

Vibrations / Modal analysis

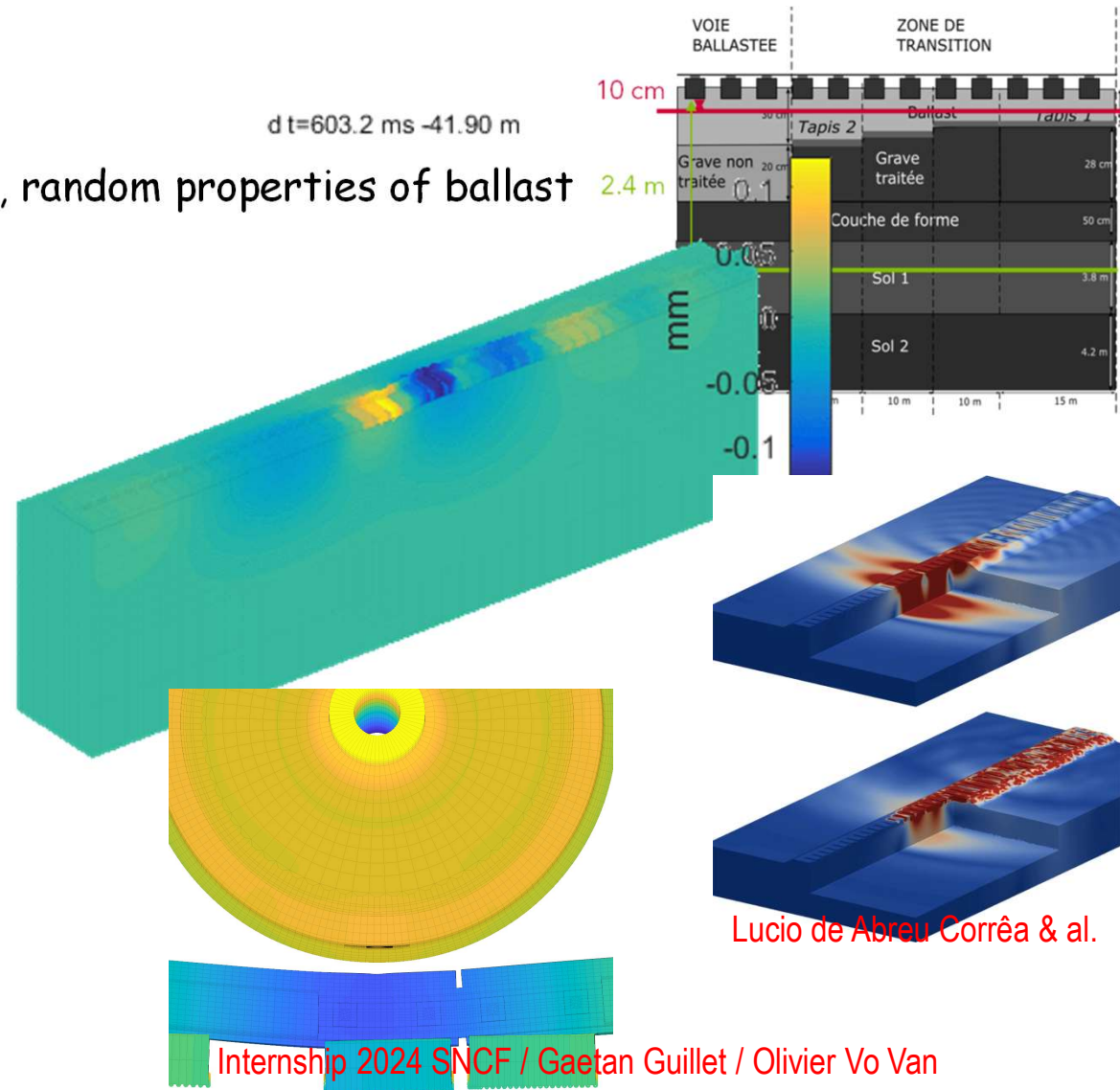
Etienne Balmes
Mathieu Corus

Ensam/PIMM, SDTools
CentraleSupélec, EDF

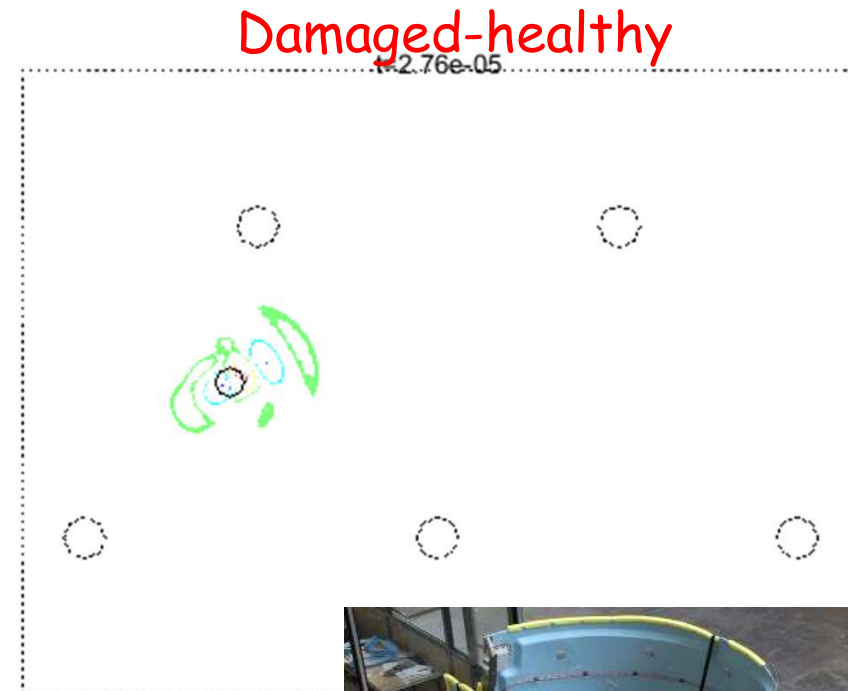
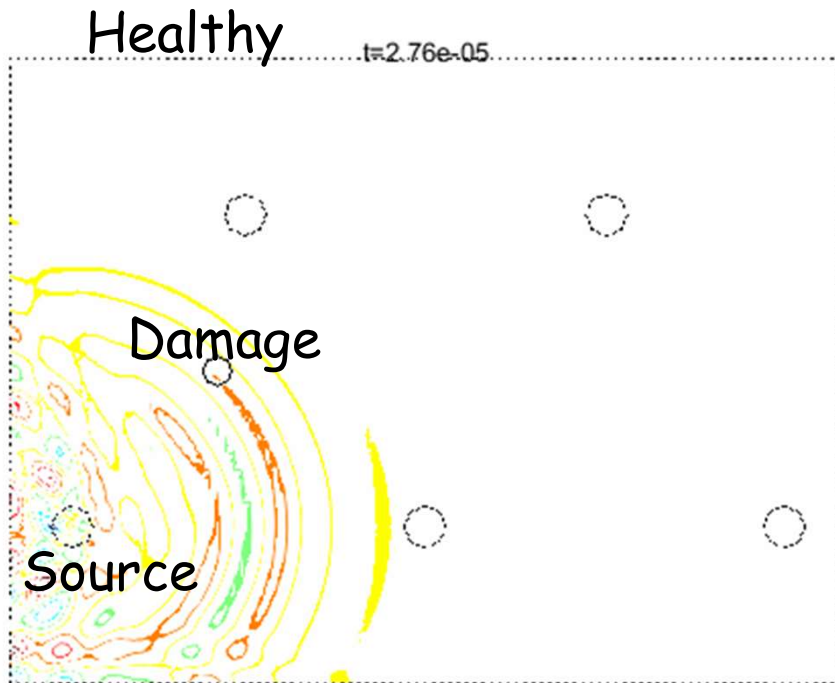
<http://savoir.ensam.eu/moodle/course/view.php?id=1874>

