

Vibrations / Modal analysis

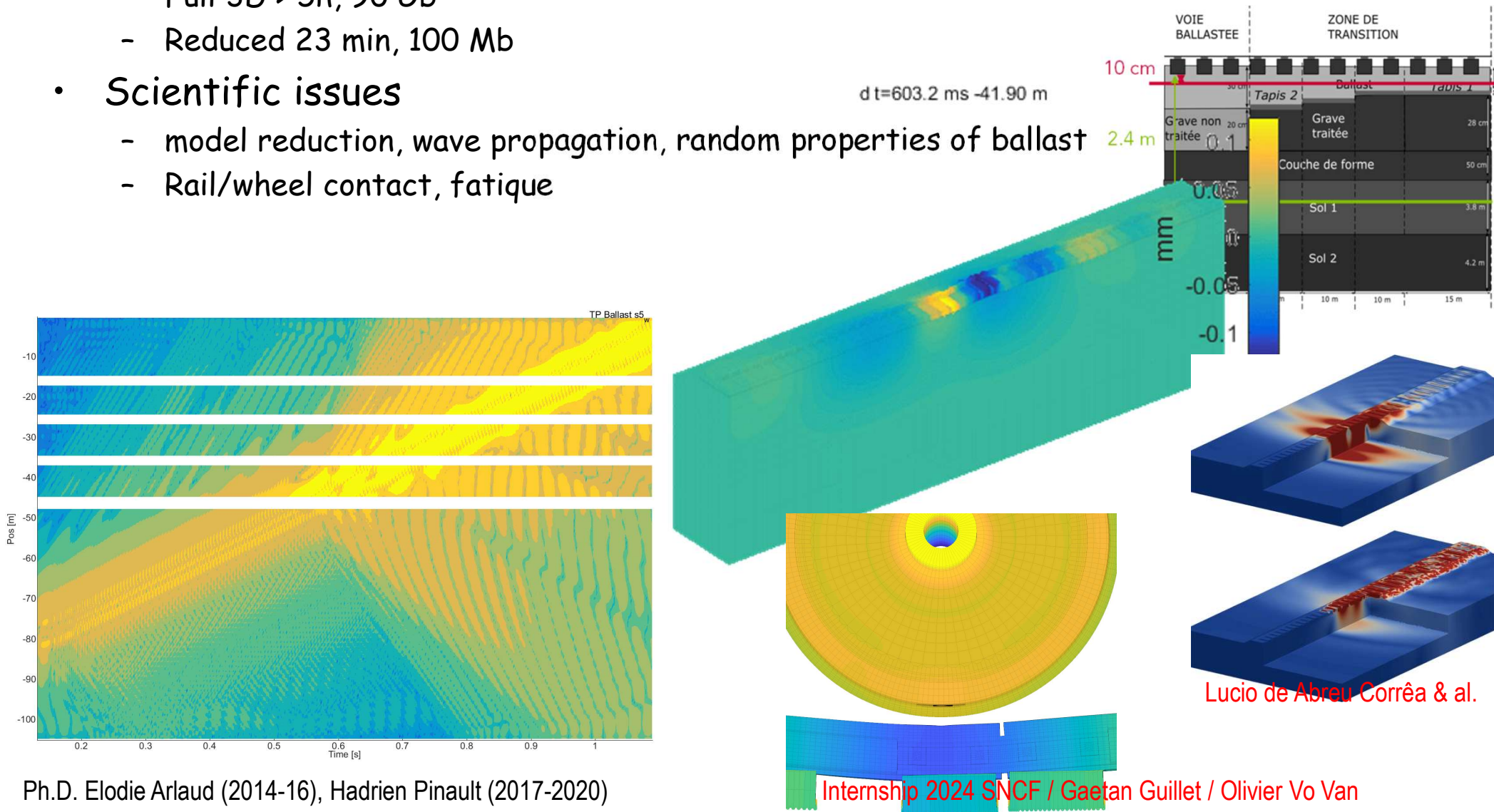
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CentraleSupélec, EDF

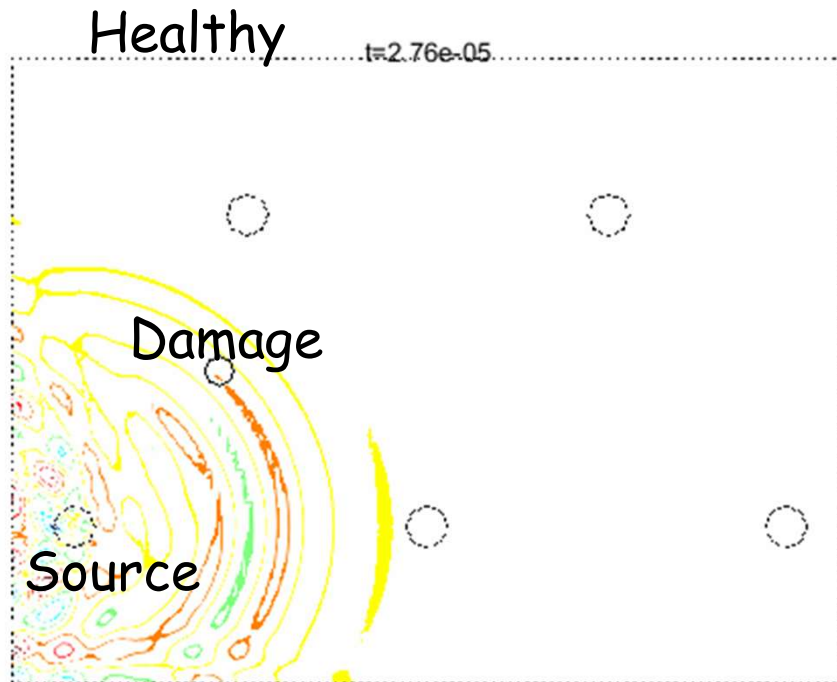
<http://savoir.ensam.eu/moodle/course/view.php?id=1874>

