# Vibrations des structures

## FIP Energies marines


Etienne Balmes,
ENSAM/PIMM, SDTools


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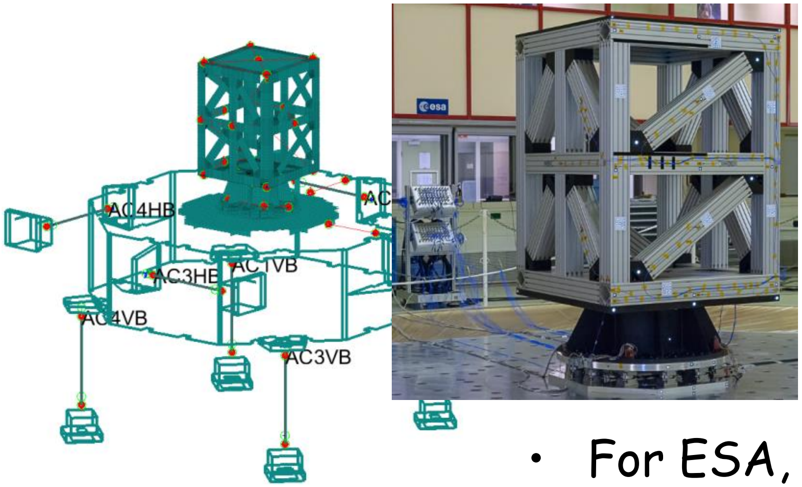
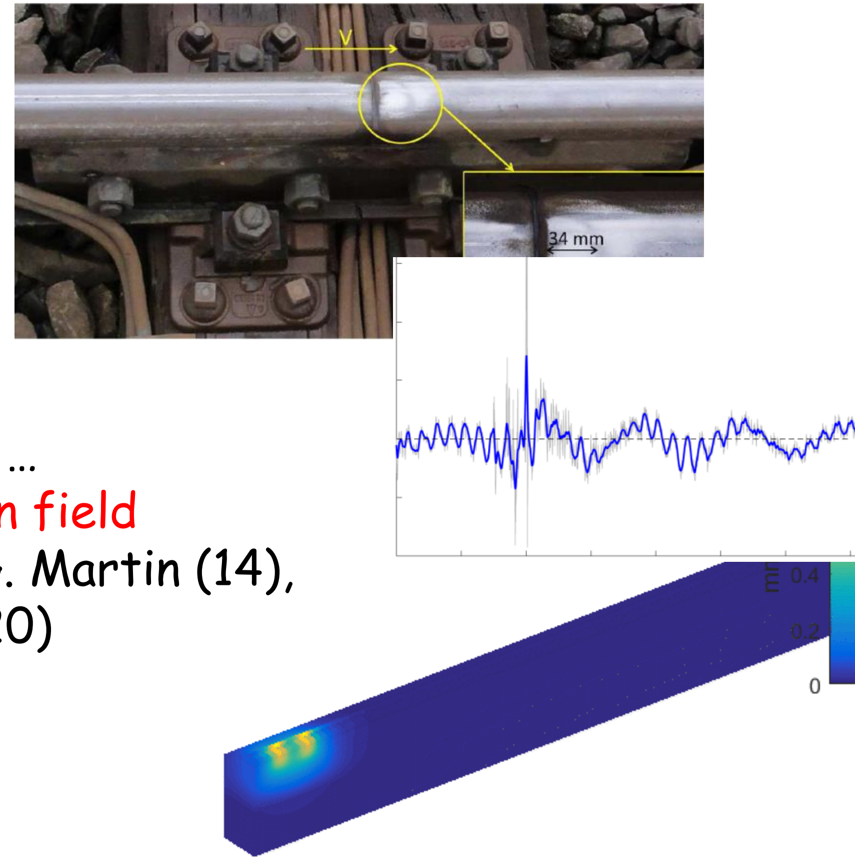
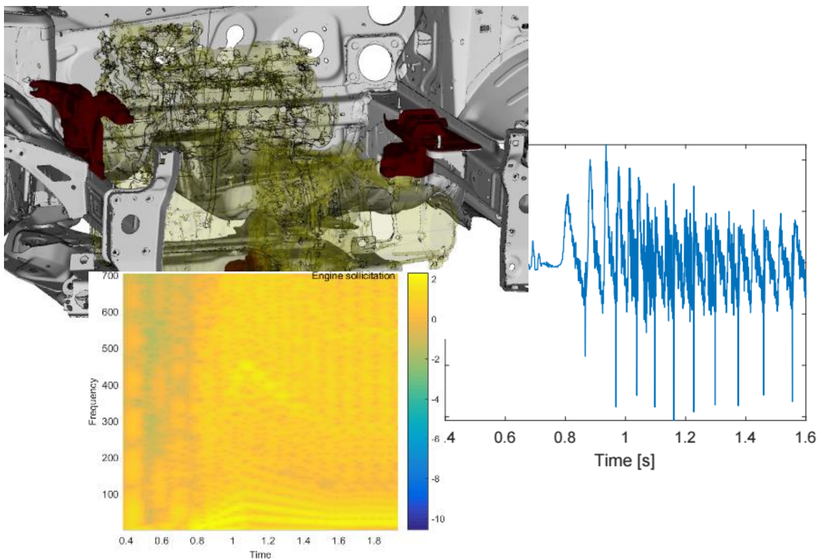
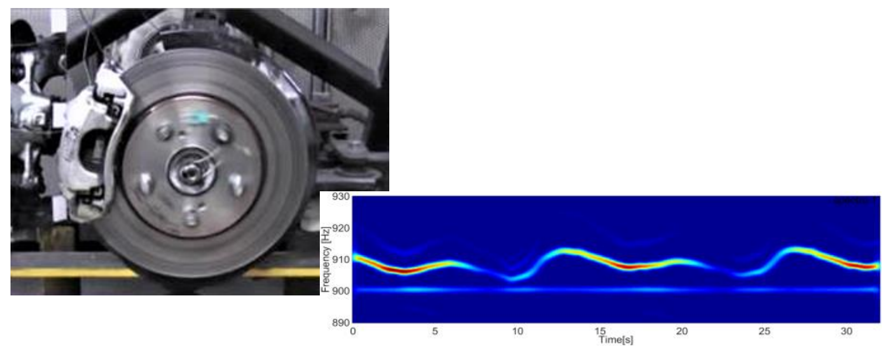
# 2021 Projects