Track/train interaction

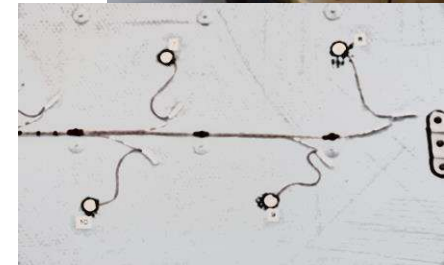
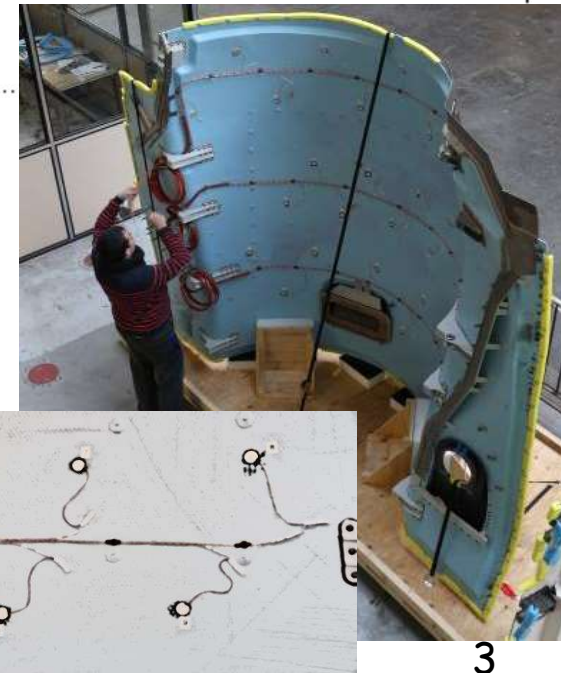
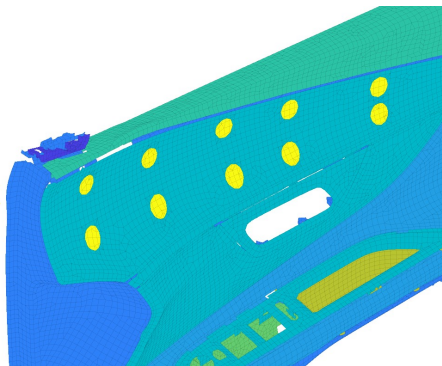
- Piece-wise periodic structure with 15^e3 node per 60cm slice
 - Full 3D > 5h, 90 Gb
 - Reduced 23 min, 100 Mb
- Scientific issues
 - model reduction, wave propagation, random properties of ballast
 - Rail/wheel contact, fatigue



SHM transients



- Electrical signals used for measurement
- Damage acts as a source in **healthy-damaged** signal



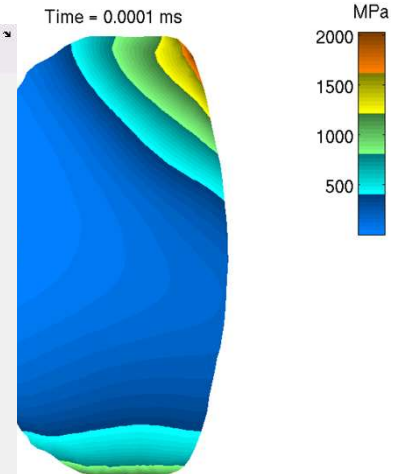
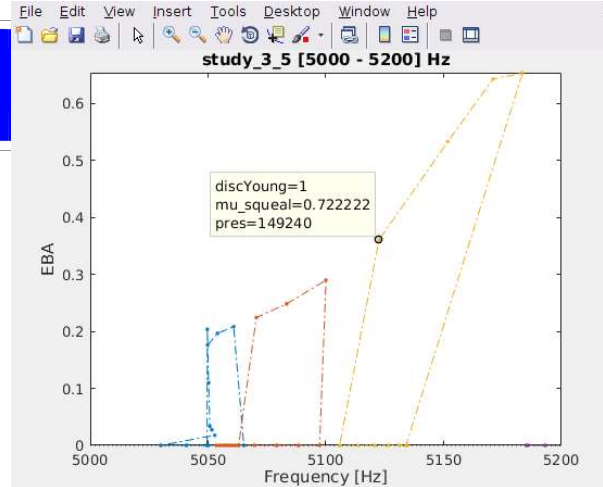
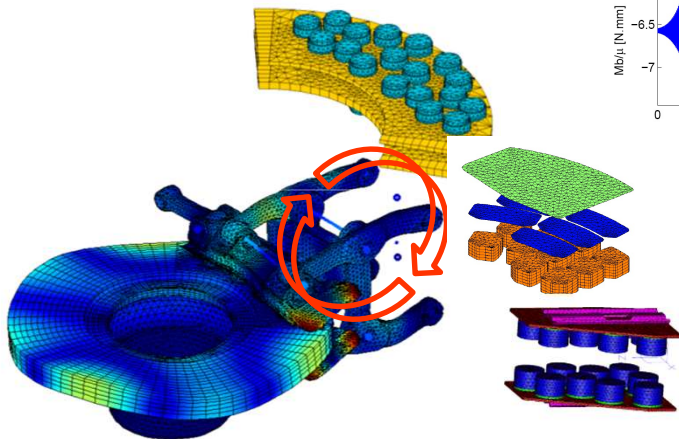
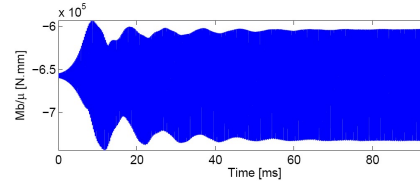
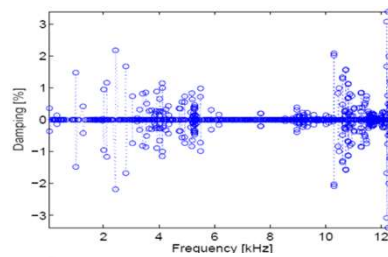
Brake squeal simulation

- 1 full time @ SDTools
(Audi, Daimler, Stellantis, CBI, ...)
- Advanced solvers in frequency & time
- Objectives
 - Industrial design tools
 - Parametric model reduction
 - Optimized transients

Project \ Mode \ ModePost \ Mode set stability \ EltSet \ QuickRun \ DoE \ PoleD \ JobHost \

A	B	C
Feplot figure	feplot(2)	
Deformation curve		stability
fmin[Hz]	0	
fmax[Hz]	15000	
Zmin[%]	-Inf	
Zmax[%]	20	
Mode indicator table		Mode table
Stability diagram		Stability
Model display	Global	
Feplot coloring	Lin	
Refresh or Reset view		Refresh
Batch report	<input type="checkbox"/>	
Print to report		Print
Make Movies		Movie