Track/train interaction

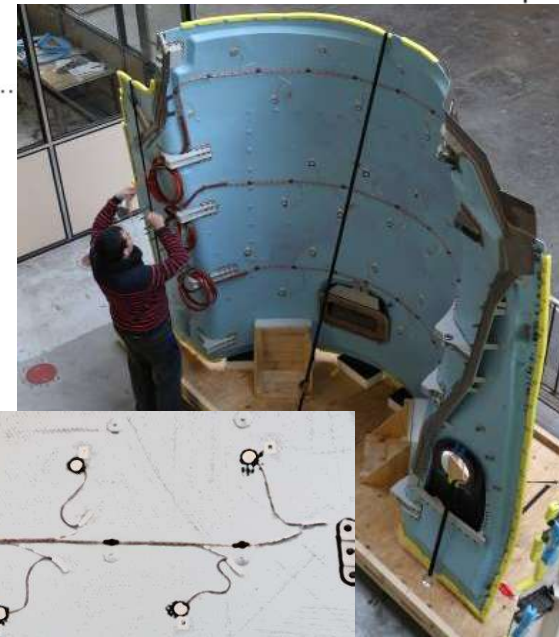
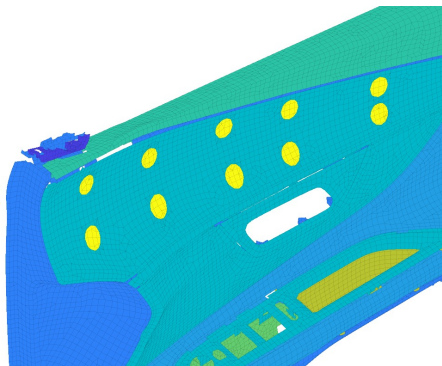
- Piece-wise periodic structure with 15^e3 node per 60cm slice
 - Full 3D > 5h, 90 Gb
 - Reduced 23 min, 100 Mb
- Scientific issues
 - model reduction, wave propagation, random properties of ballast
 - Rail/wheel contact, fatigue



SHM transients

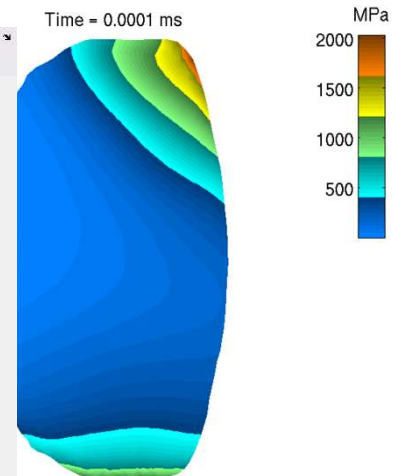
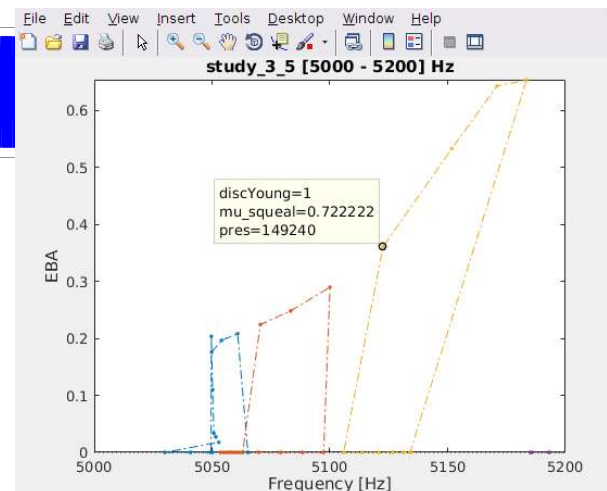
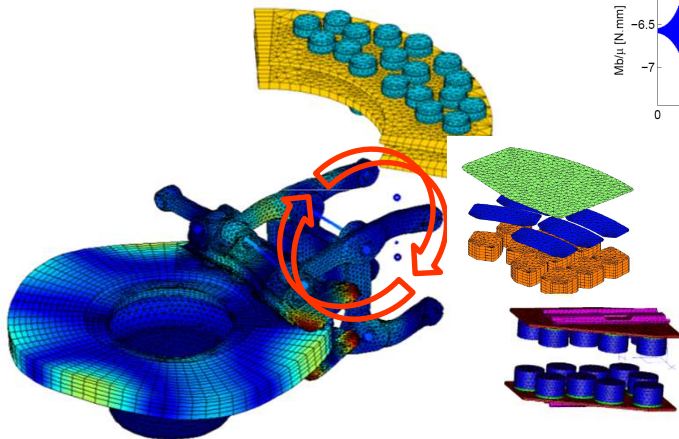
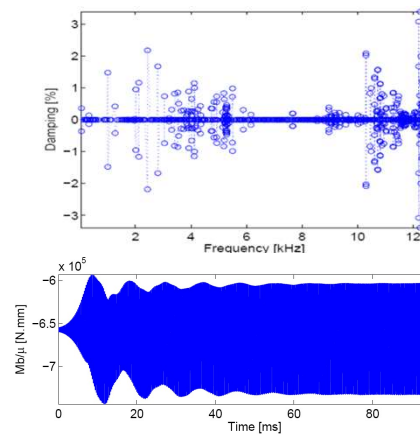
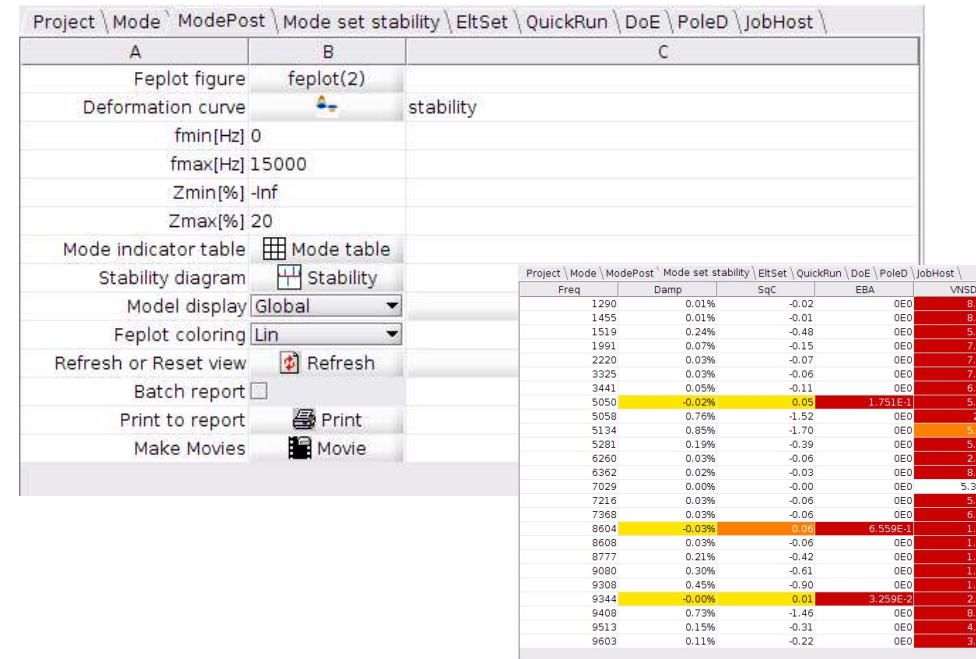


