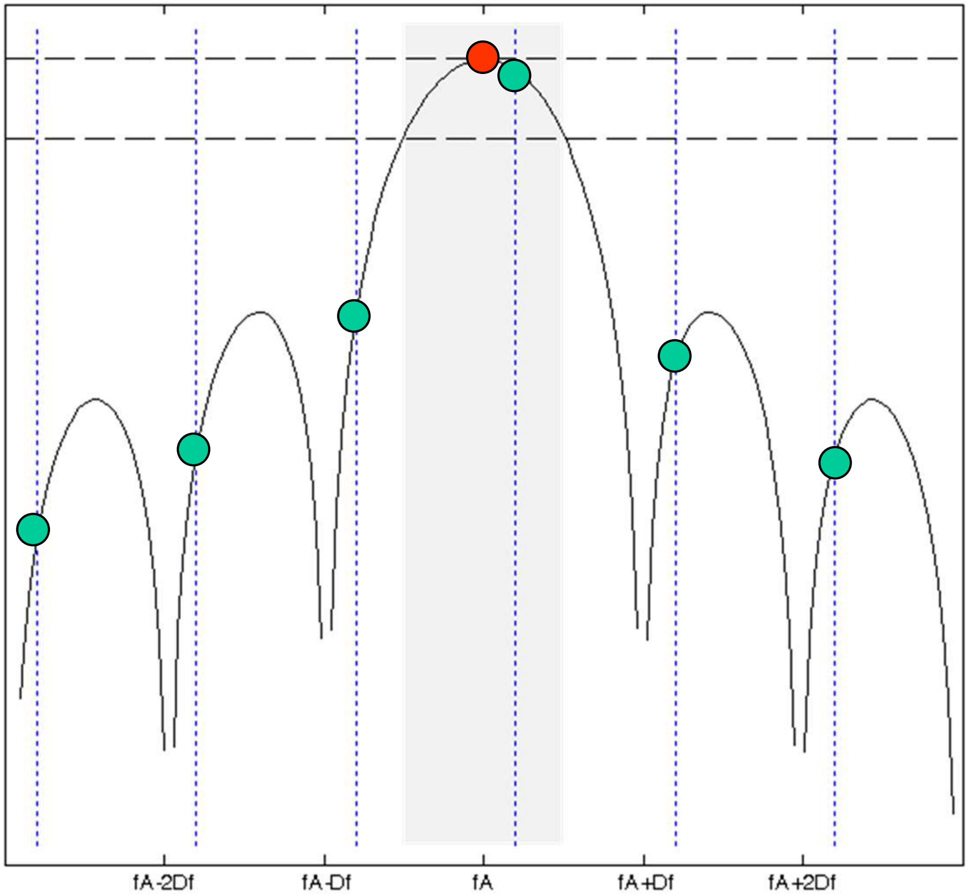
- Natural matching freq : no leakage
- Natural $\Delta f/2$: maximum error
- Hanning $\Delta f/4$ spread close but decrease faster

Time window properties



[1] K. G. McConnell, *Vibration Testing. Theory and Practice.* Wiley Interscience, New-York, 1995.
[2] W. Heylen et P. Sas, *Modal analysis theory and testing*. KUL, 2006

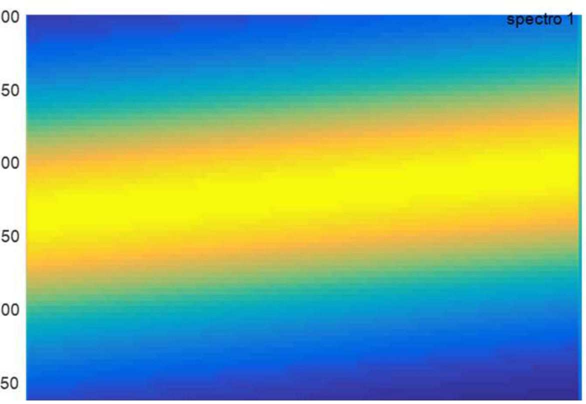
Leakage

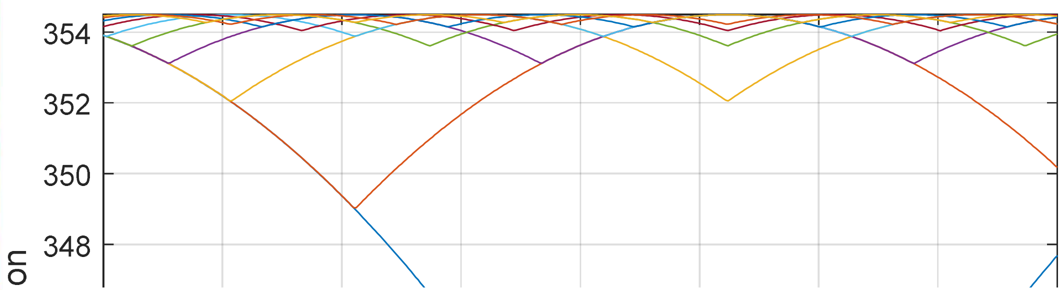


Non coincident
Sine freq.
DFT values

Coincident sine
DFT values

Leakage =
- energy at other f
- error on frequency
- error on amplitude





Below example with sine sweep

Time windows

$$w(t) = \left(\sum_{j=0}^{2} a_j \cos\left(\frac{2\pi}{N\Delta t} jt\right)\right)\left(\sum_{k=0}^{N-1} \delta(t - k\Delta t)\right)$$

Type	a0	a1	a2	a3	
Rectangular	1	0	0	0	Periodic signals
Hanning	0.5	-0.5	0	0	Continuous random signals
Hamming	0.54	-0.46	0	0	
Flat Top	0.281	-0.521	0.198	0	Low amplitude error (calibration)
Kaiser-Bessel	1	-1.298	0.244	0.003	Separate close components
Exponential					Transients of length > *T*