Project \ Mode \ ModePost \ Mode set stability \ EltSet \ QuickRun \ DoE \ PoleD \ JobHost \	Freq	Damp	SqC	EBA	PoleD	JobHost	VNSD
	1290	0.01%	-0.02	0E0			8.1
	1455	0.01%	-0.01	0E0			8.1
	1519	0.24%	-0.48	0E0			5.9
	1991	0.07%	-0.15	0E0			7.2
	2220	0.03%	-0.07	0E0			7.6
	3325	0.03%	-0.06	0E0			7.4
	3441	0.05%	-0.11	0E0			6.7
	5050	-0.02%	0.05	1.751E-1			5.6
	5058	0.76%	-1.52	0E0			2.
	5134	0.95%	-1.70	0E0			5.5
	5281	0.19%	-0.39	0E0			5.2
	6260	0.03%	-0.06	0E0			2.8
	6362	0.02%	-0.03	0E0			8.2
	7029	0.00%	-0.00	0E0			5.38
	7216	0.03%	-0.06	0E0			5.8
	7368	0.03%	-0.06	0E0			6.0
	8604	-0.03%	0.08	6.559E-1			1.3
	8608	0.03%	-0.06	0E0			1.3
	8777	0.21%	-0.42	0E0			1.0
	9080	0.30%	-0.61	0E0			1.1
	9308	0.45%	-0.90	0E0			1.5
	9344	-0.00%	0.01	3.259E-2			2.1
	9408	0.73%	-1.46	0E0			8.8
	9513	0.15%	-0.31	0E0			4.1
	9603	0.11%	-0.22	0E0			3.8



Squeal testing : combined test/FEM

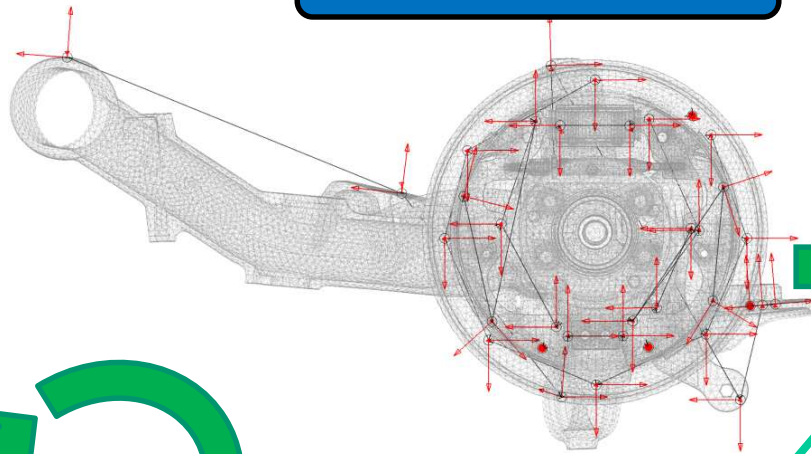
Full FEM

Update :

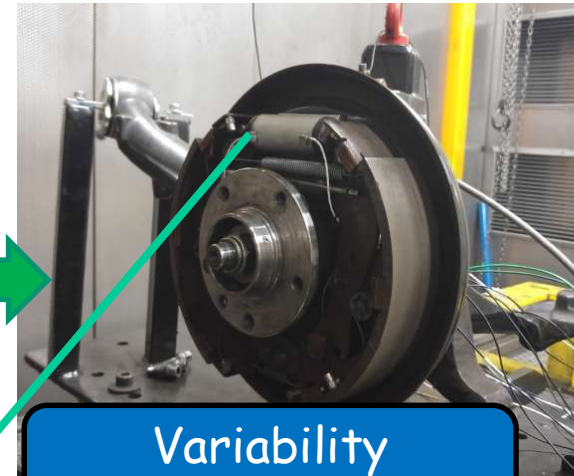
1. Geometry
2. Parameters
3. component contacts



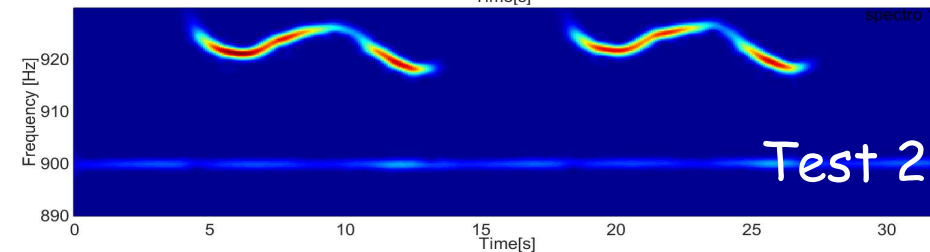
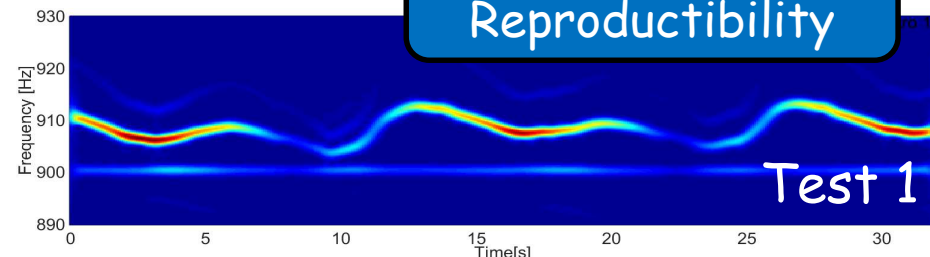
Sensor placement



Real test



Variability
Reproducibility



Squeal : time varying with wheel position

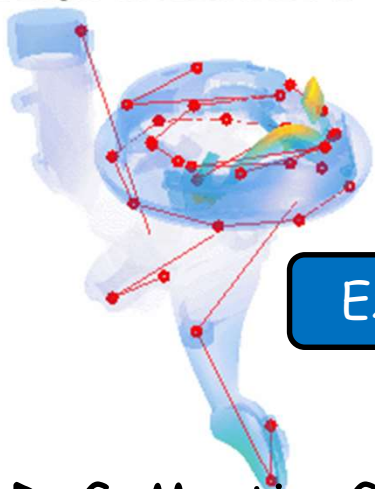
Expansion

Mode 19 at 910 Hz



Shape
extraction

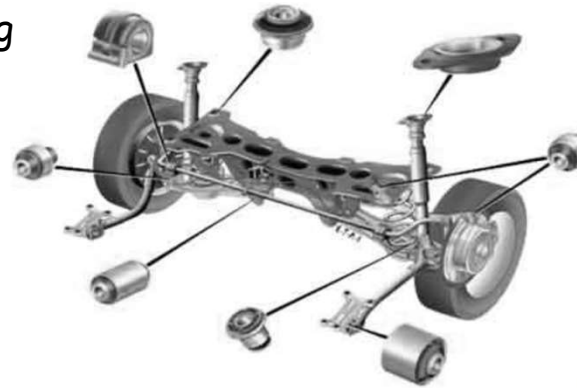
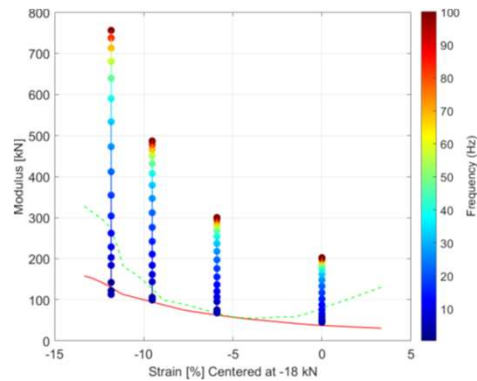
9 @ 910 Hz g1e+10, dEk 62 % dY 0 %