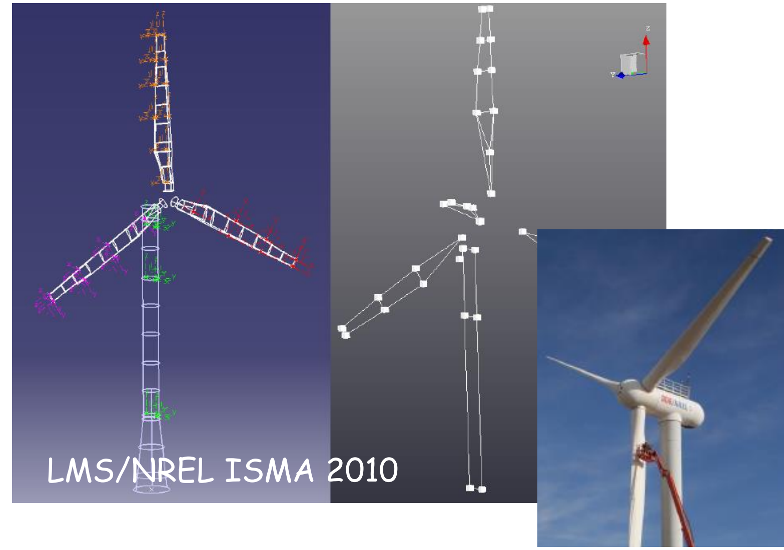

- For ESA, SNCF, PSA, CBI …
  a multi-industry application field
- With ENSAM students : G. Martin (14), R. Penas (17), A. Lemate (20)



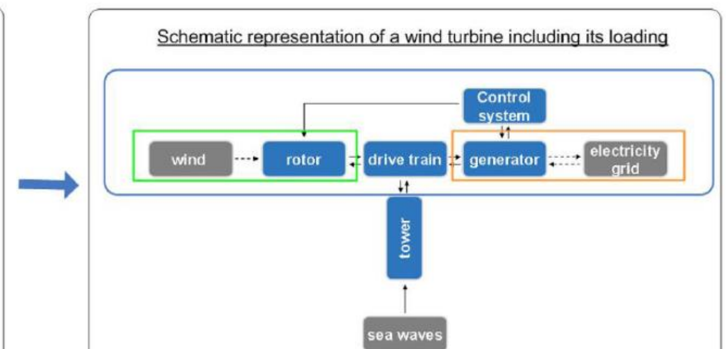



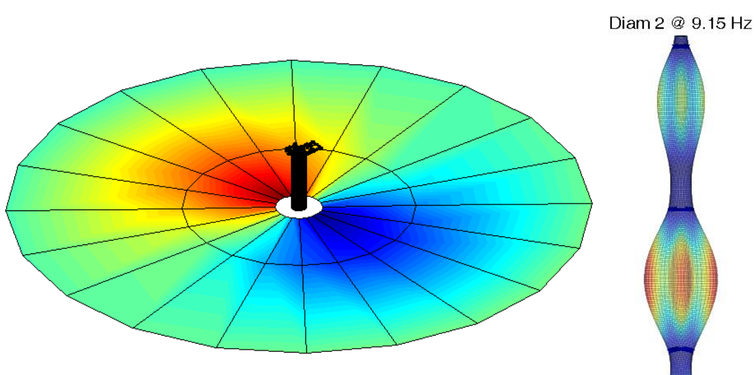
# Sample vibration problems : wind turbines

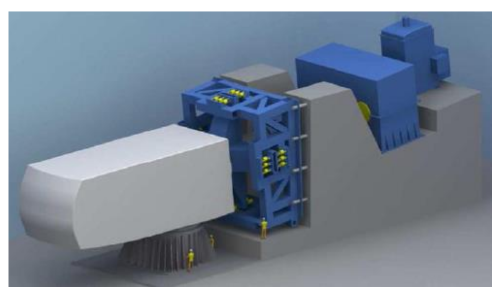
- Dynamic loads & control strategies
  - Blade orientation, energy production.
  - Testing and multi-body modeling.
- Drive train dynamics
  - limit wear, deal with emergency conditions, …
- Environment constraints
  - Noise : large radiating surfaces
  - Wave loads on base