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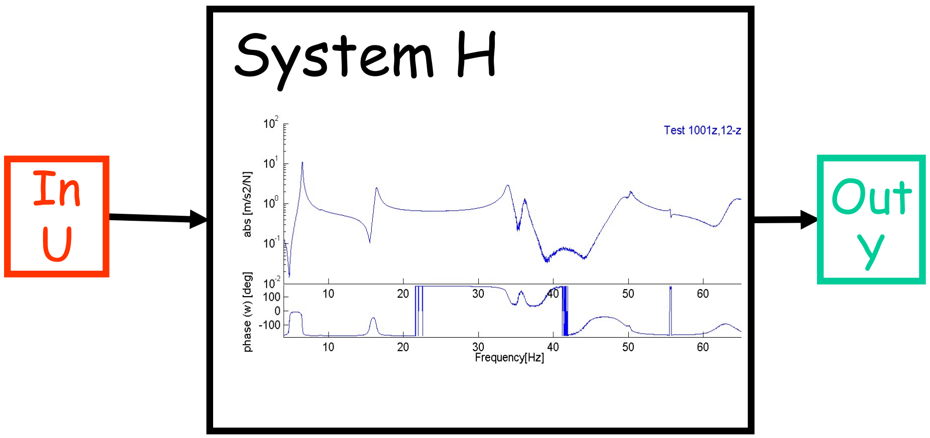
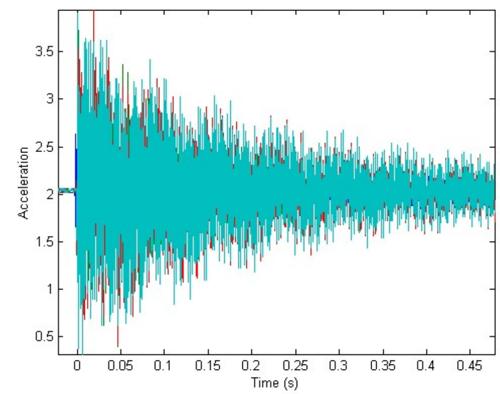
- Signal processing basics
- FRF estimation

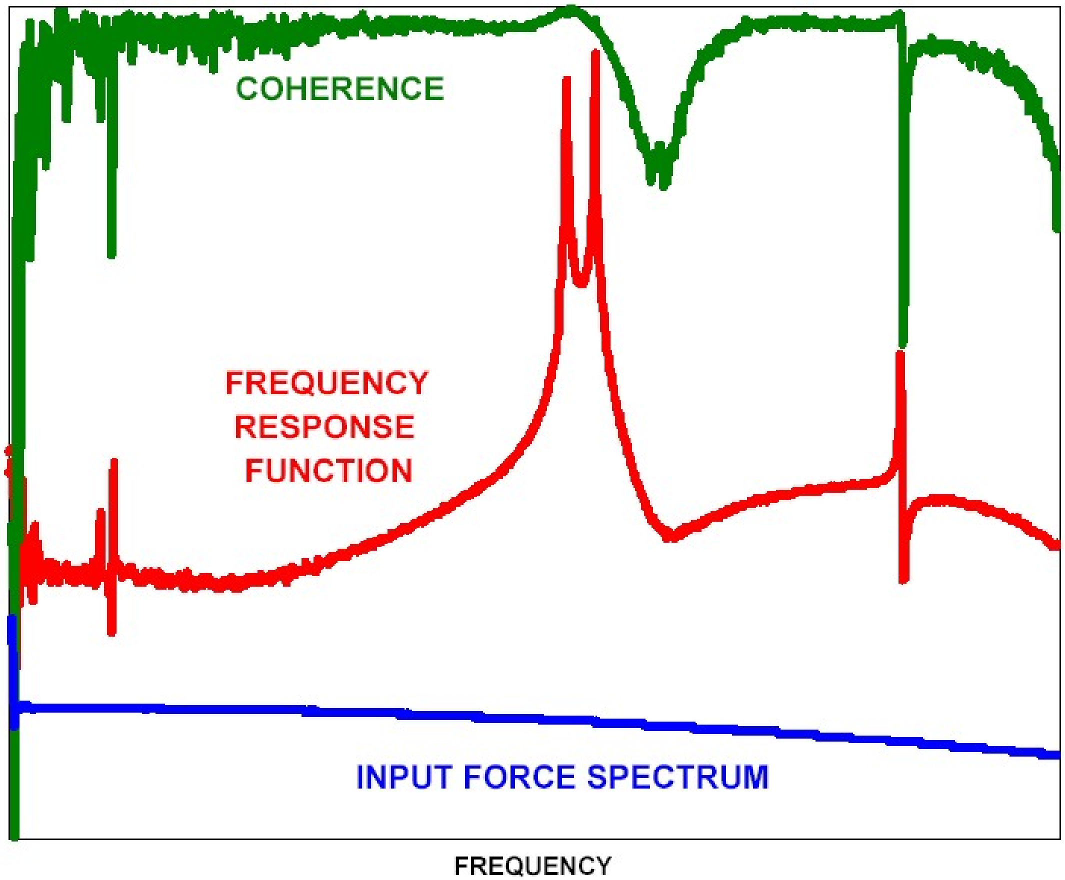




- Sensor/shaker technology

H1 estimator / coherence

$$\left[\hat{H}_1(f)\right] = \frac{\sum_{n=1}^{N} y_n(f)^H u_n(f)}{\sum_{n=1}^{N} u_n(f)^H u_n(f)} = \frac{\sum_{n=1}^{N} (G_{yu})_n}{\sum_{n=1}^{N} (G_{uu})_n} = \frac{\hat{G}_{yu}}{\hat{G}_{uu}}$$



$$\gamma^2 = |\hat{G}_{uy}|^2 / \left(\hat{G}_{uu}\hat{G}_{yy}\right)$$

[1] https://www.uml.edu/Research/SDASL/Education/Modal-Space.aspx
[2] W. Heylen, & al., Modal Analysis Theory and Testing. KUL Press, Leuven, 1997.

Data acquisition and processing

Contents

- Sensor/shaker technology
- Analyzer
- Signal processing basics
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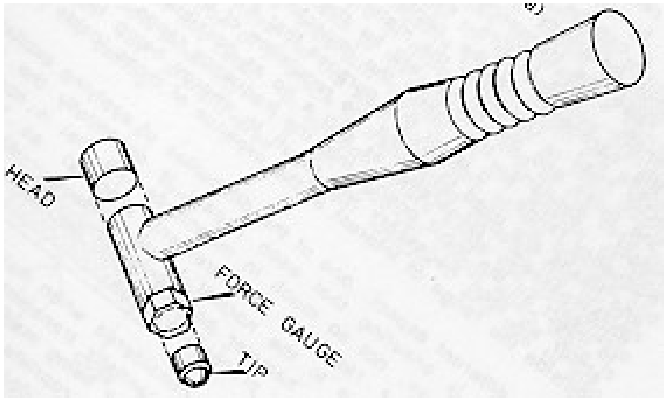
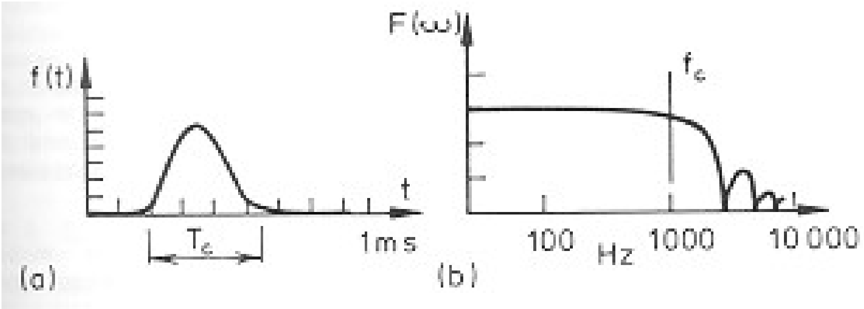
Hammer