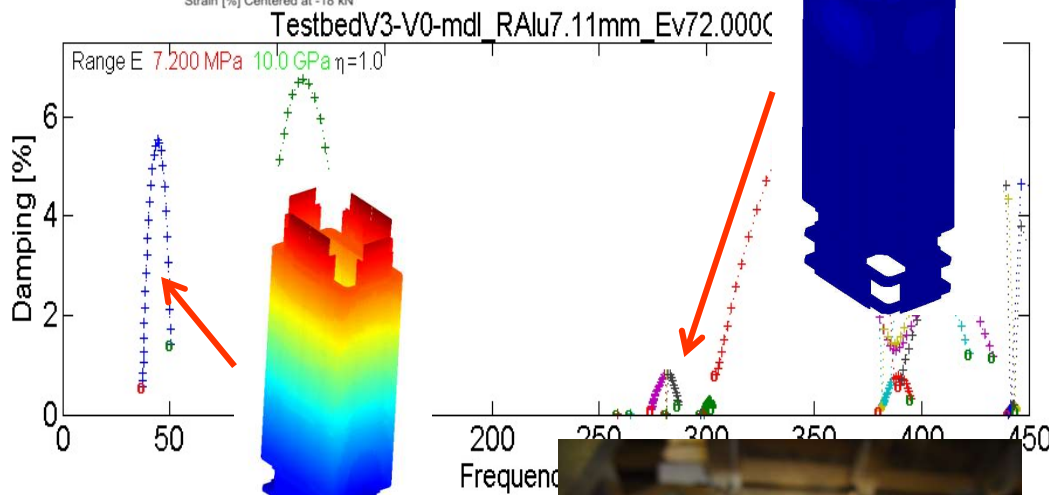
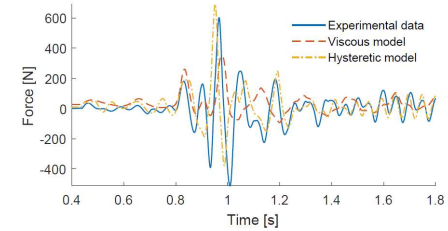
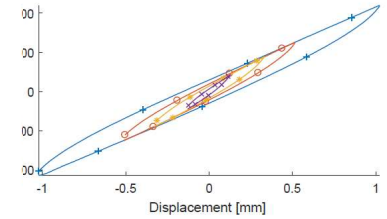
Viscoelastic damping

Scientific problems with damping

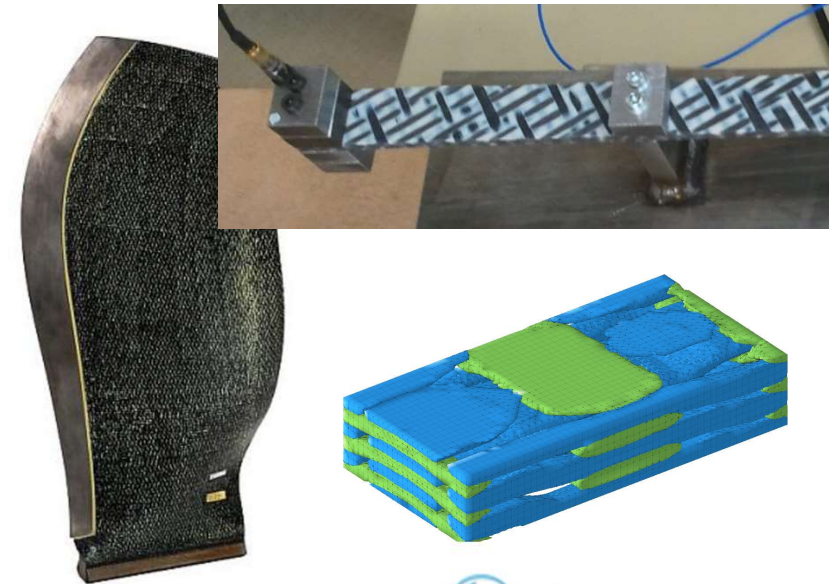
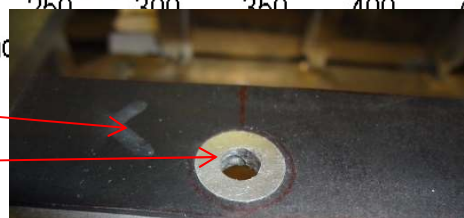
- OD (structure) vs. 3D (material + geometry) modeling
- Damping in complex structures
- Homogenization in composites



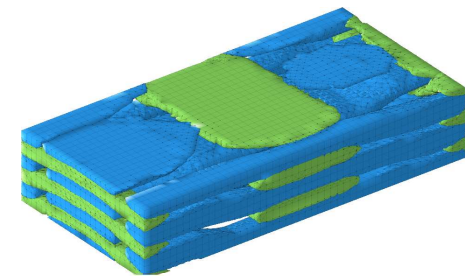
Ph.D. R. Penas 21



Visco Washer



Ph.D. F. Conejos 21



Ph.D. C. Hammami

SDT core focus

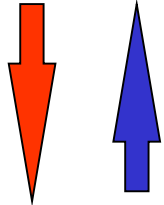
SDT (software since 1995, 700+ licenses)

SDTools (company since 2001)

4 engineers (develop + interact with clients & PhDs)

- **FEM** simulations
- System models (**model reduction**, state-space, active control)
- Experimental **modal analysis**
- Test/analysis **correlation**, model **updating**

Simulation



Validation

CAD/Meshing
FEM
Simulation
Testing

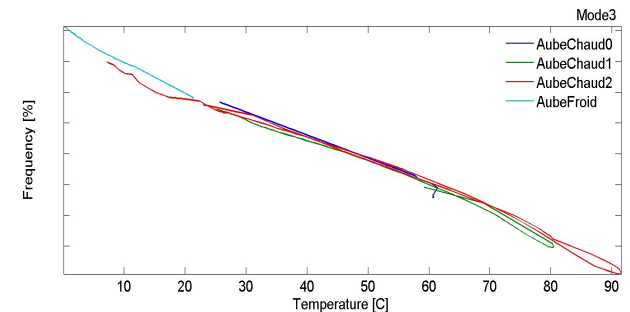
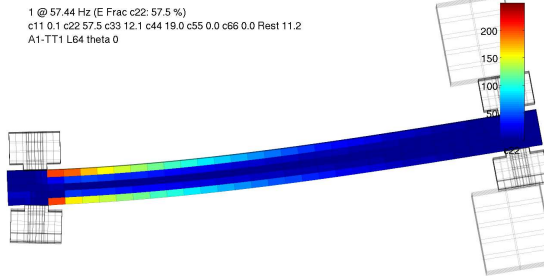
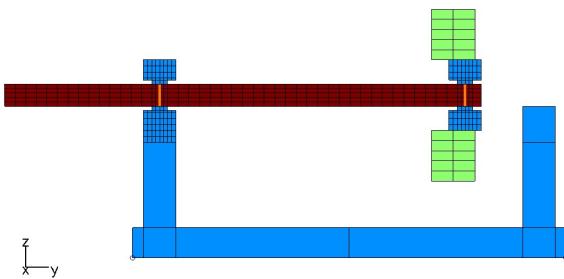
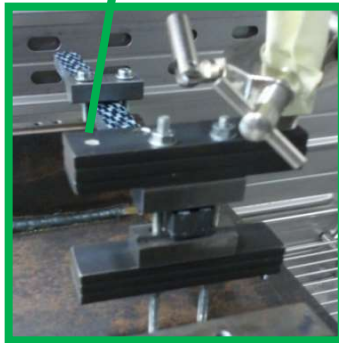
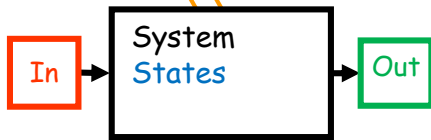
CATIA, Workbench, ...
NASTRAN, ABAQUS, ANSYS, ...
Adams, Simpack, Simulink, ...
LMS TestLab, ME-Scope, ...