- Electrical signals used for measurement
- Damage acts as a source in **healthy-damaged** signal



Brake squeal simulation

- 1 full time @ SDTools
(Audi, Daimler, Stellantis, CBI, ...)
- Advanced solvers in frequency & time
- Objectives
 - Industrial design tools
 - Parametric model reduction
 - Optimized transients



Squeal testing : combined test/FEM

Full FEM

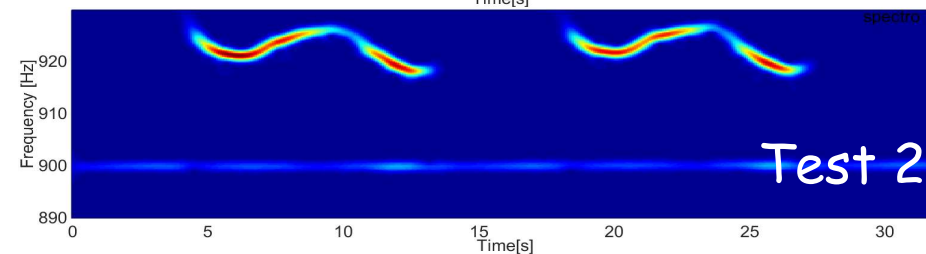
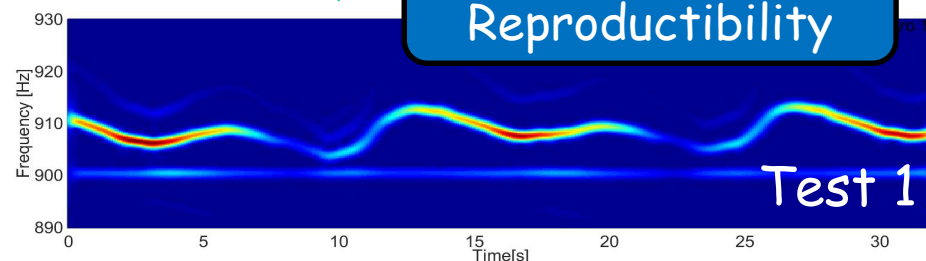
Update :

1. Geometry
2. Parameters
3. component contacts

Sensor placement

Real test

Variability
Reproducibility



Squeal : time varying with wheel position

Expansion

Mode 19 at 910 Hz

Shape
extraction

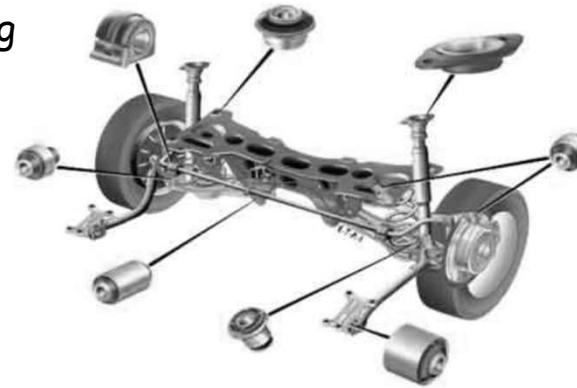
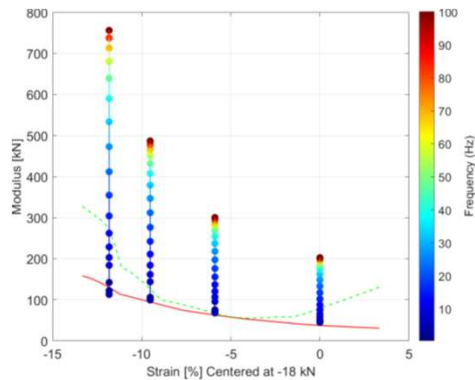
HITACHI
Inspire the Next