+ Easy to use
- Master single impact
- Low energy level



- Trigger problems
- Triboelectric problems in cables
- Max 10 kHz



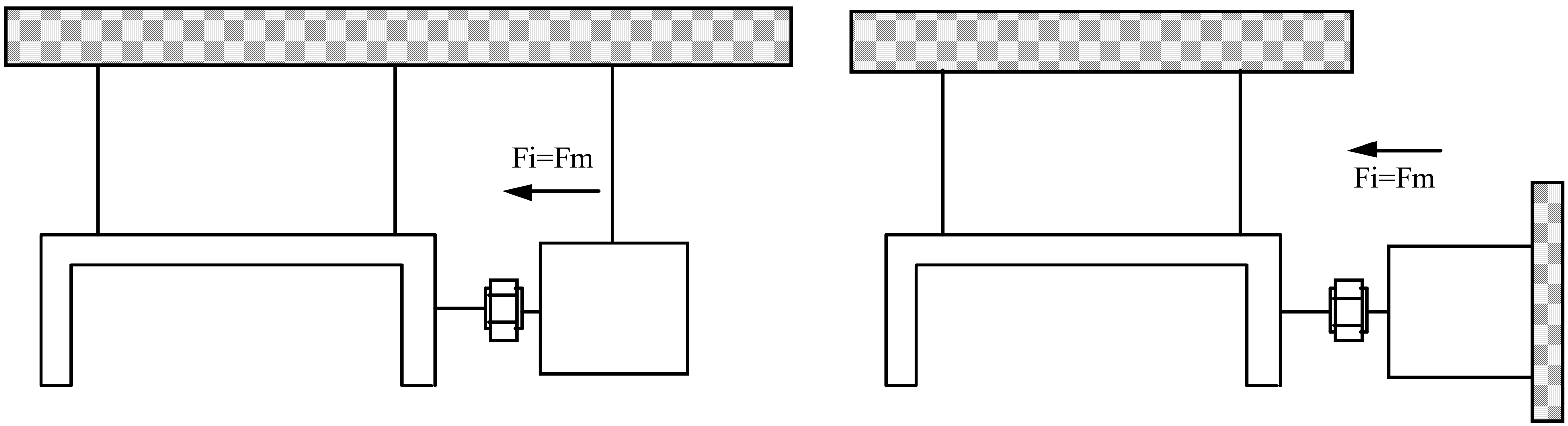
Related : cord cutting, pyrotechnic

Shakers

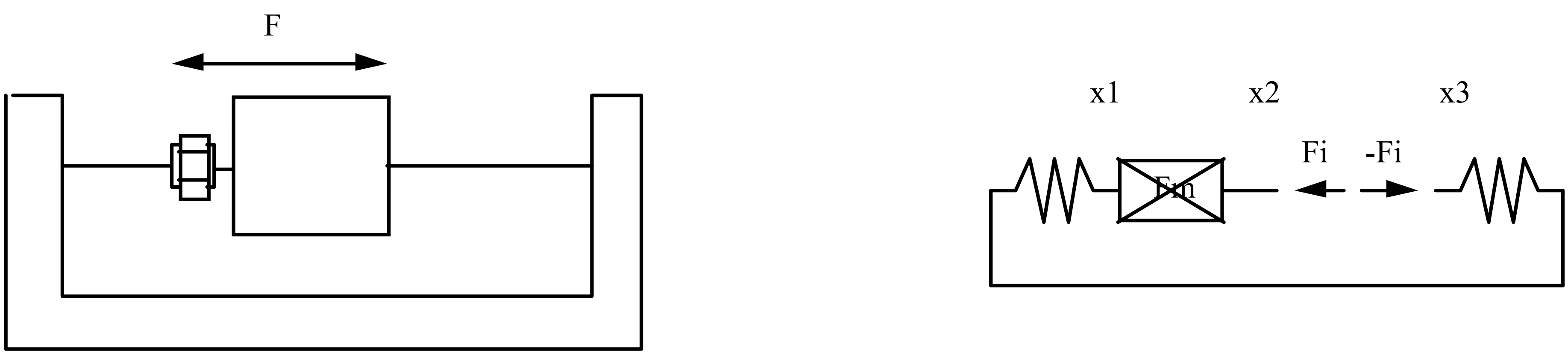
+ High energy level
+ Sustained excitation
+ Repeatable input signal
- Need attachement (modifies response)
- Enforcing acceleration is difficult

Related : piezo inertial stack & patch

Shakers setup

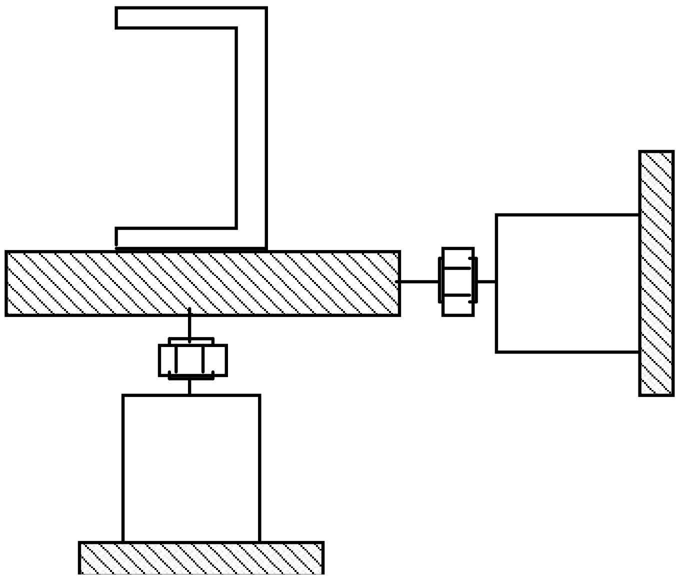


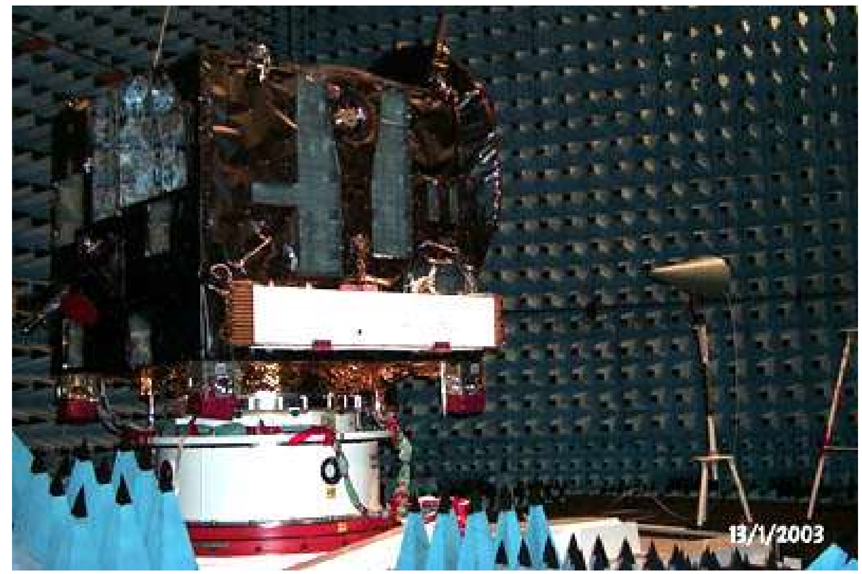
F measured ⇒ Shaker boundary conditions indifferent