- **Necessity**: programmatic access to all steps
- **Proposed solution**: flexible toolbox & custom applications
- **Base commercial library** : for quality, durability, capitalization
- **Consulting/research**

What is a system ?

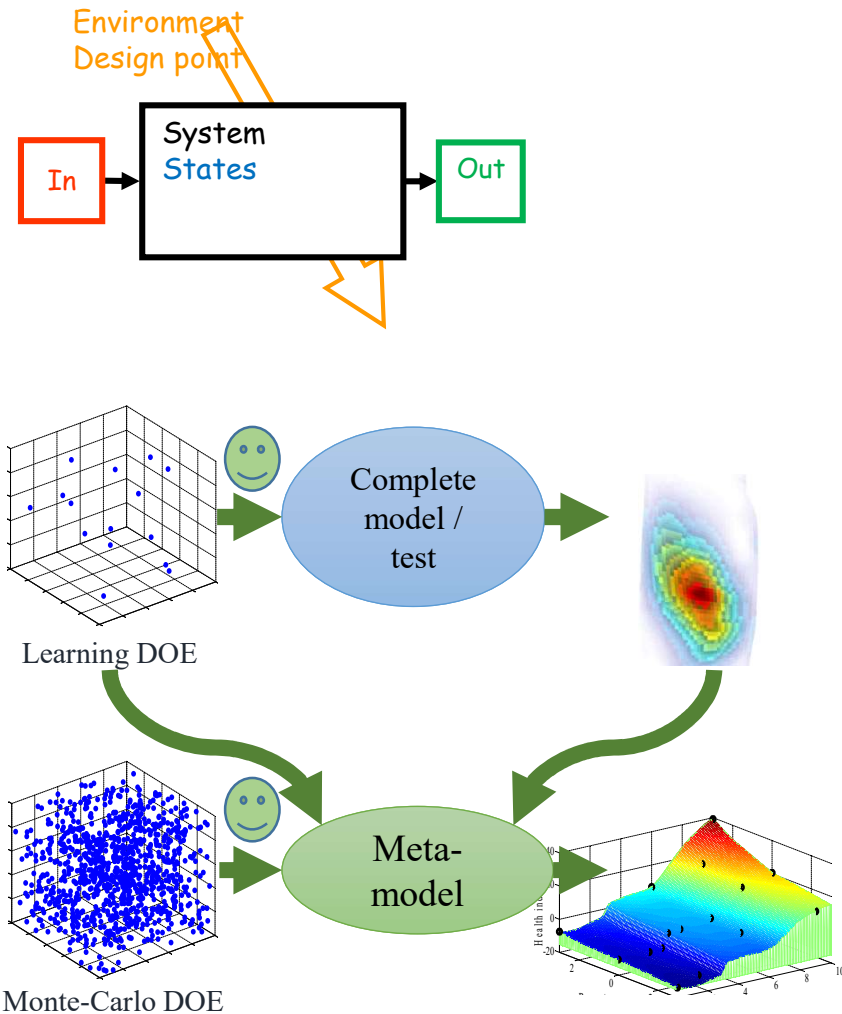
- **Inputs $u(t)$** : hammer with force measurement
- **Outputs $y(t)$**
 - Test : vibrometer on testbed
 - Computation : stresses
- **State $x(t)$**
 - Displacement & velocity field as function of time
 - $\{\dot{x}(t)\} = f(x(t), u(t), p, t)$ evolution
 - $\{y(t)\} = g(x(t), u(t), p, t)$ observation
- **Environment variables p**
 - Dimensions, test piece (design point)
 - Temperature (value of constitutive law or state of thermo-viscoelastic)
- Feature : function of output (example modal frequency)

Environment
Design point



Simple example : modified Oberst test for 3D woven composite test

System models : nature & objectives?



What is a model

- A function relating input and outputs
- For one or many parametric configurations

Model categories

- **Behavior** models (meta-models)
 - Test, constitutive laws, Neural networks
 - Difficulties : choice of parametrization, domain of validity
- **Knowledge** models
 - Physical principles, low level meta-models

Why do we need system models ?

Design

- Become predictive : understand, know limitations
- Perform sizing, optimize, deal with robustness

Certify

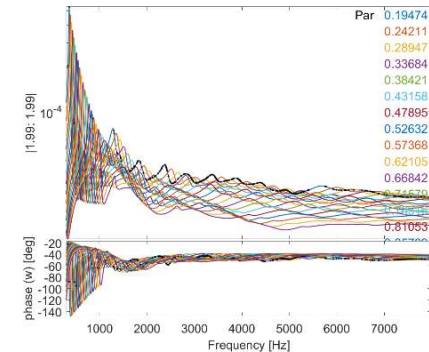
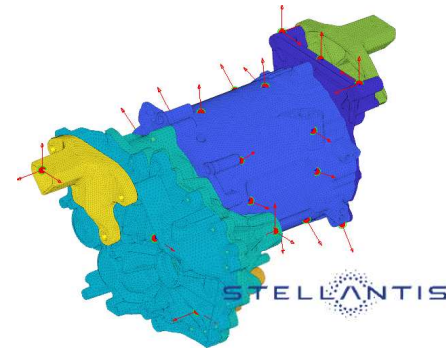
- Optimize tests : number, conditions
- Understand relation between real conditions and certification
- Account for variability

Maintain during life

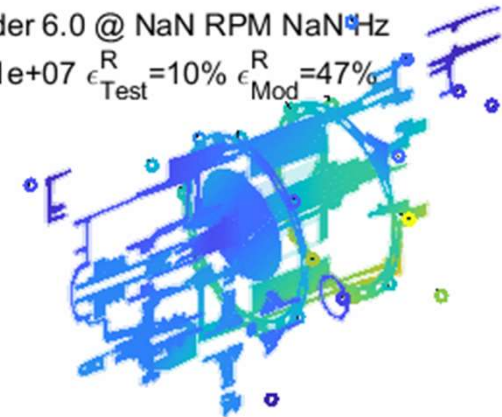
- Design full life cycle (plan maintenance)
- Use data for conditional maintenance (SHM)

A gradation of models/twins

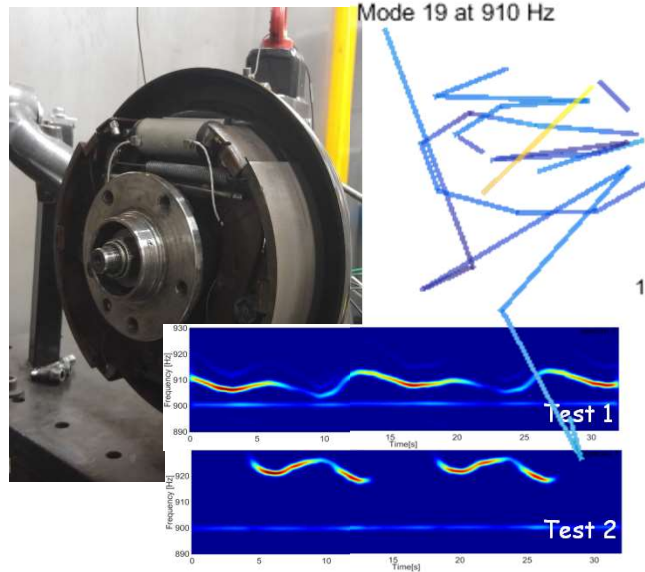
- Direct FEM simulation / virtual twin
 - Direct response, offline 🐢
- Reduced-meta model / digital twin
 - Offline generation of training data 🐢
 - Reduction (direct problem) / learning (inverse, identification, training)
 - Online 🐰 usage
- State-estimation, parameter updating / hybrid twin
 - Combine physical model & test