LMS/NREL ISMA 2010

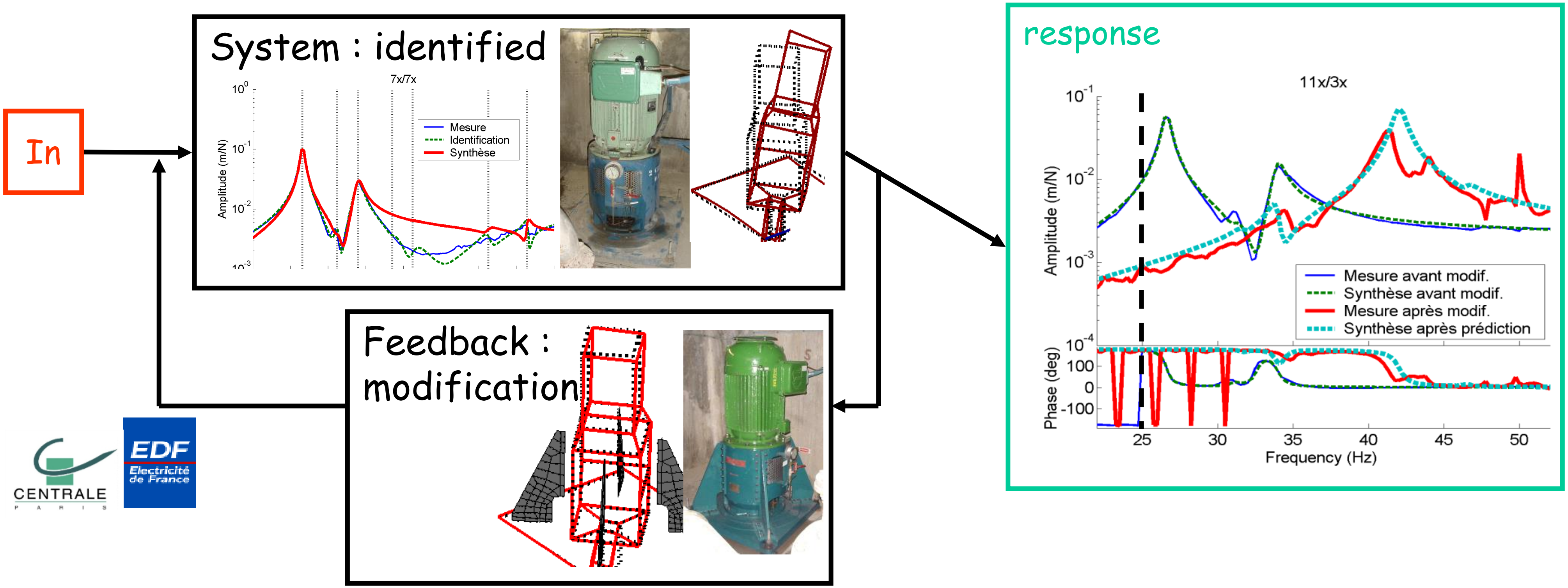



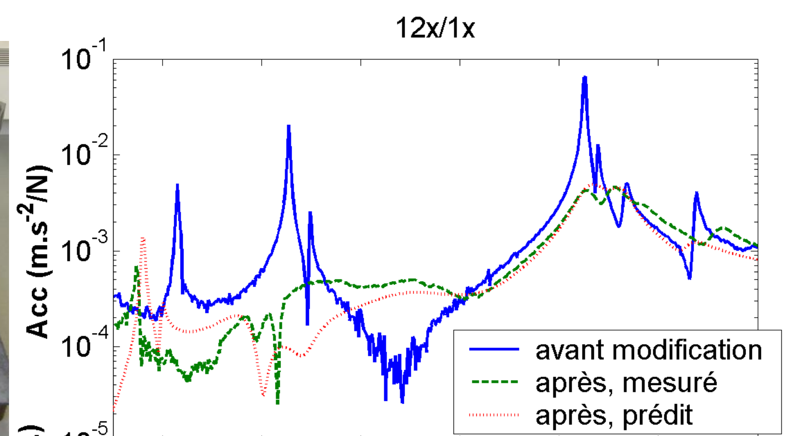
Hansen transmission/KUL ISMA 2010

# Example : structural dynamics modification

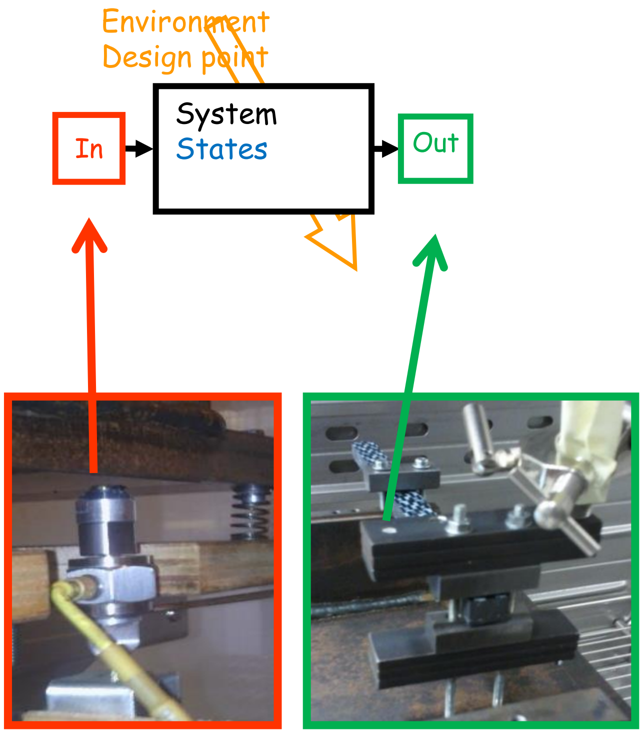



Coincidence problem

Modification : mass, stiffness or damping modifications



# What is a system ?



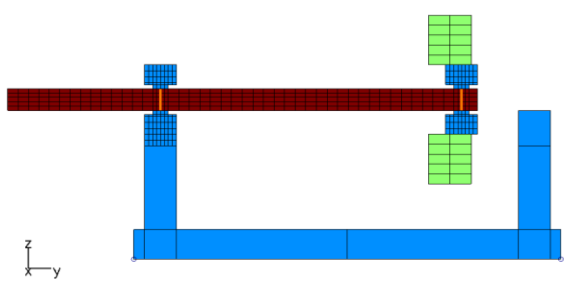

- **Inputs $u(t)$** : hammer with force measurement
- **Outputs $y(t)$**
  - Test : vibrometer on testbed
  - Computation : stresses
- **State $x(t)$**
  - Displacement & velocity field as function of time

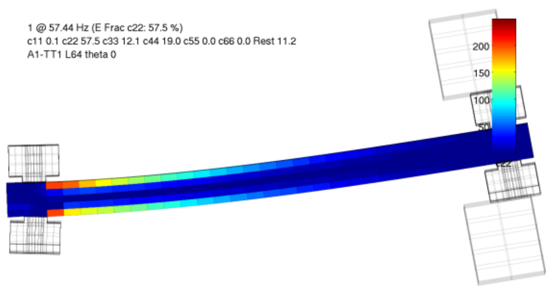
$$\{\dot{x}(t)\} = f(x(t), u(t), p, t) \qquad \text{evolution}$$

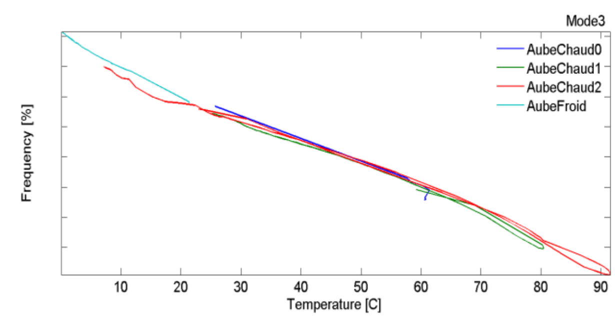
$$\{y(t)\} = g(x(t), u(t), p, t) \qquad \text{observation}$$

- **Environment variables $p$**
  - Dimensions, test piece (design point)
  - Temperature (value of constitutive law or state of thermo-viscoelastic)

- Feature : function of output (example modal frequency)

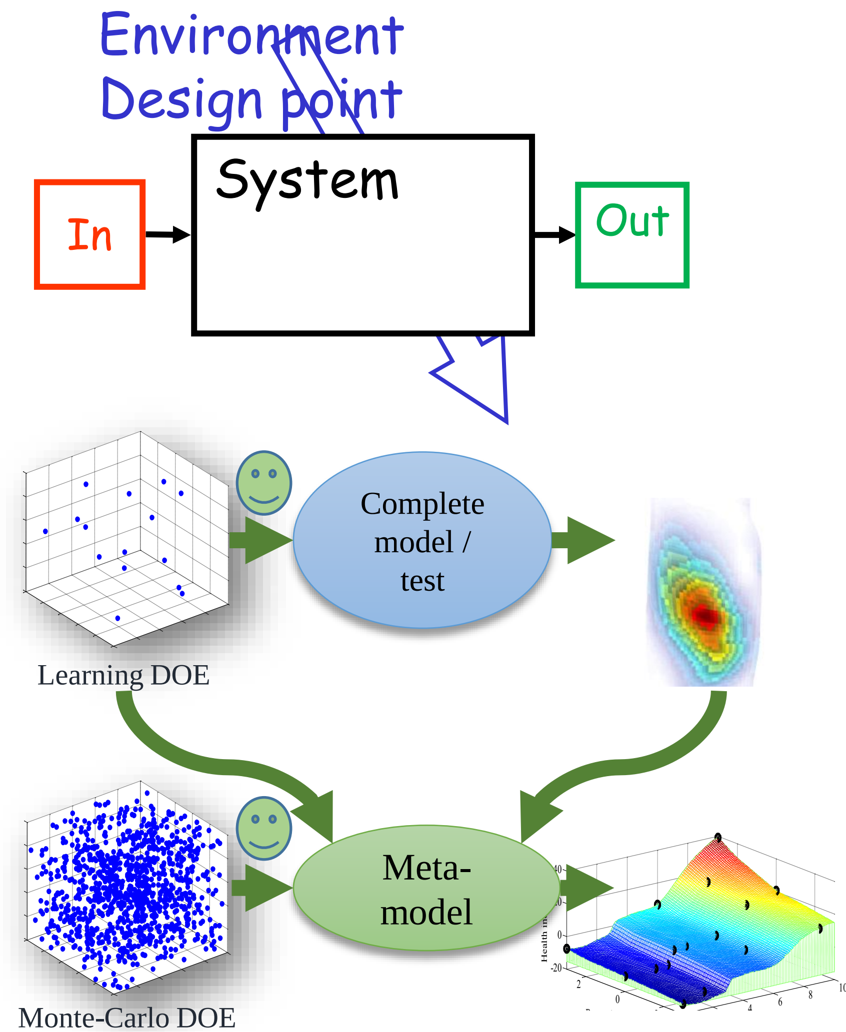



Simple example : modified Oberst test for 3D weaved composite test

# System models : nature & objectives?




## What is a model

- A function relating input and outputs
- For one or many parametric configurations