5/49

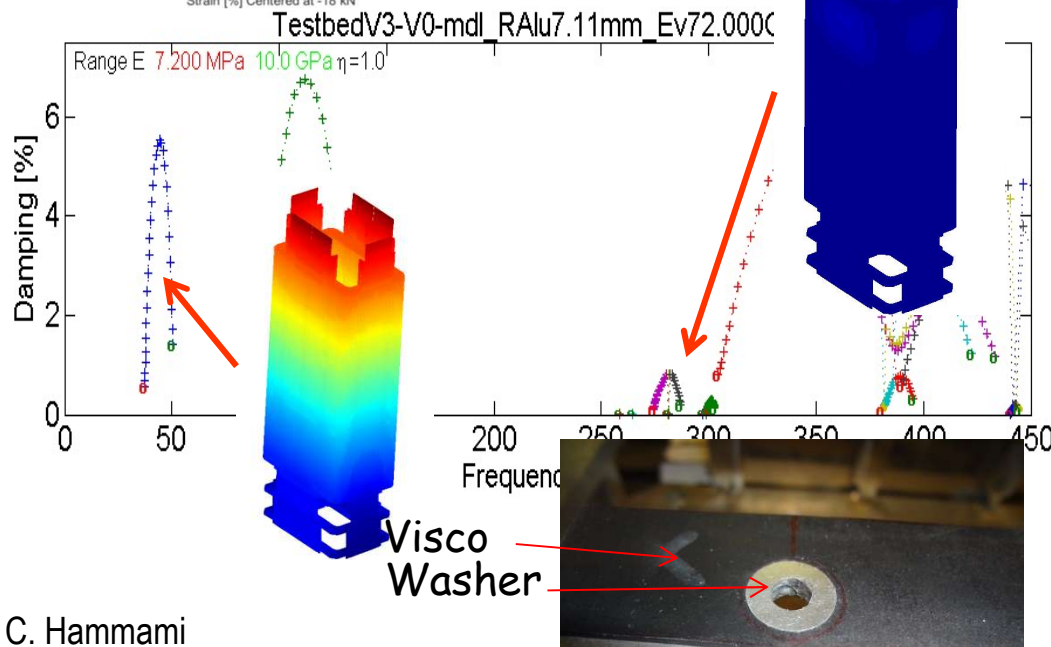
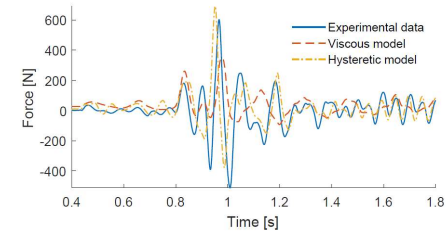
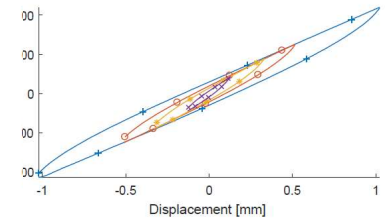
Viscoelastic damping

Scientific problems with damping

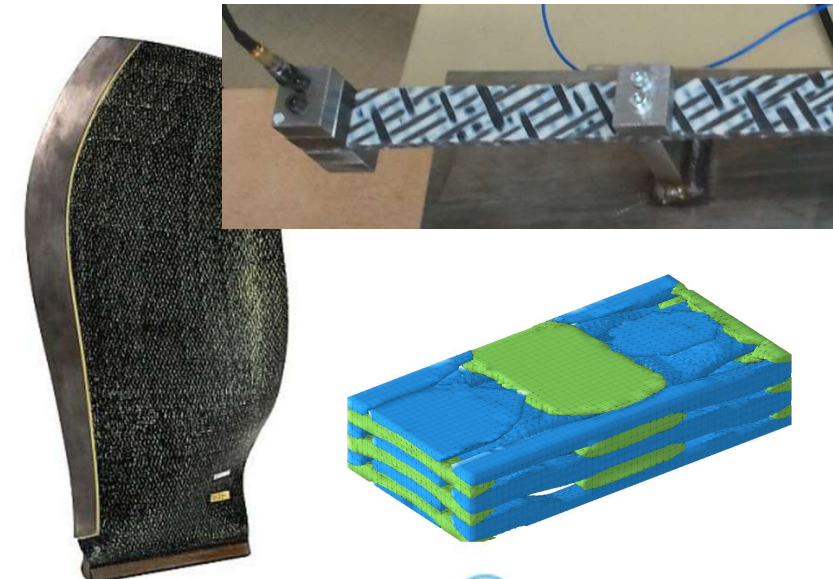
- OD (structure) vs. 3D (material + geometry) modeling
- Damping in complex structures
- Homogeneization in composites