Internal force application ⇒ improper test

Shaker table





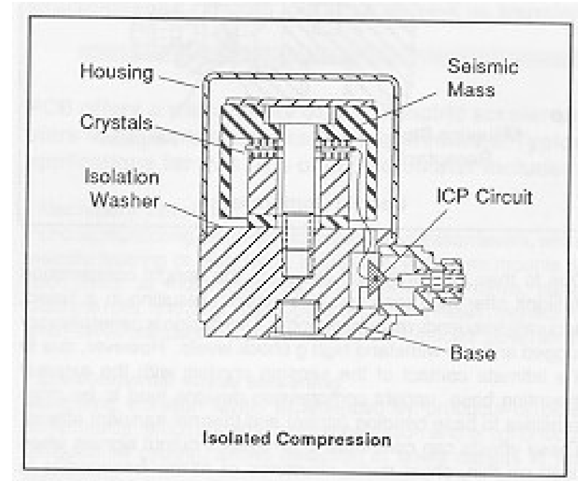
$$\begin{bmatrix} K_{ii}(s) & K_{ic}(s) \\ K_{ci}(s) & K_{cc}(s) \end{bmatrix} \begin{Bmatrix} \langle q_i \rangle \\ q_c \end{Bmatrix} = \begin{Bmatrix} f_i \\ 0 \end{Bmatrix}$$

$$y = c_i q_i + c_c K_{cc}^{-1} \left[-K_{ci} q_i \right] = c_i q_i + c_c \left[\sum_j \frac{\phi_{cj} \phi_{cj}^T}{s^2 + 2s\zeta\omega_{cj} + \omega_{cj}^2} \right] \left[-s^2 M_{ci} q_i \right]$$

Accelerometers

Piezoelectric: measure deformation which is linked to acceleration of seismic mass

Capacitive : measure change in gap



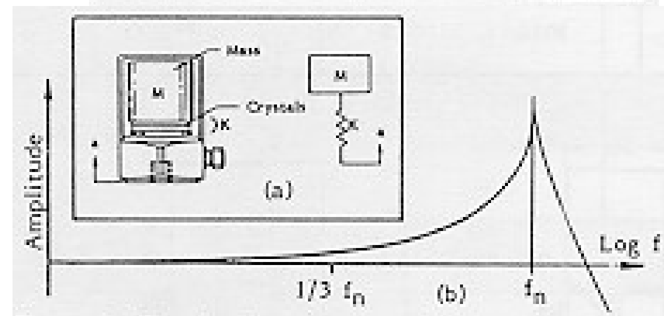
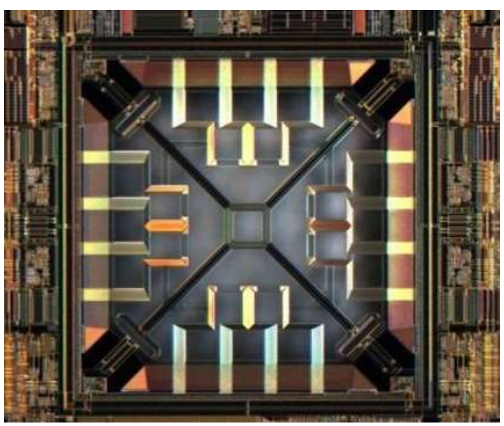
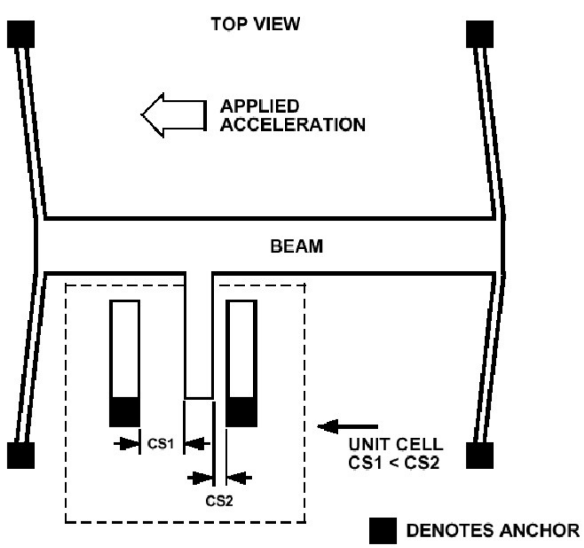
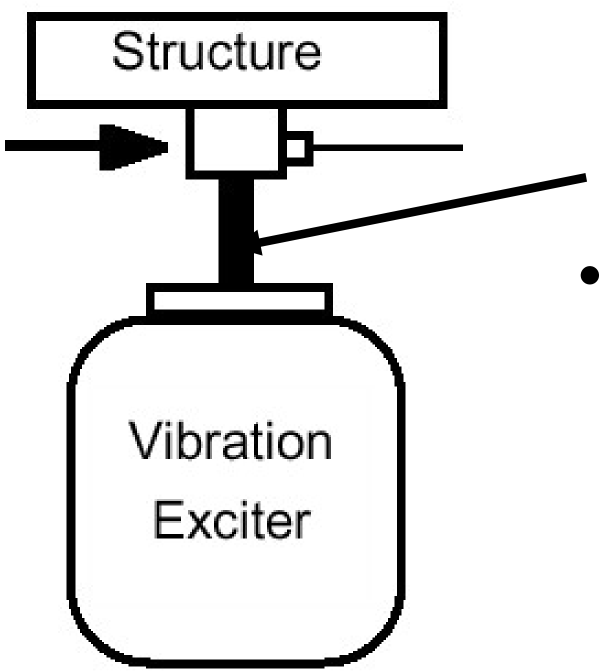