## Model categories

- Behavior models (meta-models)
  - Test, constitutive laws, Neural networks
  - Difficulties : choice of parametrization, domain of validity
- Knowledge models
  - Physical principles, low level meta-models

## Why do we need system models ?

**Design**

- Become predictive : understand, know limitations
- Perform sizing, optimize, deal with robustness

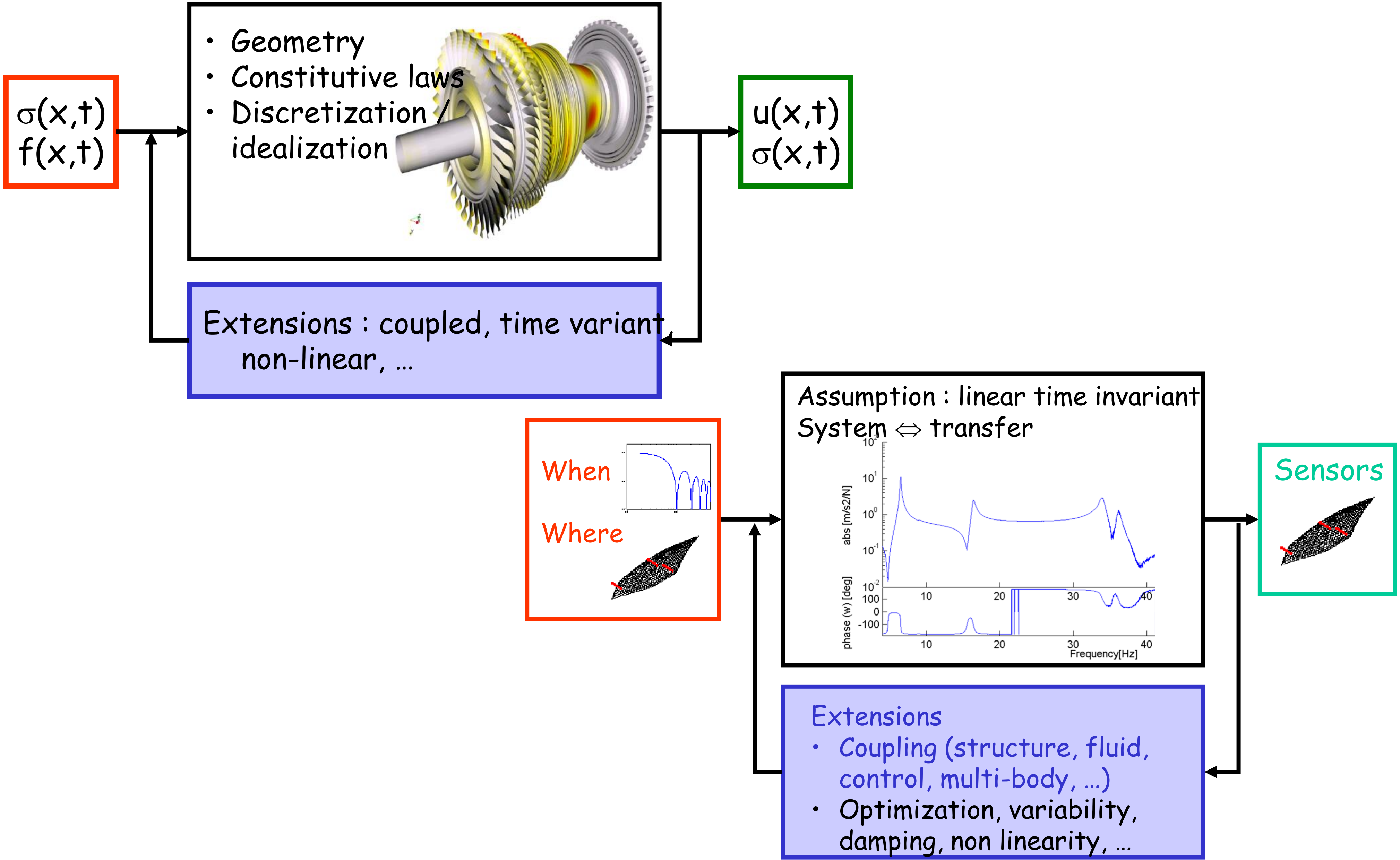
**Certify**

- Optimize tests : number, conditions
- Understand relation between real conditions and certification
- Account for variability