Ph.D. R. Penas 21



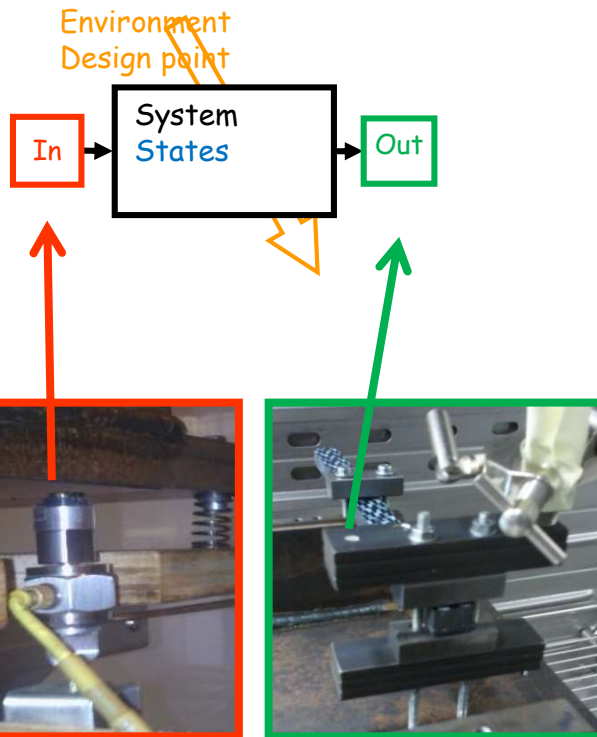
Ph.D. C. Hammami



Ph.D. F. Conejos 21



What is a system ?



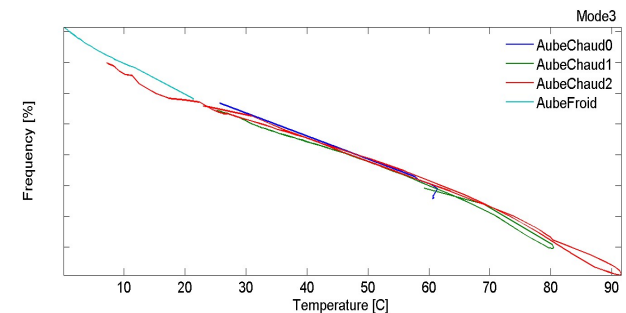
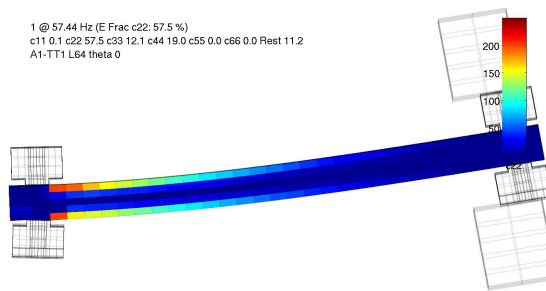
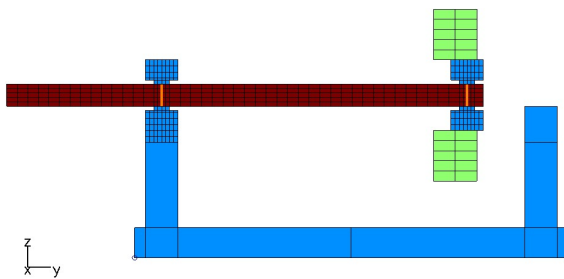
- **Inputs $u(t)$** : hammer with force measurement
- **Outputs $y(t)$**
 - Test : vibrometer on testbed
 - Computation : stresses
- **State $x(t)$**
 - Displacement & velocity field as function of time