Figure 1: The Accelerometer as a Spring-Mass System



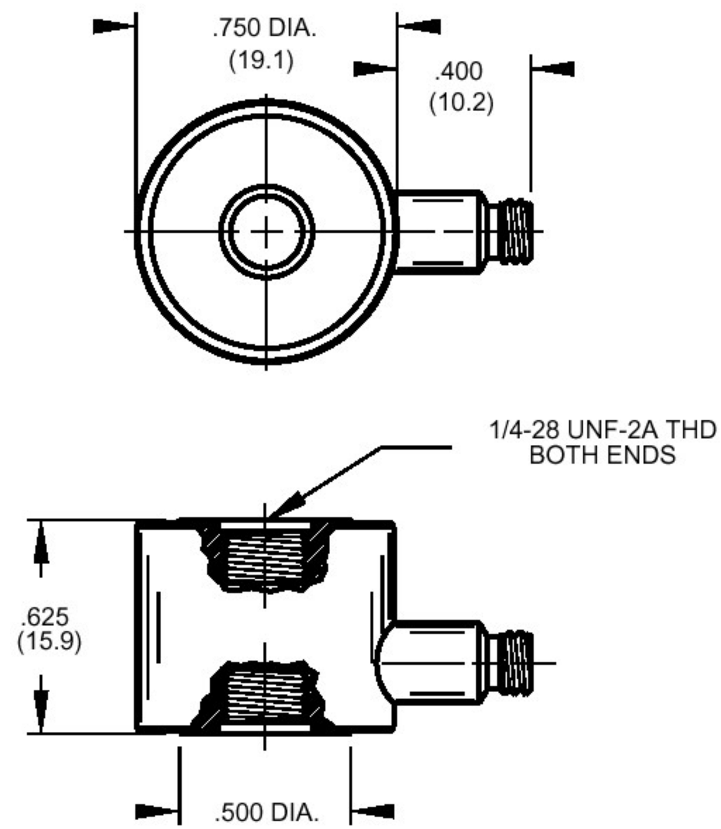


Load cells



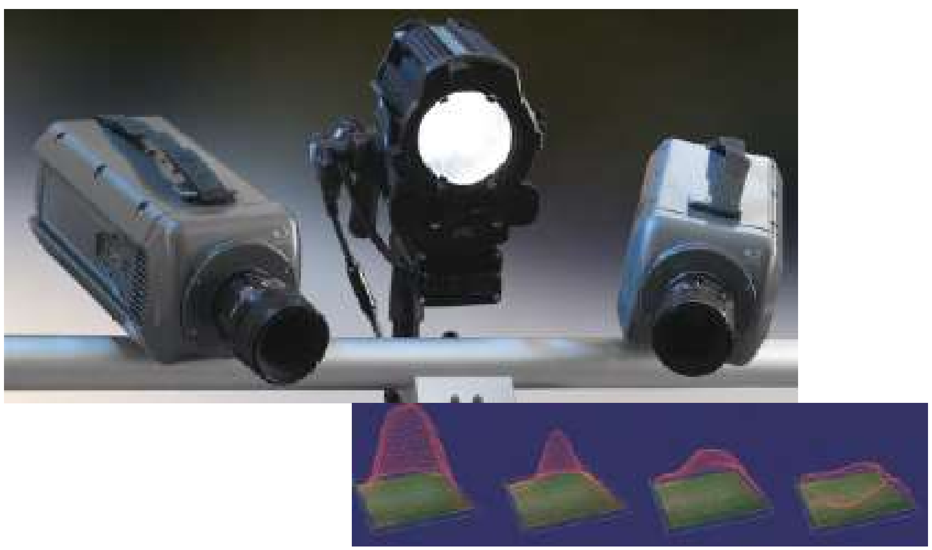
- Measure deformation to estimate force
- Rod to minimize moment transmision

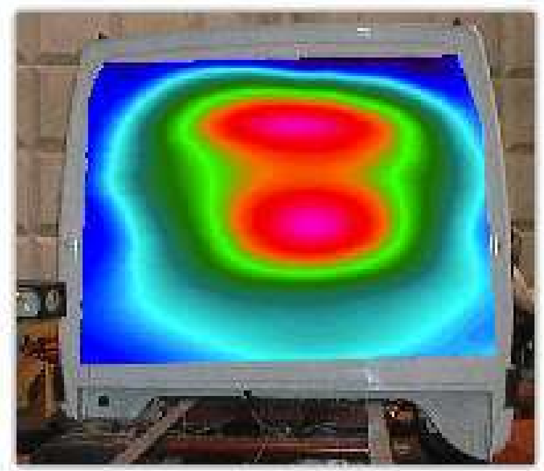
Actual size



Optical techniques

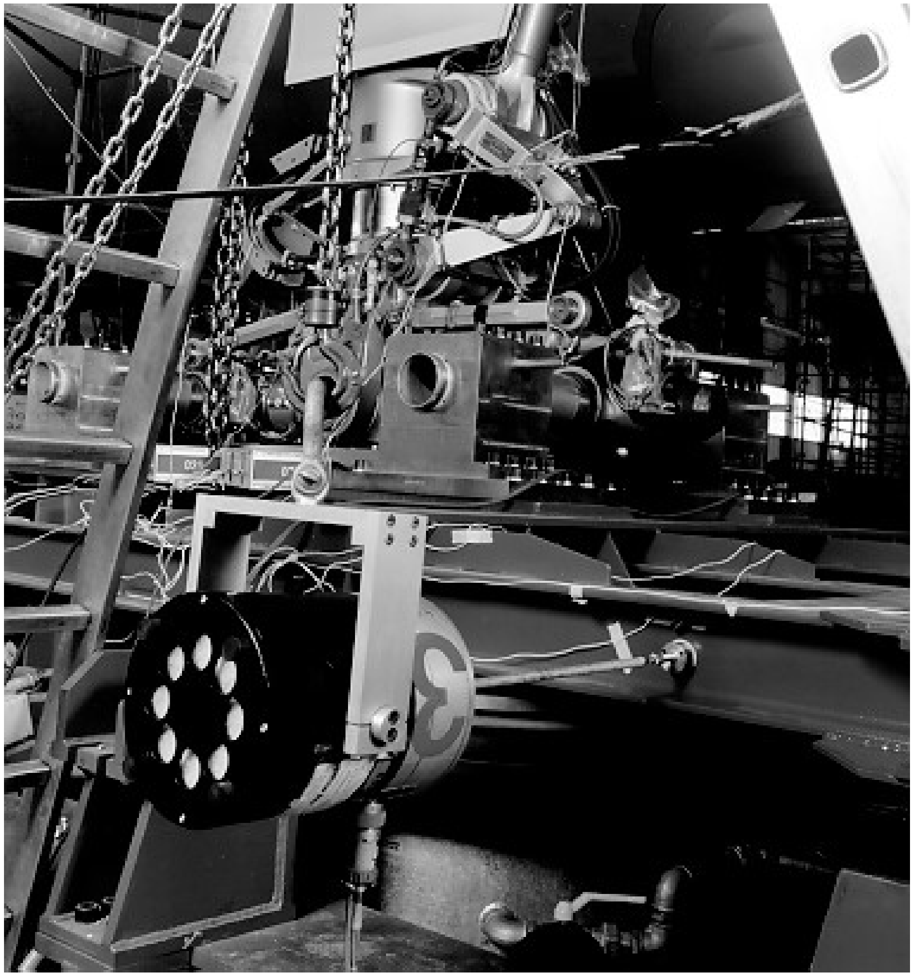
- Laser Doppler Vibrometers
  high amplitude resolution, point scanning
- Images (high number of simultaneous points, smaller amplitude resolution)
  - ESPI (Electronic Speckle Pattern Interferometer)
  - Image correlation white light