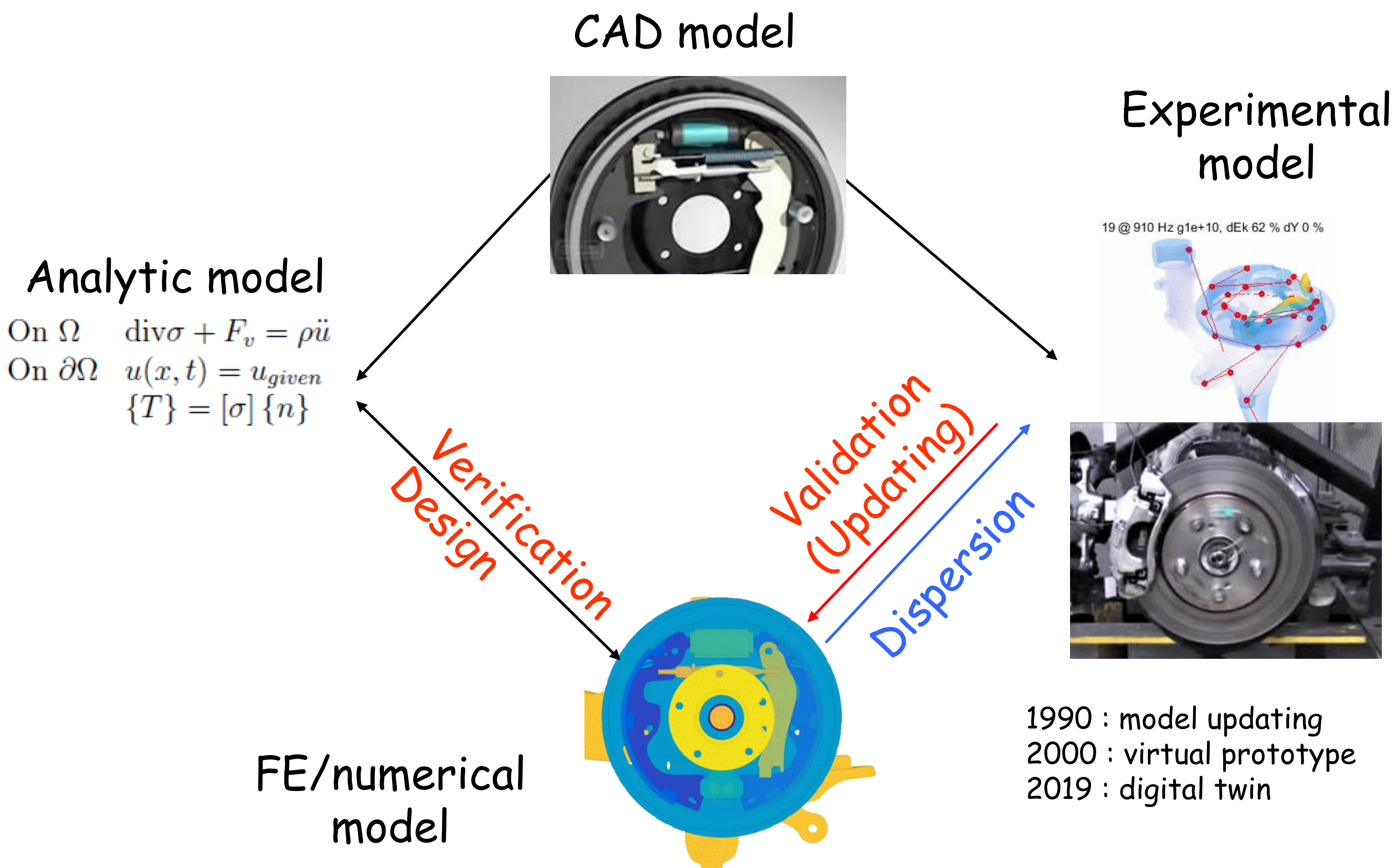
**Maintain during life**

- Design full life cycle (plan  maintenance)
- Use data for conditional maintenance (SHM)

# FEM model / system model



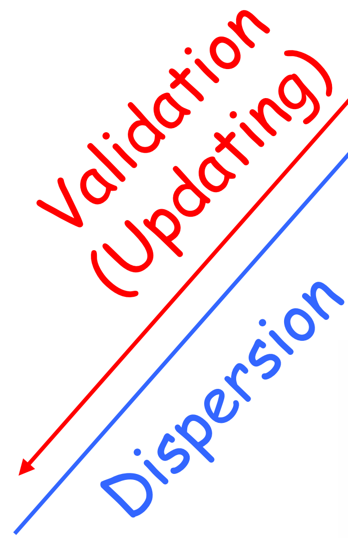
# Model validation and verification

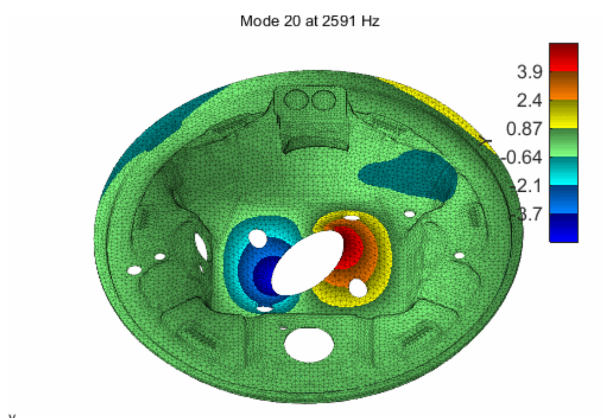


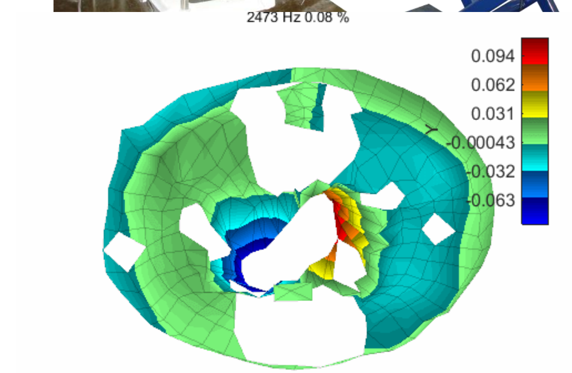
# Objectives of lab work

FEM model
1. Mesh and properties
2. Solve for modes
3. Predict modes at sensors
4. Predict transfers
5. Predict frequency shifts

Experimental model
1. Build prototype
2. Measure vibrations
3. Extract comparable information
   - Transfers (non-parametric ID)
   - Modes at sensors (parametric ID)



# Course outline

- Introduction
- How are modes measured
  - Mode ≈ resonance ≈ 1 DOF (degree of freedom) system
  - Transfer (series of modal contributions)
- How are modes predicted
  - Modes, inputs, outputs, damping
- Test / analysis correlation
  - Identification
  - Topology correlation
  - MAC / Updating
- Vibration design / conclusion