$$\{\dot{x}(t)\} = f(x(t), u(t), p, t)$$

evolution

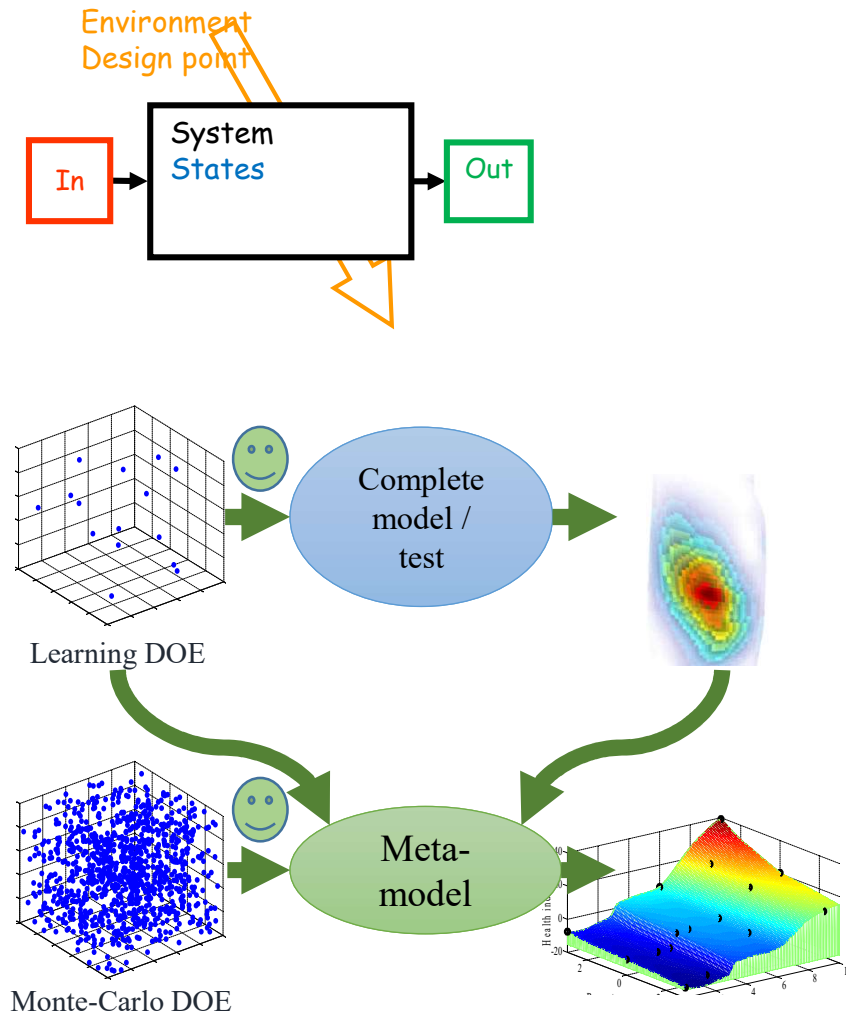
$$\{y(t)\} = g(x(t), u(t), p, t)$$

observation
- **Environment variables p**
 - Dimensions, test piece (design point)
 - Temperature (value of constitutive law or state of thermo-viscoelastic)
- Feature : function of output (example modal frequency)



Simple example : modified Oberst test for 3D weaved composite test

System models : nature & objectives?



What is a model

- A function relating input and outputs
- For one or many parametric configurations

Model categories

- **Behavior** models (meta-models)
 - Test, constitutive laws, Neural networks
 - Difficulties : choice of parametrization, domain of validity
- **Knowledge** models
 - Physical principles, low level meta-models

Why do we need system models ?

Design

- Become predictive : understand, know limitations
- Perform sizing, optimize, deal with robustness

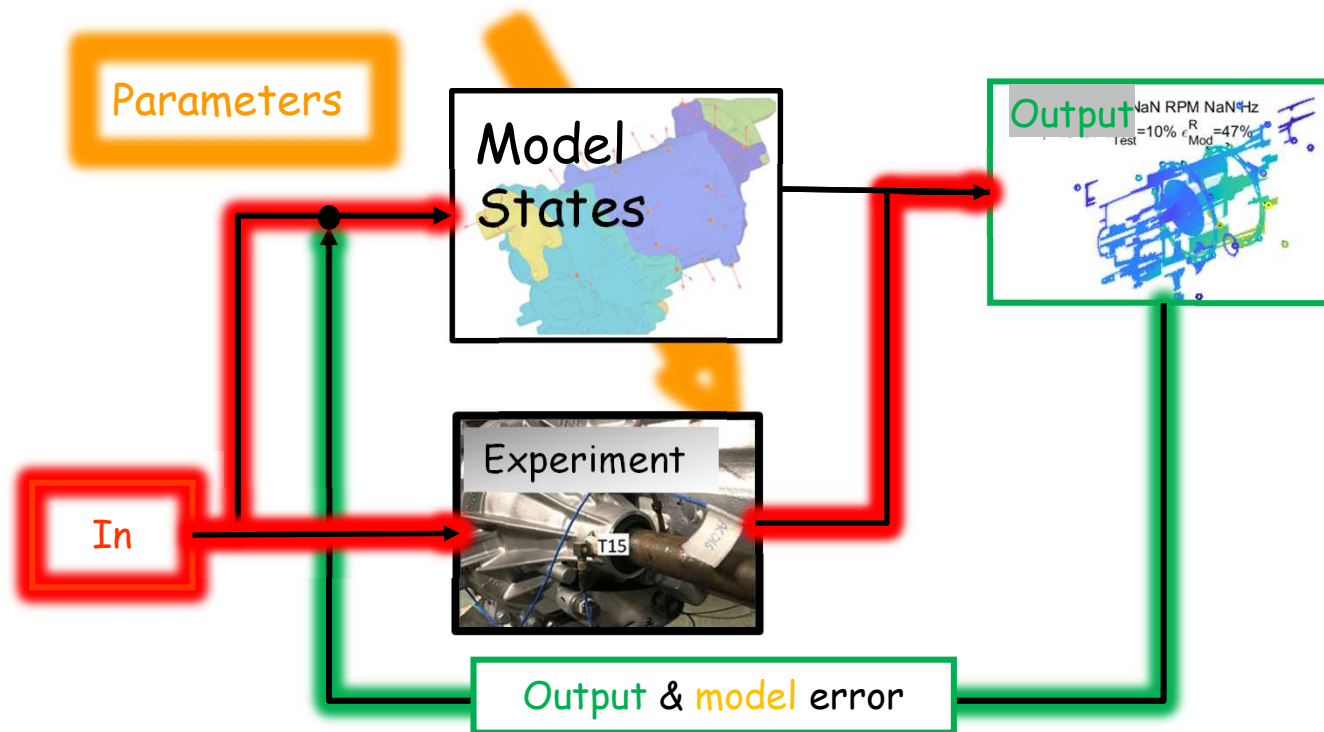
Certify

- Optimize tests : number, conditions
- Understand relation between real conditions and certification
- Account for variability

Maintain during life

- Design full life cycle (plan maintenance)
- Use data for conditional maintenance (SHM)