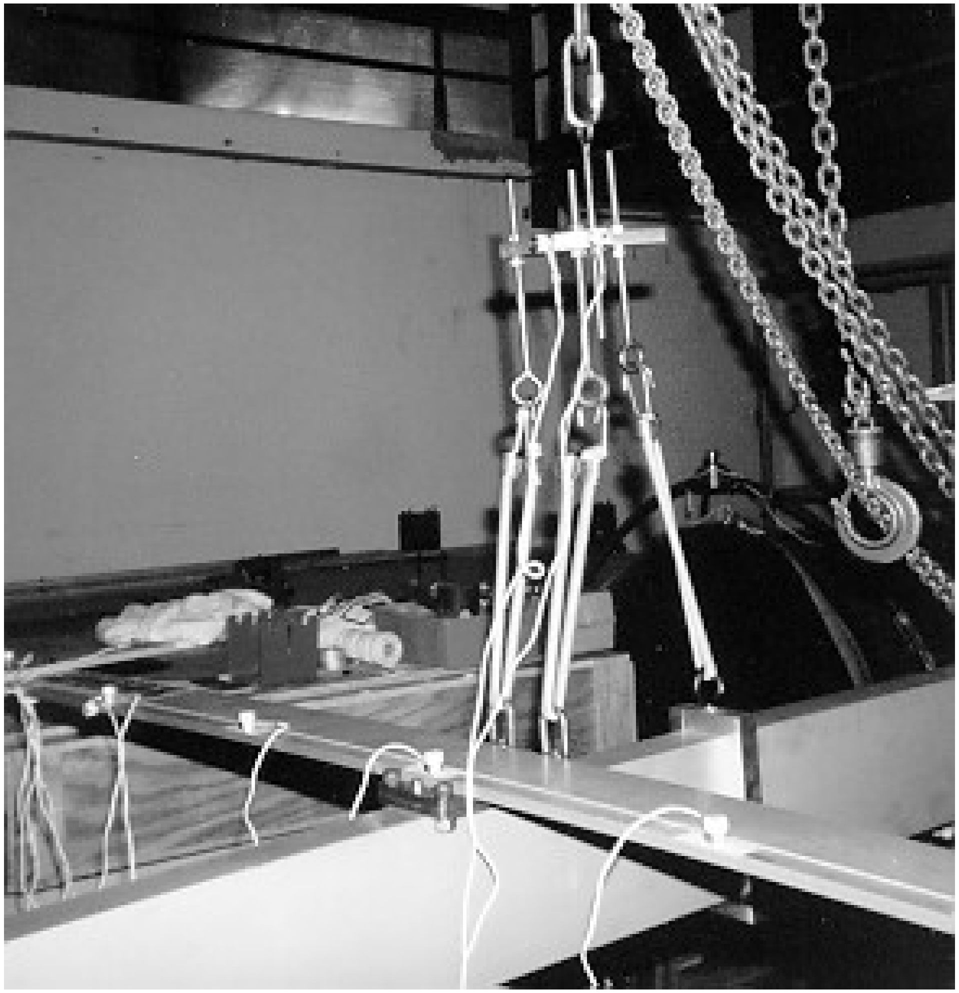




Boundary conditions

Suspended, fixed …



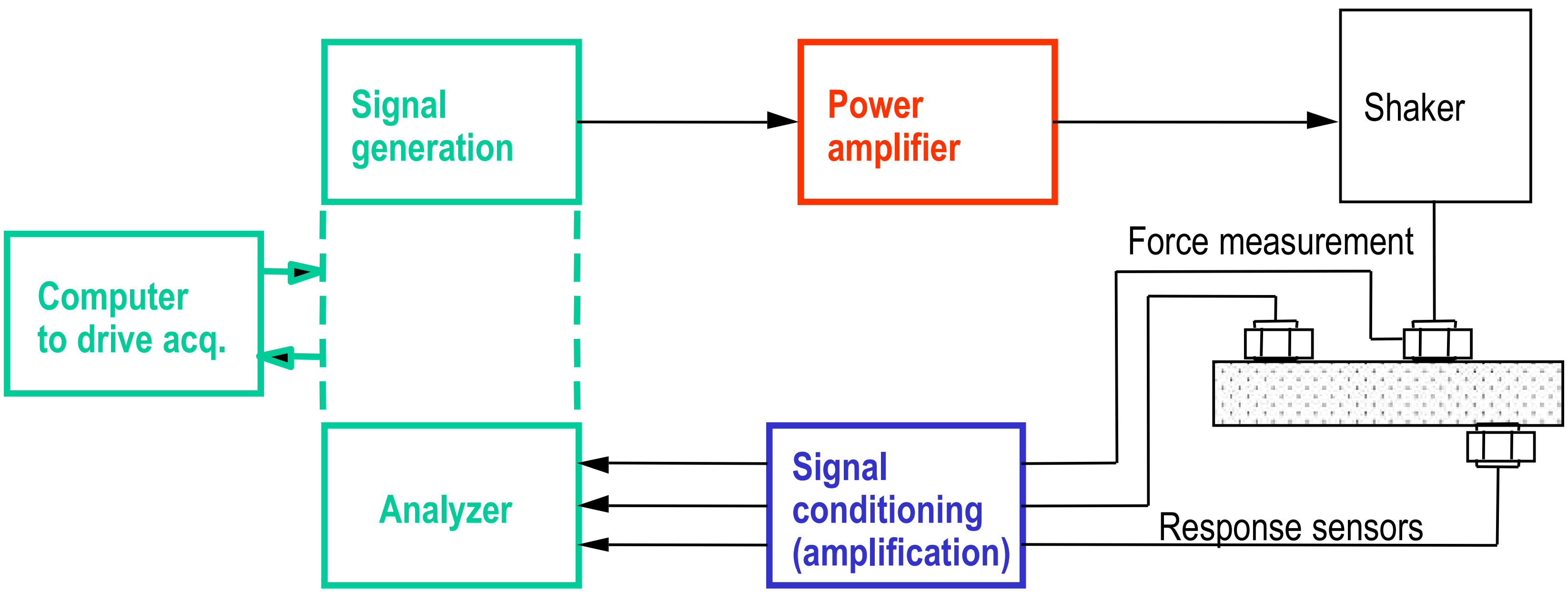


Signal conditioning

Analyzers measure/generate voltages

Sensors generate charges (charge amplifier external or ICP)