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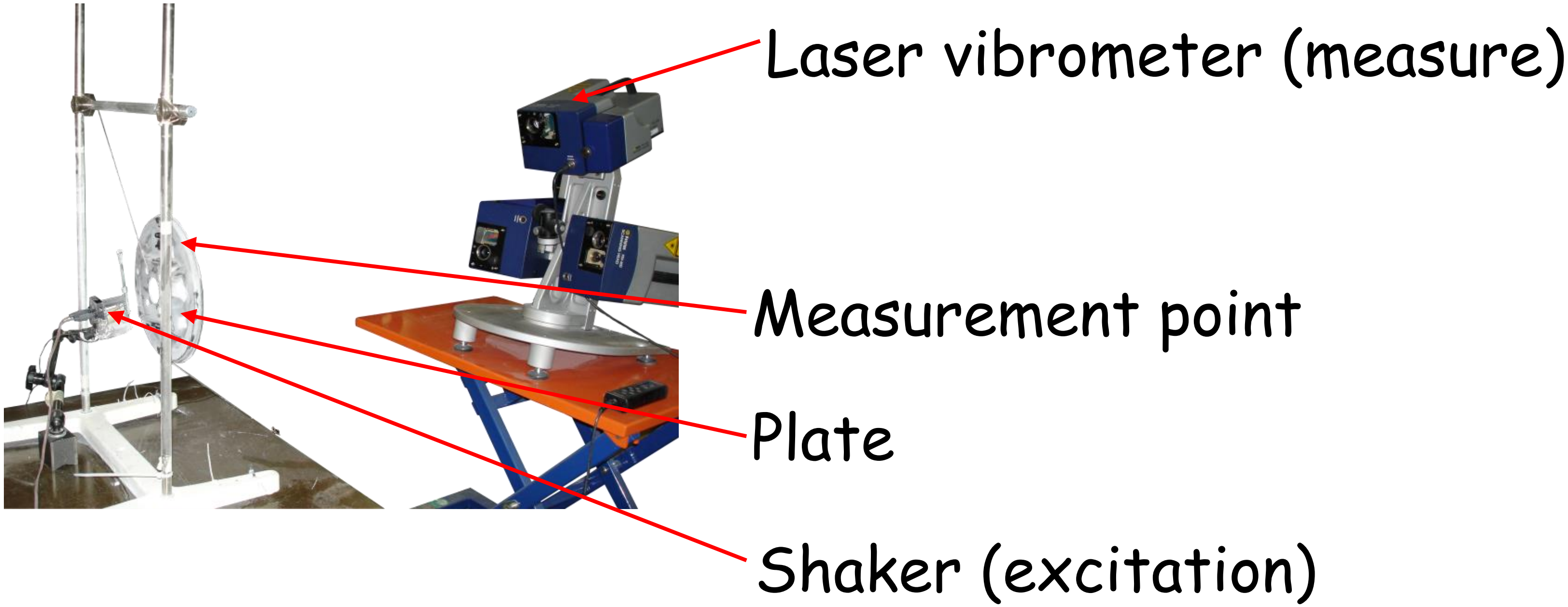
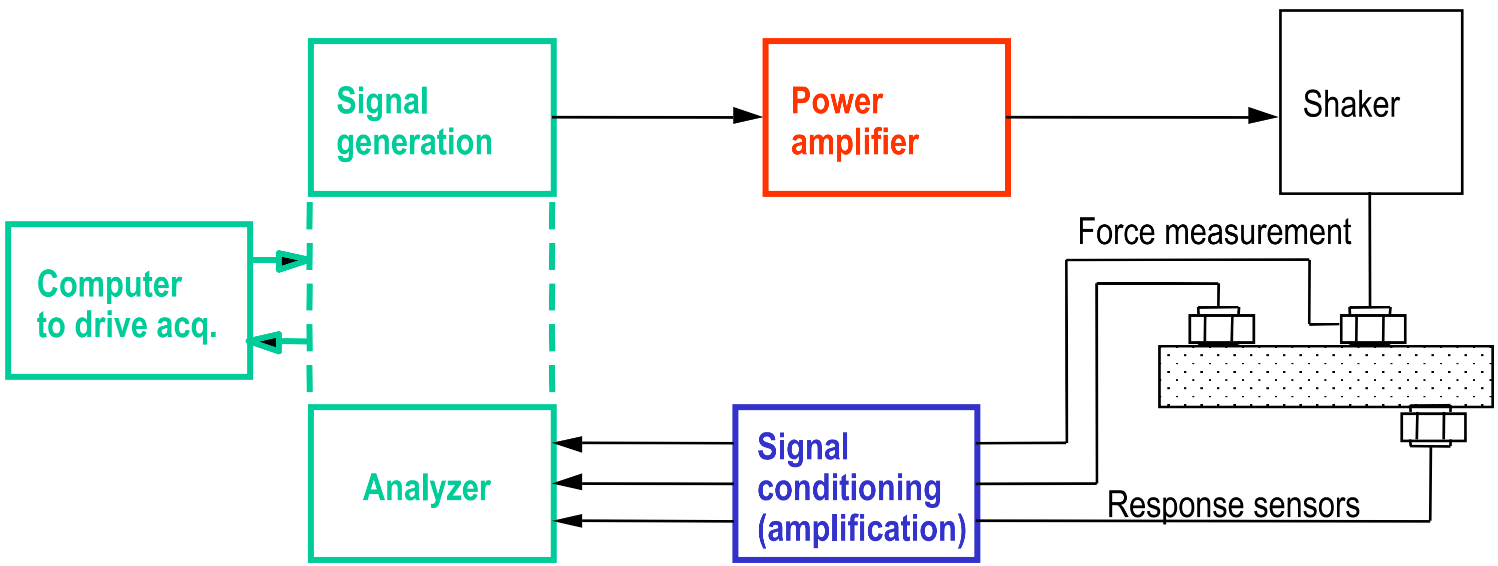
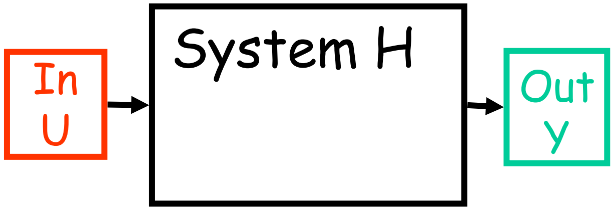
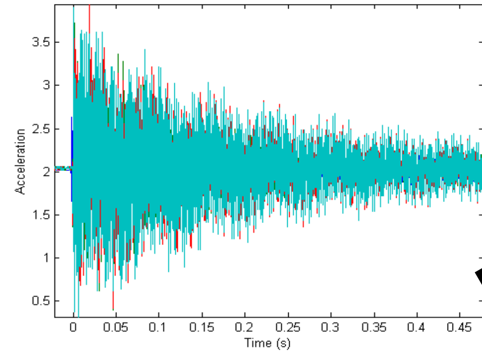
# Experimental modal analysis : measurements