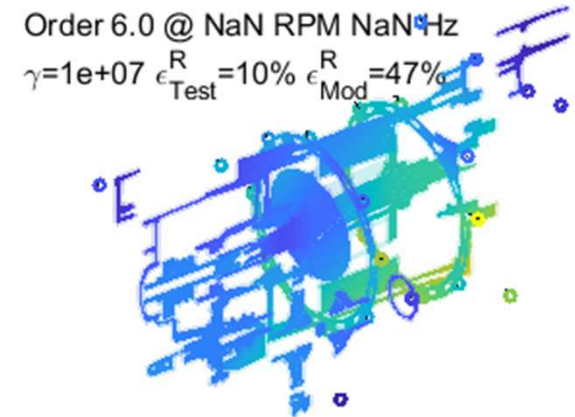
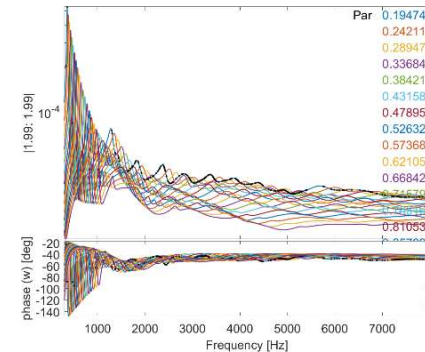
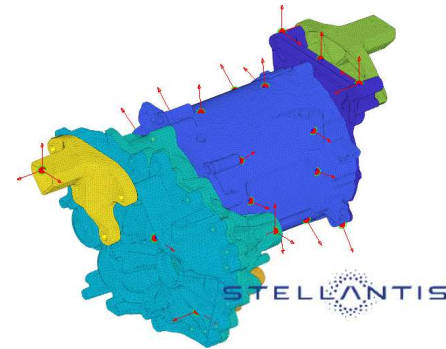
Graphical course outline



1. **Test characterization : input / system, output (space & time & frequency)**
C3: signal, C5: Experimental modal analysis
2. **Parametric system models for online use**
C1-C2: modes and system models, C4: model reduction, C6: sensitivity, C7: reanalysis, C9: damping, C10: component mode synthesis, C11: subspace methods
3. **Hybrid test/FEM models**
C8 : test/analysis correlation, expansion, C12 updating

A gradation of models/twins

- Direct FEM simulation / virtual twin
 - Direct response, offline 🐢
- Reduced-meta model / digital twin
 - Offline generation of training data 🐢
 - Reduction (direct problem)
/ learning (inverse, identification, training)
 - Online 🐰 usage
- State-estimation, parameter updating
/ hybrid twin
 - Combine physical model & test



Lab work / evaluation

Lab work (with Mathieu Corus)

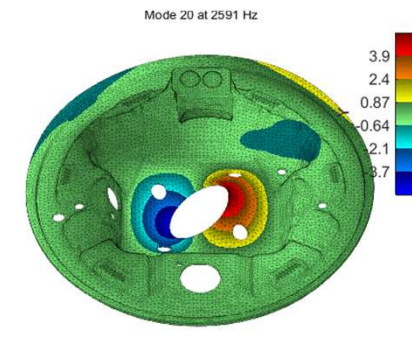
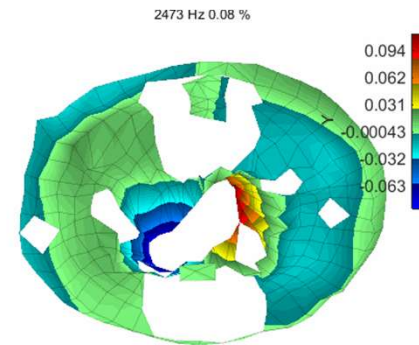
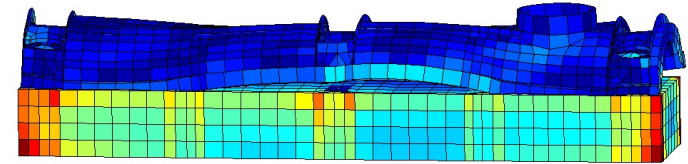
- 1 : code verification, signal, 1 DOF
- 2 : transfers, time/frequency
- 3 : identification, test/analysis correlation
- 4 : reduction and parametric models, updating

TP2-4 MATLAB +SDT : www.sdtools.com/sdtcur

Evaluation

- **oral** (PPT no interactive MATLAB), 30 mn (< 20 slides)
equal weight for 1-2-3-4 (address most questions)
- Work as pairs (not 3)
- Evaluation on how you **expose & comment** results
(5 pt per lab)

Data : <http://savoir.ensam.eu/moodle/course/view.php?id=1874>



MATLAB 2025a-26a has incompatibilities (dev SDT version)
easier with 2024b or earlier

2026 Planning

- 15/9 Course1 : Intro, 1DOF, system
C2 : modes & synthesis, base of reduction, spectral decomposition
- 22/9 C3 : signal for vibration (continuous vs. discrete, aliasing, windowing)
C4 : Ritz and learning. Historical : McNeal, Craig-Bampton, ...
- 29/09 C5 : Model parameterization, sensitivity computations
C6 : experimental modal analysis : from test to a system model. Inverse problem.
- Mercredi 30/09 PM Lab1 : 1 DOF, state-space, signal, NL
- 06/10 C7 : Reduction for reanalysis. Validity and error control.
C8: Test/analysis correlation. Topology correlation. Observation, MAC, expansion. Start of model updating.
- 13/10 Lab2 : modal base frequency domain, transfers, transient, signal processing
- 20/10 10h-13h : Lab3 : identification, sub-space, test/analysis
- 03/11 C9 : damping : devices, physical mechanisms, numeric tools
C10 : CMS (Component Mode Synthesis), coupling models, reduction for coupling
- 17/11 : C11-12 subspace methods, current issues with non-linear systems, updating
- 24/11 : Lab4 : Parametric models, reduction, damping, updating
- xxx/12 : Oral

To go further

Course material (notes, slides)