Order 6.0 @ NaN RPM NaN Hz
 $\gamma = 1e+07$ $\epsilon_{Test}^R = 10\%$ $\epsilon_{Mod}^R = 47\%$

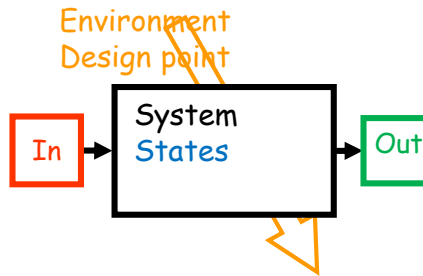
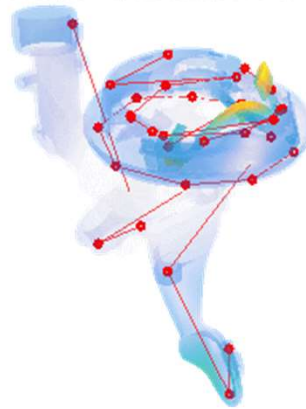


A system = I/O representation



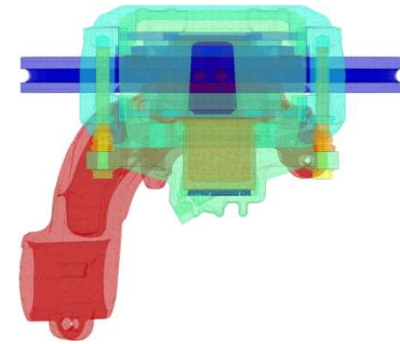
Prototype

- ☺ all physics (no risk on validity)
- ☺ in operation response
- ☹ limited test inputs
- ☹ measurements only
- ☹ few designs
- ☹ Cost : build and operate

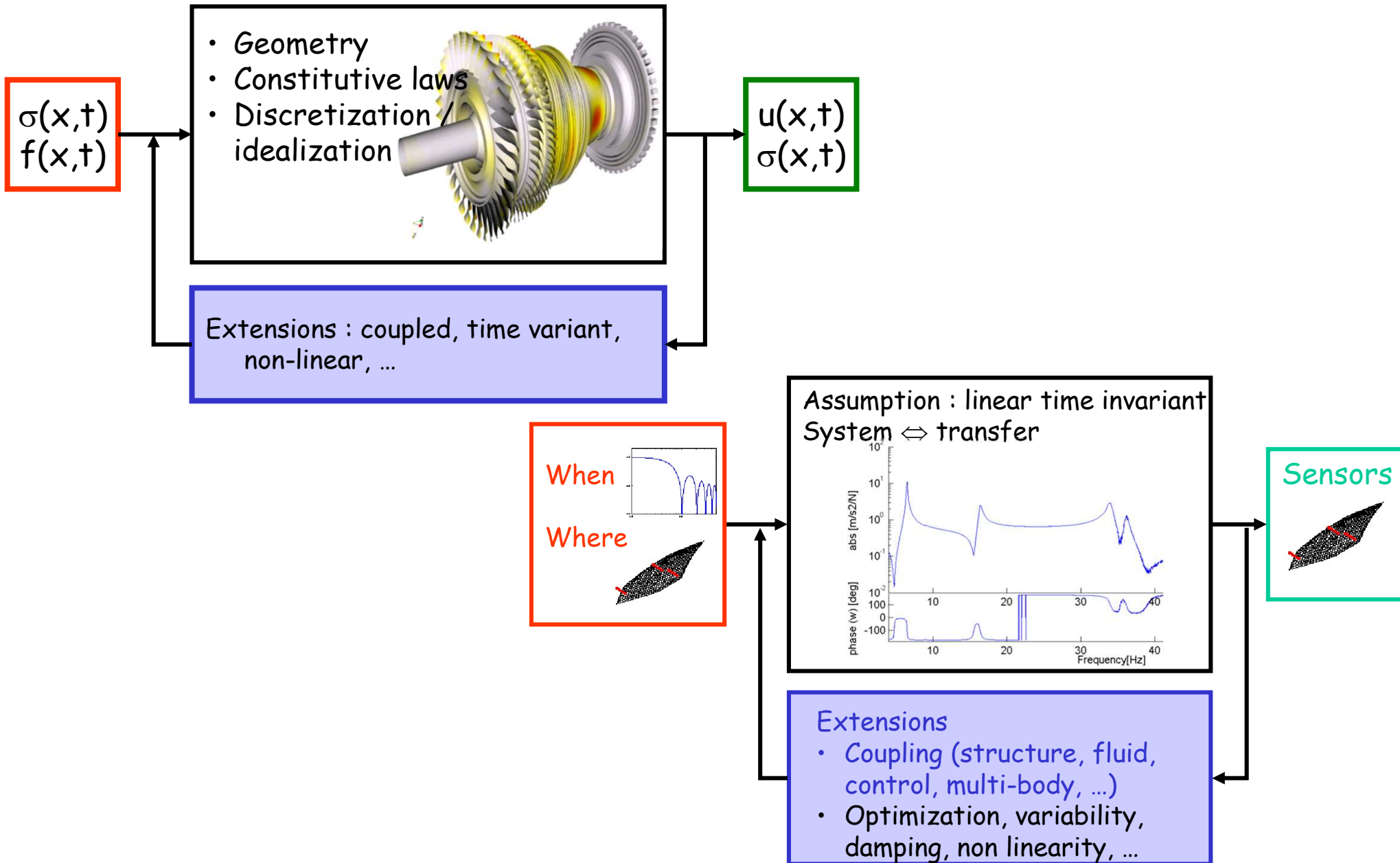


Virtual prototype

- ☹ limited physics (unknown & long CPU)
- ☹ design loads
- ☺ user chosen loads
- ☺ all states known
- ☺ multiple (but 1 hour, 1 night, several days, ... thresholds)
- ☹ Cost : setup, manipulate



FEM model / system model



Model verification & validation (ISO)

CAD Model



Experimental model

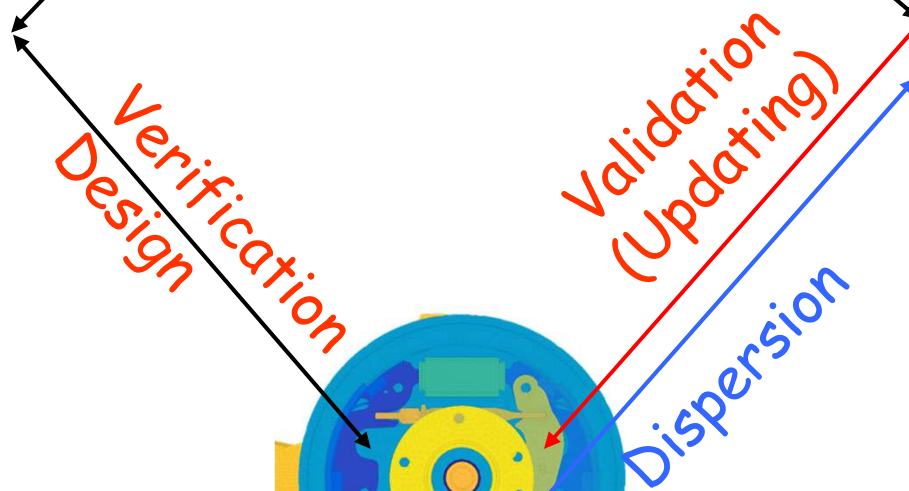
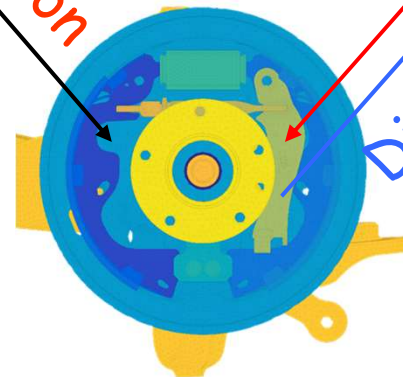
19 @ 910 Hz g1e+10, dEk 62 % dY 0 %