# Modal analysis : transfers



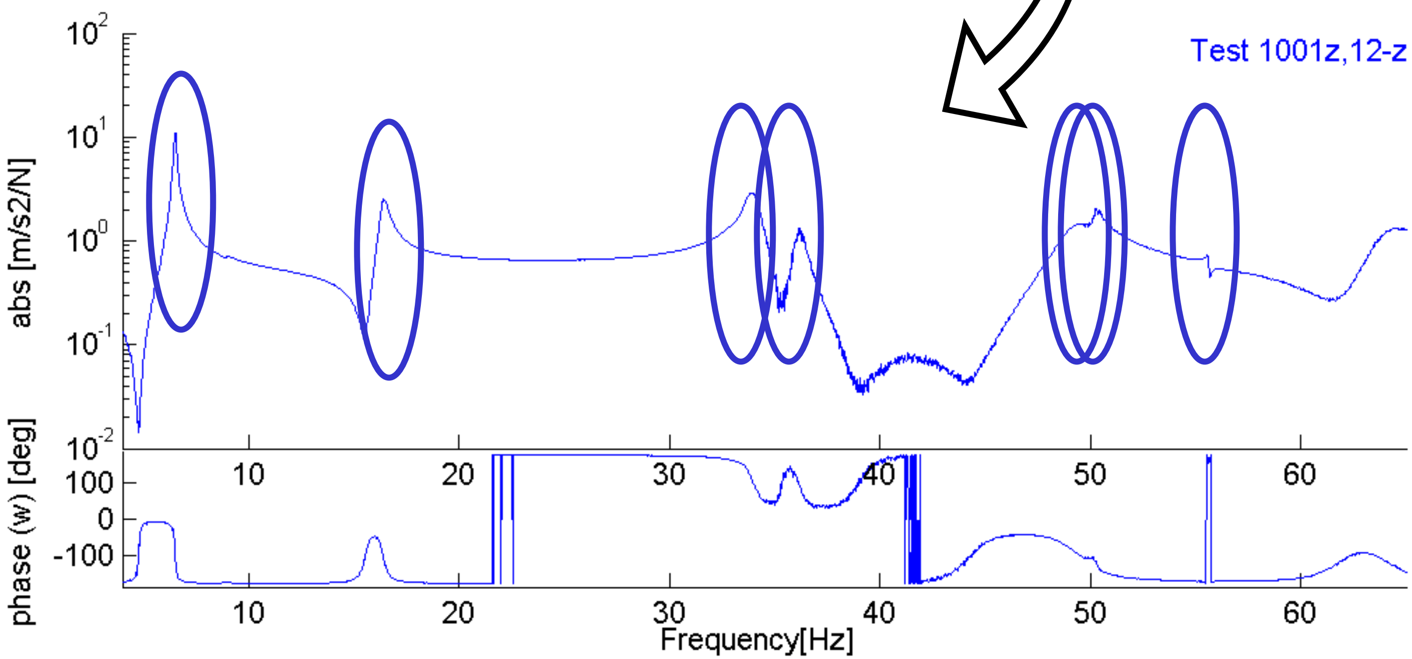




Transfers estimated from time response

ONE input

ONE output

$$\{Y(\omega)\}=[H(\omega)]\{U(\omega)\}$$



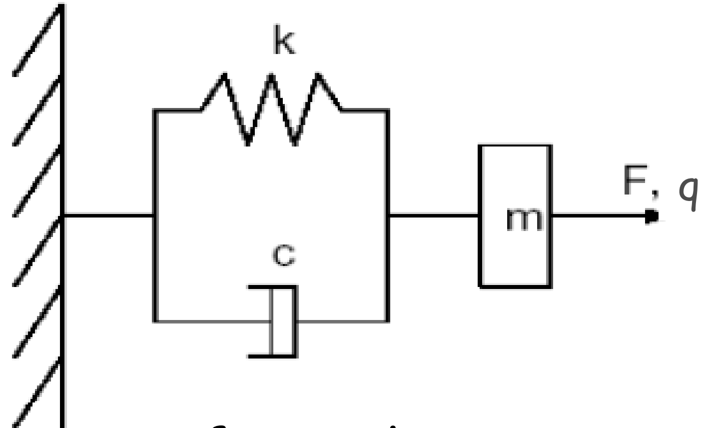

MANY resonances

Bode plot : visualization of transfer function

# Resonance (1 DOF oscillator)

Dynamic equation :

$$m\ddot{q}(t) + c\dot{q}(t) + kq(t) = F(t)$$




Harmonic excitation

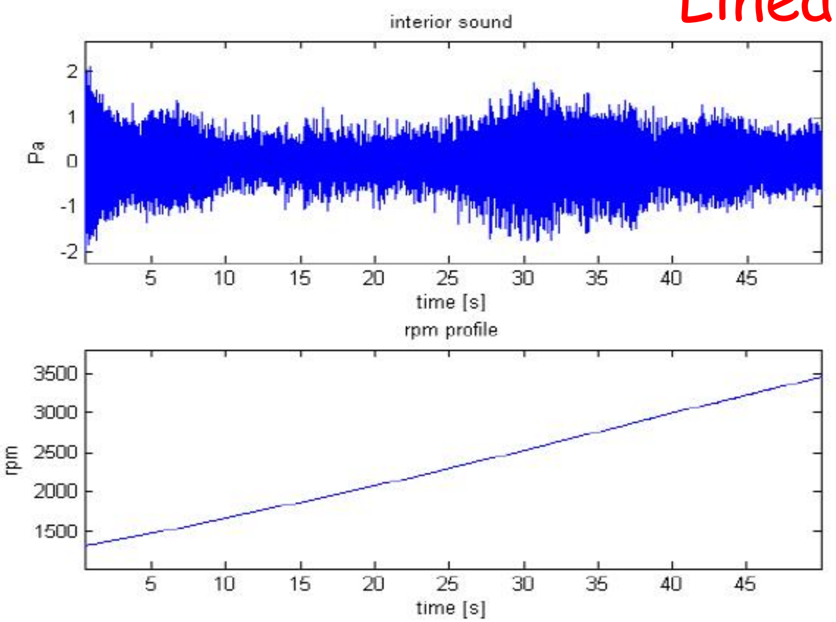
$$F(t) = \mathrm{Re}\left(F(\omega)e^{\imath\omega t}\right)$$

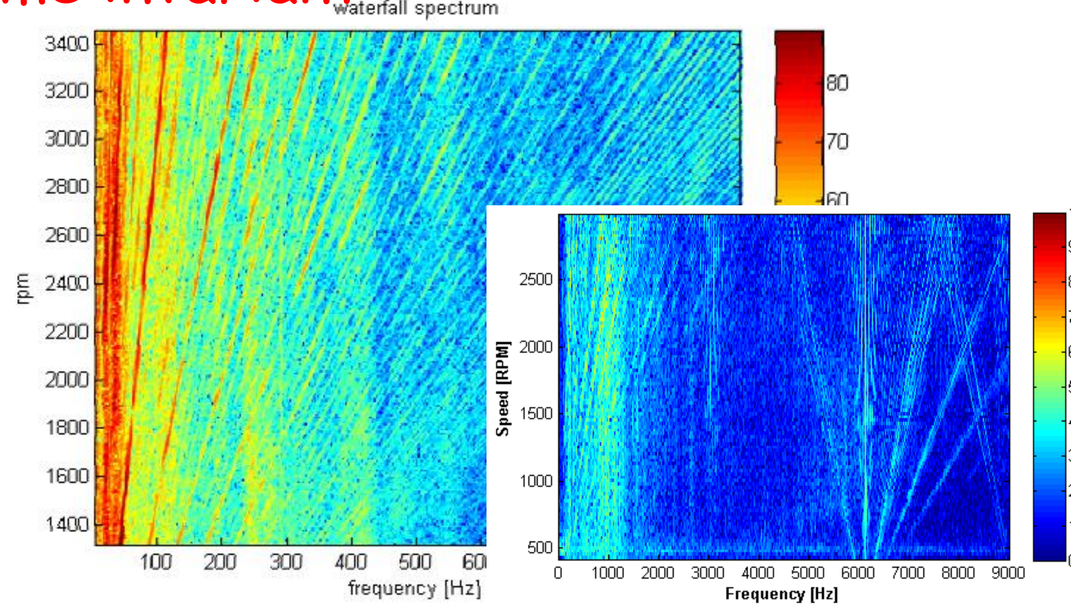
Hyp ?