<https://savoir.ensam.eu/moodle/mod/folder/view.php?id=19444>

For experimental modal analysis

- [1] W. Heylen, S. Lammens, and P. Sas, Modal Analysis Theory and Testing. KUL Press, Leuven, Belgium, 1997.
- [2] D. Ewins, Modal Testing: Theory and Practice. John Wiley and Sons, Inc., New York, NY, 1984.
- [3] <https://www.uml.edu/Research/SDASL/Education/Modal-Space.aspx>
- [4] K. G. McConnell, Vibration Testing. Theory and Practice. Wiley Interscience, New-York, 1995

Numerical aspects

- [5] D. Inman, Engineering Vibration. Prentice-Hall, Englewood Clis, N.J., 1994.
- [6] M. Geradin and D. Rixen, Mechanical Vibrations. Theory and Application to Structural Dynamics. Editions 1993 & 2015
- [7] R. J. Craig and A. Kurdila, Fundamentals of Structural Dynamics. Wiley, 2006.
- [8] <https://www.code-aster.org> : (English & French)

Keywords : 1 DOF / signal

After intro course: ([chapter1](#) and [section 2.1](#), [modal.pdf](#))

1. Transfer (time, Fourier $i\omega$ /Laplace s , asymptotic prop, NL)
2. Poles, resonance, damping ratio -3 dB method
3. State-space, poles
4. 1 DOF time exponential, convolution, logarithmic decrement
5. Strategies for transient in time & frequency
6. Equivalent power

Measurement and signal processing (CM3, [signal.pdf](#))

1. DFT f_k , relation δt , T , δf , linearity, dilatation, ([section 3.1](#))
2. Aliasing : Shannon's theorem, when, mitigate, ... ([s3.2.1](#))
3. Leakage & windowing : continuous vs. DFT ([s3.2.2](#))
4. Transfer function estimate (H1, coherence) ([s3.3](#))
5. Technology : sensors, actuators, acquisition

TP1 : code verification for 1DOF, integration, signal

Keywords : modes & synthesis, reduction

Modes & synthesis (CM2, see also second part of [Modal.pdf](#) slides)

1. Inputs/outputs, IO shape matrix, disp, resultants, ... ([s2.1](#))
2. Discrete modes (harmonic solution without input), orthogonality ([section 2.2.1](#))
3. Ritz/Galerkin principles
4. Ritz/modal coordinates, PPV, series & state-space, time/freq strategies
5. Modal & **Rayleigh damping**, modal damping in physical coord ([s 2.2.x](#))
6. Peak visibility, truncation, effective contributions ([s 2.2.x](#))

Reduction (course 4, see also [Reduction.pdf](#) slides)

1. Ritz/Galerkin & learning
2. Modes + Residual flexibility ([section 4.3](#))
3. McNeal= Ritz with "residual vectors", pre-filter low frequency modes
4. Residual vectors in presence of flexible modes
5. Guyan, Craig-Bampton = enforced displacement & bandwidth ([s 4.3.2](#))

Left for other course : from vector set to basis

Keywords : parametric models

Sensitivity / extended uses of modes (co. 5, [SensitivityReanalysis.pdf](#))

1. Parametrization ([s 9.1](#))
2. Sensitivity of static response, adjunct state ([s 9.2](#))
3. Sensitivity of frequencies : relation with energy distribution ([s 9.3](#))
4. Sens. of mode-shapes : modal crossing + numerical strategy for inverse of underdetermined problem

Parametric studies (course 6)

1. Reanalysis example in modal basis (start by continuous case of spring on tensioned wire). Generalization to ΔK .
2. Multi-model and nominal + residual methods
3. Illustrations (damping/Updating)
4. Error control, iterative basis refinement
5. Orthogonalization strategies, GS/GSM/IGSM, Mseq

TP2 : modes & synthesis, signal

Keywords : experimental modal analysis

Identification (course 7, [EMA.pdf](#))

1. Identification demo
2. Inverse problems : model forms, data ([s 6.1](#))
3. Model forms for identification, discussion of residual terms
4. Frequency domain least-squares solution, implicit NL
5. Evaluation of results ([ch 7](#))

Test/analysis correlation (course 8, [Correlation.pdf](#))

1. Topology correlation ([s 8.1](#))
2. Measuring distance between test & analysis. Shape correlation : MAC, pairing issues ([s 8.2](#))
3. Static condensation/expansion, reduced mass, orthogonality on sensors ([s 8.3](#))
4. Hybrid models

Current trends 1

Damping (course 9, [damping.pdf](#))

1. Sample damping devices
2. Notion of coupling & impact on damping
3. Viscoelasticity/complex modulus, MSE
4. Real modes/modal damping & separation
complex modes & enhanced reduction
5. Internal states (// with friction)

Substructuring (Component mode synthesis) (course 10)

1. Coupling conditions : energy or continuity
2. constraints : elimination, Lagrange, Penalization
3. contact & locking/stress concentration
4. Reduction for CMS : classical Craig-Bampton, CMT
5. The "problem" of large interfaces

TP3 : parametric models, damping, identification, correlation

Current trends 2

Updating (course 11, [updating.pdf](#))

1. Typical errors : property, geometry, contact, model
2. Physical and equivalent models
3. Least squares and conditioning, SVD (link with TP3)
4. Error localization
5. Sample applications

Features in vibration behavior (course 12)

1. SVD for mechanics : principal loads, modal energy coordinates, interface DOFs
2. Modal coordinates, physical and macro-models models of junctions with contact/friction
3. Geometrically periodic systems (engine, track)