Continuous model
+interfaces

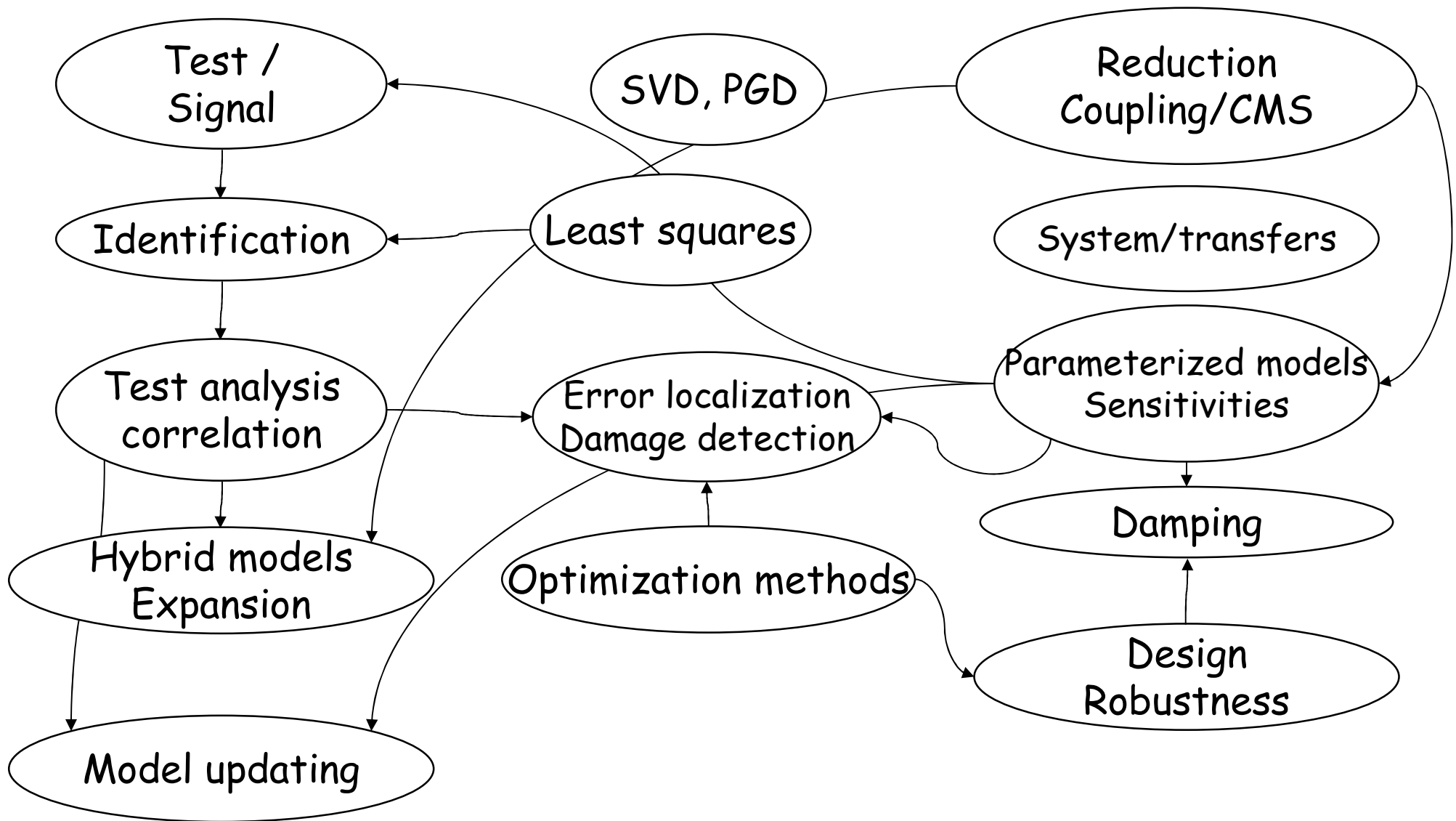
$$\begin{aligned} \text{On } \Omega & \quad \text{div} \sigma + F_v = \rho \ddot{u} \\ \text{On } \partial \Omega & \quad u(x, t) = u_{\text{given}} \\ & \quad \{T\} = [\sigma] \{n\} \end{aligned}$$

FE/numerical
model



1990 : model updating
2000 : virtual prototype
2019 : digital twin

Methods considered in the course



Lab work / evaluation

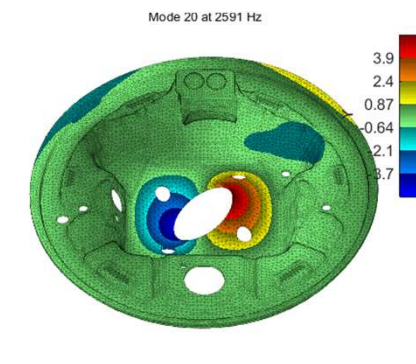
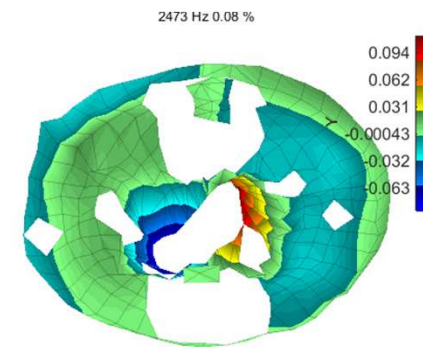
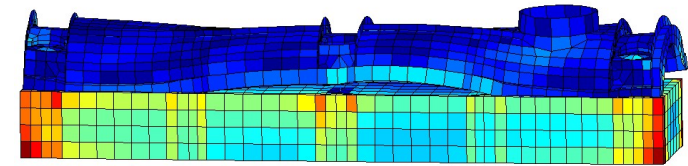
Lab work (with Mathieu Corus)

- 1 : code verification, signal, 1 DOF
- 2 : transfers, time/frequency
- 3 : identification, test/analysis correlation
- 4 : reduction and parametric models, updating

TP2-4 MATLAB+SDT : www.sdtools.com/sdtcur

Evaluation

- **oral** (PPT no interactive MATLAB), 30 mn (< 20 slides)
equal weight for 1-2-3-4 (address most questions)
- Work as pairs (not 3)
- Evaluation on how you **expose & comment** results
(5 pt per lab)