Shakers need power



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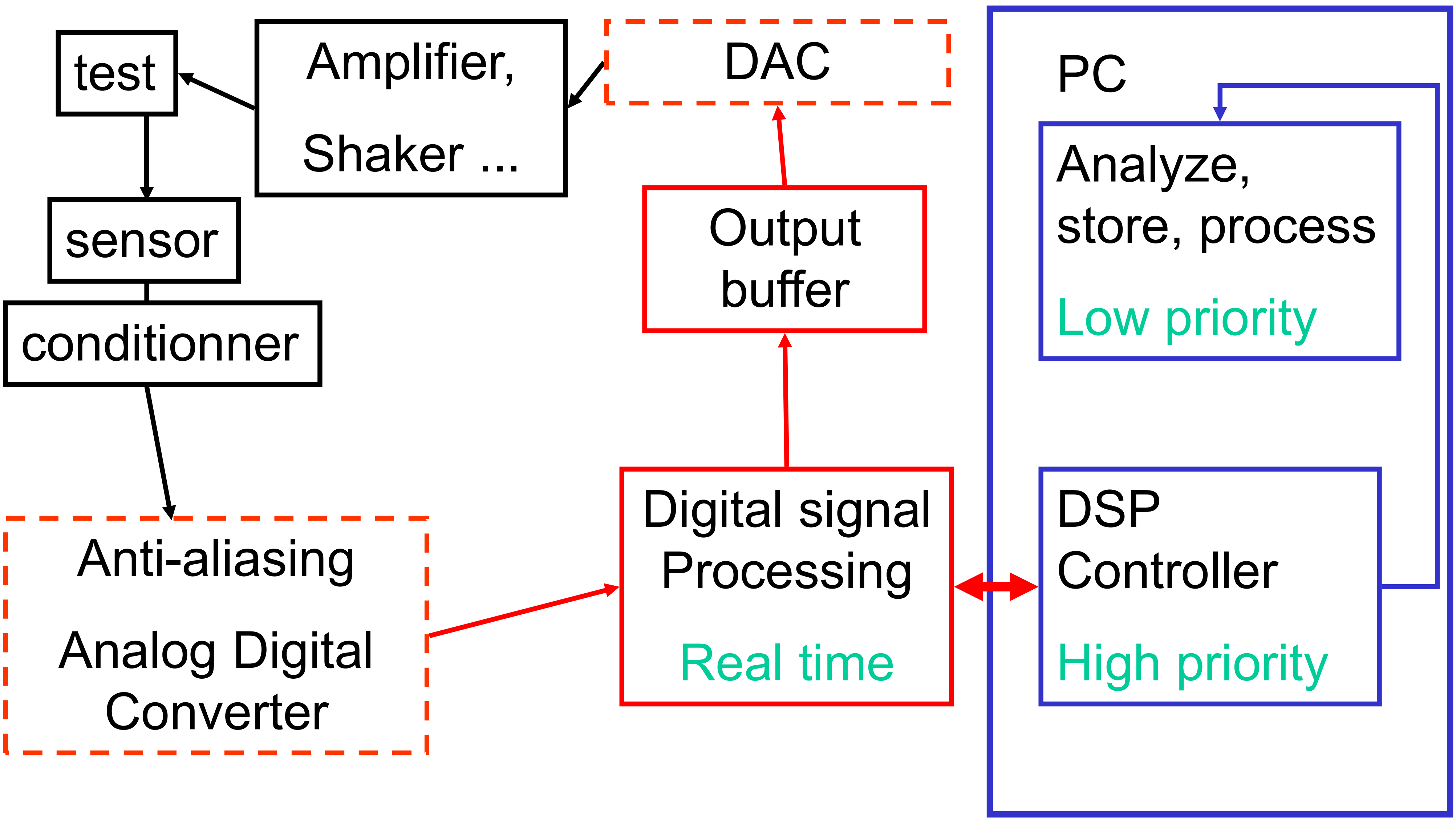
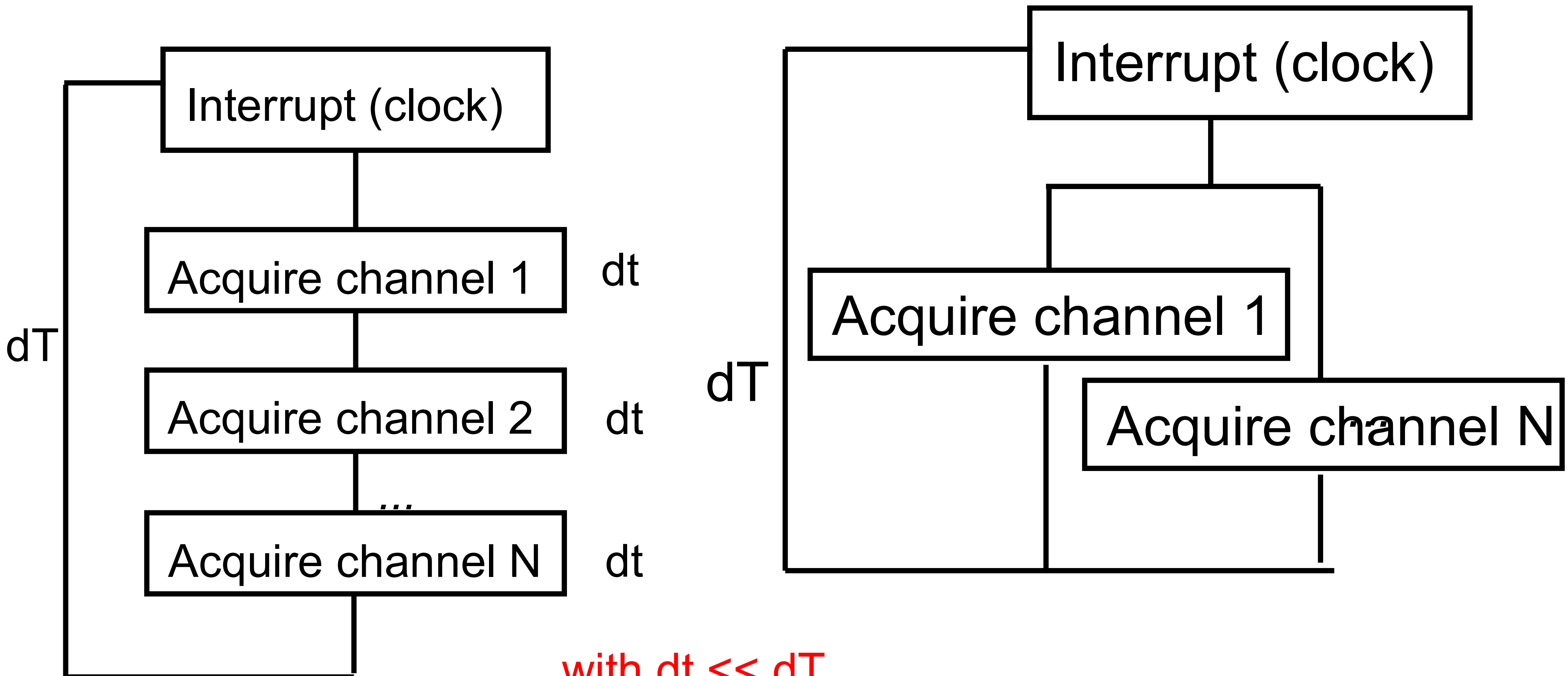
Acquisition and processing



Sampling technology

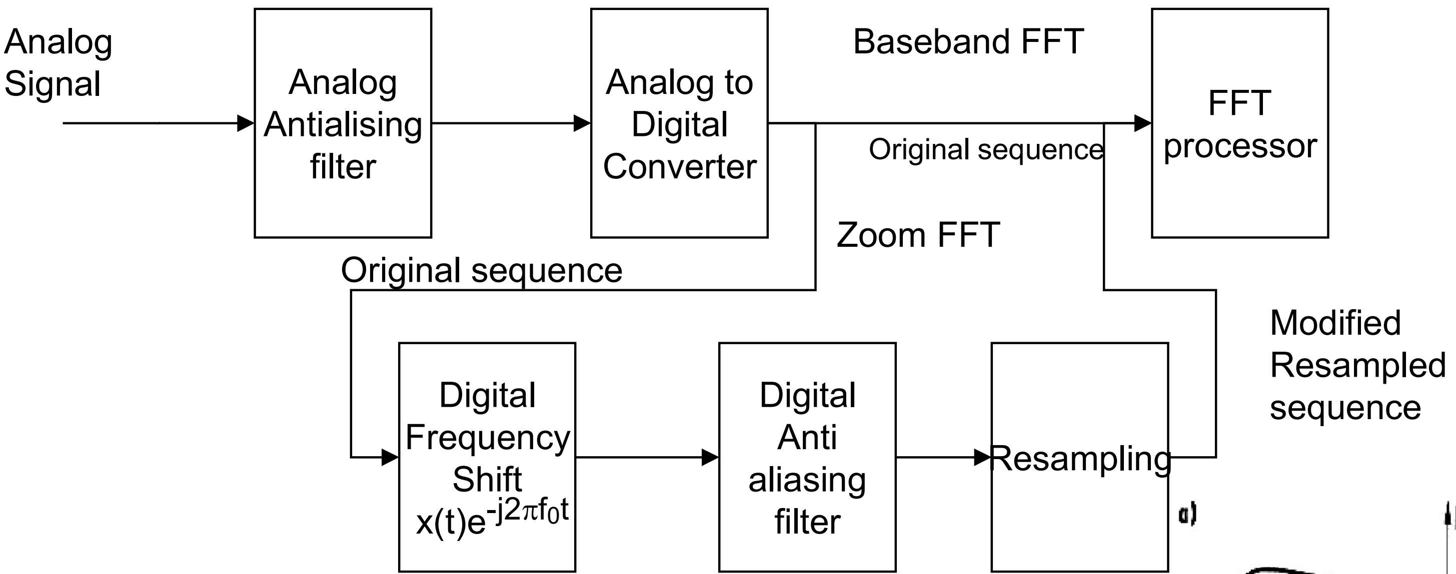


with dt << dT

DAC : Precision = RangeV / 2^nBits

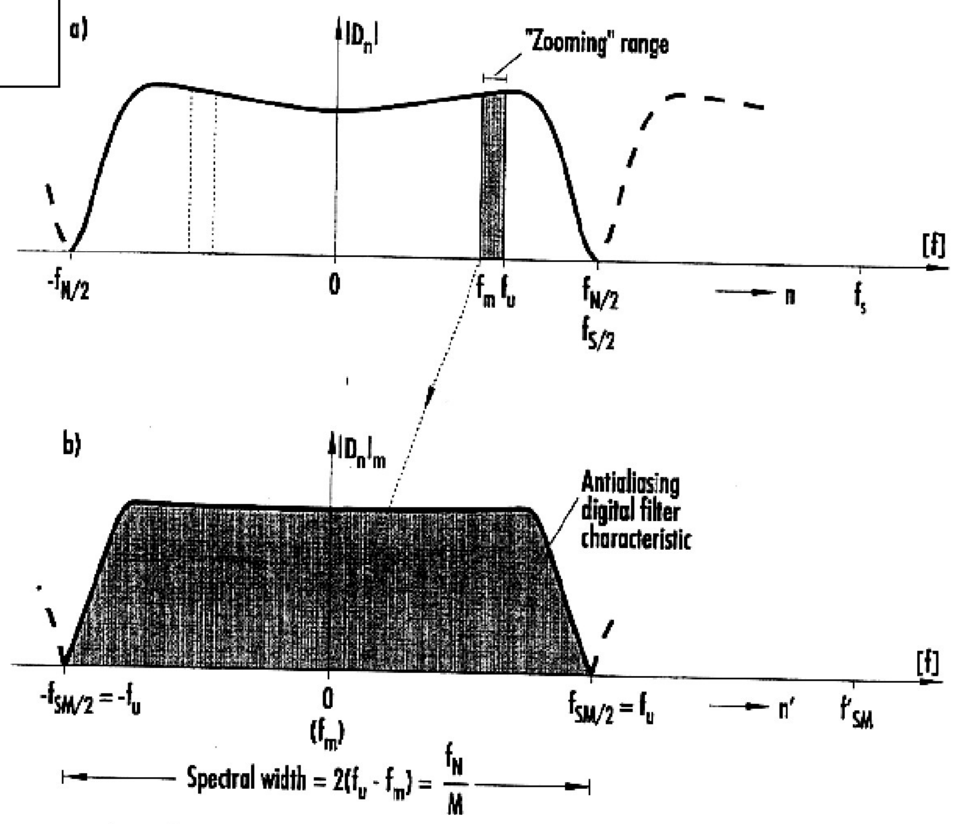
Example : 12 bits *P* = 20V/4096, 24 bits *P* = 20/16777216

FFT Zoom



Based on spectrum shift property

$$\mathcal{F}\left(y(t)e^{-j2\pi f t_0}\right) = Y(f - f_0)$$




```
Y=fft(y.*exp(-sqrt(-1)*2*pi*ftarg*t));  % shift and transform
N=length(t)/Nsub; Y(N:end-N)=0;      % filter
Y(end:-1:end-N+1)=conj(Y(2:N+1)); % make symmetric
Y(1)=real(Y(1));
x=real(ifft(Y));
tx=t(1:Nsub:end); x=x(1:Nsub:end);    % resample
fx=[1/diff(tx(1:2))*[0:length(tx)-1]'/length(tx)]+ftarg;
Y=fft(x)*Nsub;
```

Hardware / software