Data : <http://savoir.ensam.eu/moodle/course/view.php?id=1874>

2024 Planning

- 17/9 Course1 : Intro, 1DOF, system
C2 : modes & synthesis, base of reduction, spectral decomposition
- 24/9 C3 : signal for vibration (continuous vs. discrete, aliasing, windowing)
C4 : Ritz and learning. Historical : McNeal, Craig-Bampton, ...
- 01/10 Lab1 : 1 DOF, state-space, signal, NL
- 08/10 C5 : experimental modal analysis : from test to a system model. Inverse problem.
C6 : Model parameterization, sensitivity computations
- 15/10 C7 : Reduction for reanalysis. Validity and error control.
- C8: Test/analysis correlation. Topology correlation. Observation, MAC, expansion. Start of model updating.
- 22/10 Lab2 : modal base frequency domain, transfers, transient, signal processing
- 05/11 C9 : damping : devices, physical mechanisms, numeric tools
C10 : CMS (Component Mode Synthesis), coupling models, reduction for coupling
- 12/11 : Lab3 : identification, sub-space, test/analysis
19/11 : Lab4 : Parametric models, reduction, damping, updating
- 26/11 : C11-12 subspace methods, current issues with non-linear systems, updating
- 10/12 : Oral

To go further

Course material (notes, slides)

<https://savoir.ensam.eu/moodle/mod/folder/view.php?id=19444>

For experimental modal analysis

- [1] W. Heylen, S. Lammens, and P. Sas, Modal Analysis Theory and Testing. KUL Press, Leuven, Belgium, 1997.
- [2] D. Ewins, Modal Testing: Theory and Practice. John Wiley and Sons, Inc., New York, NY, 1984.
- [3] <https://www.uml.edu/Research/SDASL/Education/Modal-Space.aspx>
- [4] K. G. McConnell, Vibration Testing. Theory and Practice. Wiley Interscience, New-York, 1995

Numerical aspects

- [5] D. Inman, Engineering Vibration. Prentice-Hall, Englewood Clis, N.J., 1994.
- [6] M. Geradin and D. Rixen, Mechanical Vibrations. Theory and Application to Structural Dynamics. John Wiley & Wiley and Sons, 1994, also in French, Masson, Paris, 1993.
- [7] R. J. Craig and A. Kurdila, Fundamentals of Structural Dynamics. Wiley, 2006.
- [8] <https://www.code-aster.org> : (English & French)

Keywords : 1 DOF / signal

After intro course: ([chapter1](#) and [section 2.1](#), [modal.pdf](#))

1. Transfer (time, Fourier $i\omega$ /Laplace s , asymptotic prop, NL)
2. Poles, resonance, damping ratio -3 dB method
3. State-space, poles
4. 1 DOF time exponential, convolution, logarithmic decrement
5. Strategies for transient in time & frequency
6. Equivalent power

Measurement and signal processing (CM3, [signal.pdf](#))

1. DFT f_k , relation δt , T , δf , linearity, dilatation, ([section 3.1](#))
2. Aliasing : Shannon's theorem, when, mitigate, ... ([s3.2.1](#))
3. Leakage & windowing : continuous vs. DFT ([s3.2.2](#))
4. Transfer function estimate (H1, coherence) ([s3.3](#))
5. Technology : sensors, actuators, acquisition

TP1 : code verification for 1DOF, integration, signal

Keywords : modes & synthesis, reduction

Modes & synthesis (CM2, see also second part of [Modal.pdf](#) slides)

1. Inputs/outputs, IO shape matrix, disp, resultants, ... ([s2.1](#))
2. Discrete modes (harmonic solution without input), orthogonality ([section 2.2.1](#))
3. Ritz/Galerkin principles
4. Ritz/modal coordinates, PPV, series & state-space, time/freq strategies
5. Modal & **Rayleigh damping**, modal damping in physical coord ([s 2.2.x](#))
6. Peak visibility, truncation, effective contributions ([s 2.2.x](#))

Reduction (course 4, see also [Reduction.pdf](#) slides)

1. Ritz/Galerkin & learning
2. Modes + Residual flexibility ([section 4.3](#))
3. McNeal= Ritz with "residual vectors", pre-filter low frequency modes
4. Residual vectors in presence of flexible modes
5. Guyan, Craig-Bampton = enforced displacement & bandwidth ([s 4.3.2](#))

Left for other course : from vector set to basis

Keywords : parametric models

Sensitivity / extended uses of modes (co. 5, [SensitivityReanalysis.pdf](#))

1. Parametrization (s 9.1)
2. Sensitivity of static response, adjunct state (s 9.2)
3. Sensitivity of frequencies : relation with energy distribution (s 9.3)
4. Sens. of mode-shapes : modal crossing + numerical strategy for inverse of underdetermined problem

Parametric studies (course 6)

1. Reanalysis example in modal basis (start by continuous case of spring on tensioned wire). Generalization to ΔK .
2. Multi-model and nominal + residual methods
3. Illustrations (damping/updating)
4. Error control, iterative basis refinement
5. Orthogonalization strategies, GS/GSM/IGSM, Mseq

TP2 : modes & synthesis, signal

Keywords : experimental modal analysis

Identification (course 7, [EMA.pdf](#))

1. Identification demo
2. Inverse problems : model forms, data ([s 6.1](#))
3. Model forms for identification, discussion of residual terms
4. Frequency domain least-squares solution, implicit NL
5. Evaluation of results ([ch 7](#))

Test/analysis correlation (course 8, [Correlation.pdf](#))

1. Topology correlation ([s 8.1](#))
2. Measuring distance between test & analysis. Shape correlation : MAC, pairing issues ([s 8.2](#))
3. Static condensation/expansion, reduced mass, orthogonality on sensors ([s 8.3](#))
4. Hybrid models

Current trends 1

Damping (course 9, [damping.pdf](#))

1. Sample damping devices
2. Notion of coupling & impact on damping
3. Viscoelasticity/complex modulus, MSE
4. Real modes/modal damping & separation
complex modes & enhanced reduction
5. Internal states (// with friction)

Substructuring (Component mode synthesis) (course 10)

1. Coupling conditions : energy or continuity
2. constraints : elimination, Lagrange, Penalization
3. contact & locking/stress concentration
4. Reduction for CMS : classical Craig-Bampton, CMT
5. The "problem" of large interfaces

TP3 : parametric models, damping, identification, correlation

Current trends 2

Updating (course 11, [updating.pdf](#))

1. Typical errors : property, geometry, contact, model
2. Physical and equivalent models
3. Least squares and conditioning, SVD (link with TP3)
4. Error localization
5. Sample applications

Features in vibration behavior (course 12)

1. SVD for mechanics : principal loads, modal energy coordinates, interface DOFs
2. Modal coordinates, physical and macro-models models of junctions with contact/friction
3. Geometrically periodic systems (engine, track)

2023 Planning

- 12/9 Course1 : Intro, 1DOF, system
C2 : modes & synthesis, base of reduction, spectral decomposition
- 19/9 C3 : signal for vibration (continuous vs. discrete, aliasing, windowing)
C4 : Ritz and learning. Historical : McNeal, Craig-Bampton, ...
- 26/9 Lab1 : 1 DOF, state-space, signal, NL
- 03/10 C5 : experimental modal analysis : from test to a system model. Inverse problem.
C6 : Model parameterization, sensitivity computations
- 10/10 C7 : Reduction for reanalysis. Validity and error control.
- C8: Test/analysis correlation. Topology correlation. Observation, MAC, expansion. Start of model updating.
- 17/10 Lab2 : modal base frequency domain, transfers, transient, signal processing
- 24/10 C9 : damping : devices, physical mechanisms, numeric tools
C10 : CMS (Component Mode Synthesis), coupling models, reduction for coupling
- 07/11 : Lab3 : identification, sub-space, test/analysis
14/11 : Lab4 : Parametric models, reduction, damping, updating
- 21/11 : C11-12 subspace methods, current issues with non-linear systems, updating