Harmonic forced response

$$q(t) = \mathrm{Re}\left(q\left(\omega\right)e^{i\omega t}\right)$$

Linear time invariant



# 1 DOF frequency domain / transfer

Dynamic equation

$$\mathrm{Re}\left(\left(-\omega^2 m + i\omega c + k\right) q\left(\omega\right) e^{i\omega t} - F\left(\omega\right) e^{i\omega t}\right) = 0$$

Transfer function

$$H\left(\omega\right) = \frac{q\left(\omega\right)}{F\left(\omega\right)} = \frac{1}{-\omega^2 m + i\omega c + k}$$

Fourier / Laplace transform

$$\mathcal{F}(y) = Y(\omega) = \int_{-\infty}^{+\infty} y(t) e^{-i\omega t} dt$$

$$\mathcal{F}(\dot{y}) = i\omega \mathcal{F}(y)$$

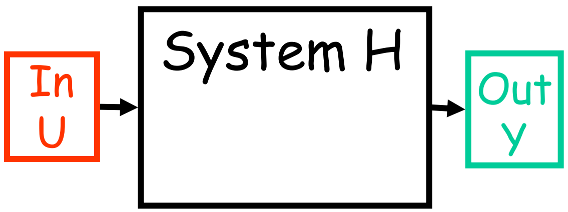
$$Y(s) = \int_{0}^{+\infty} y(t) e^{-st} dt$$

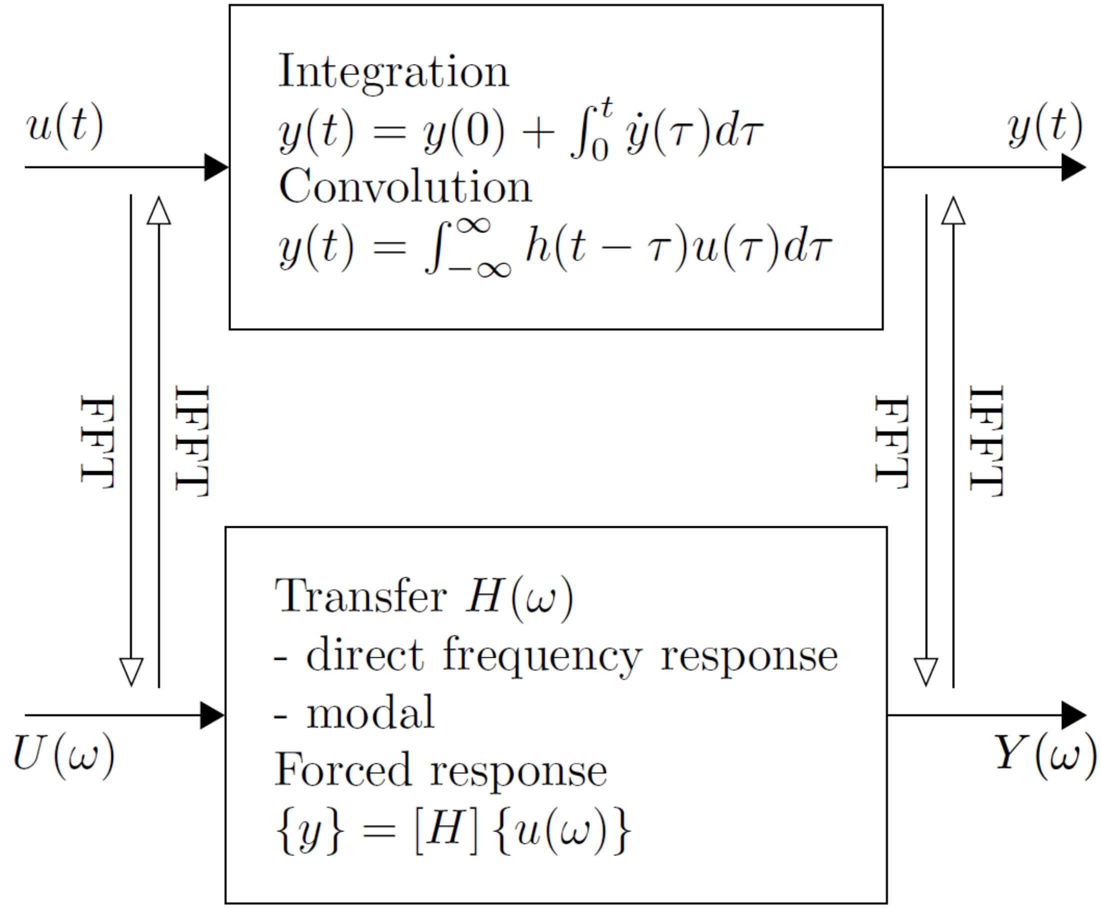
$$H(s) = \frac{q}{F} = \frac{1}{ms^2 + cs + k}$$

$s = i\omega$

Laplace/Fourier

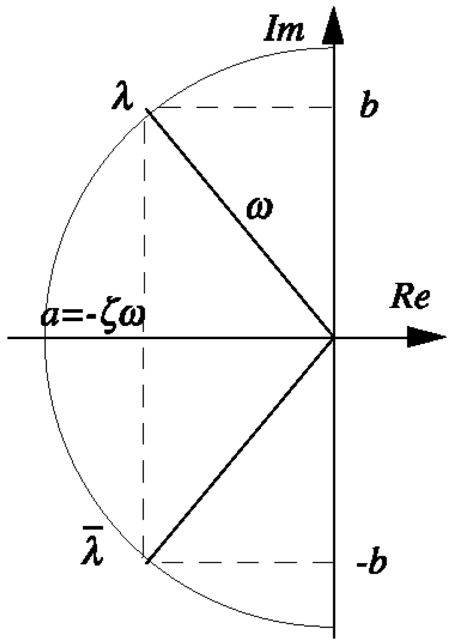
# Frequency / time responses of systems




⚠ Transfer = assume linear time invariant

# 1 DOF (Bode plot)



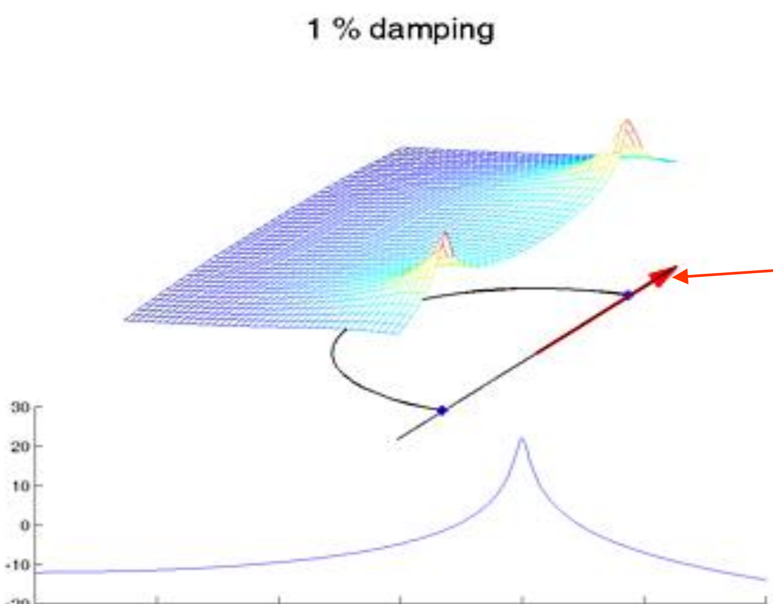

$$H(s) = \frac{1}{s^2 m + cs + k} = \frac{1}{m}\left(\frac{\beta}{s-\lambda} + \frac{\bar{\beta}}{s-\bar{\lambda}}\right)$$

Poles

$$\lambda = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2} \quad , \quad \omega_d = \omega_n\sqrt{1-\zeta^2}$$

$$\omega_n = \sqrt{k/m} = |\lambda| \quad , \quad \zeta = \frac{c}{c_{crit}} = \frac{c}{2\sqrt{km}} = \frac{-Re(\lambda)}{|\lambda|}$$

Damping ratio
quality




$s = i\omega$
Laplace/Fourier

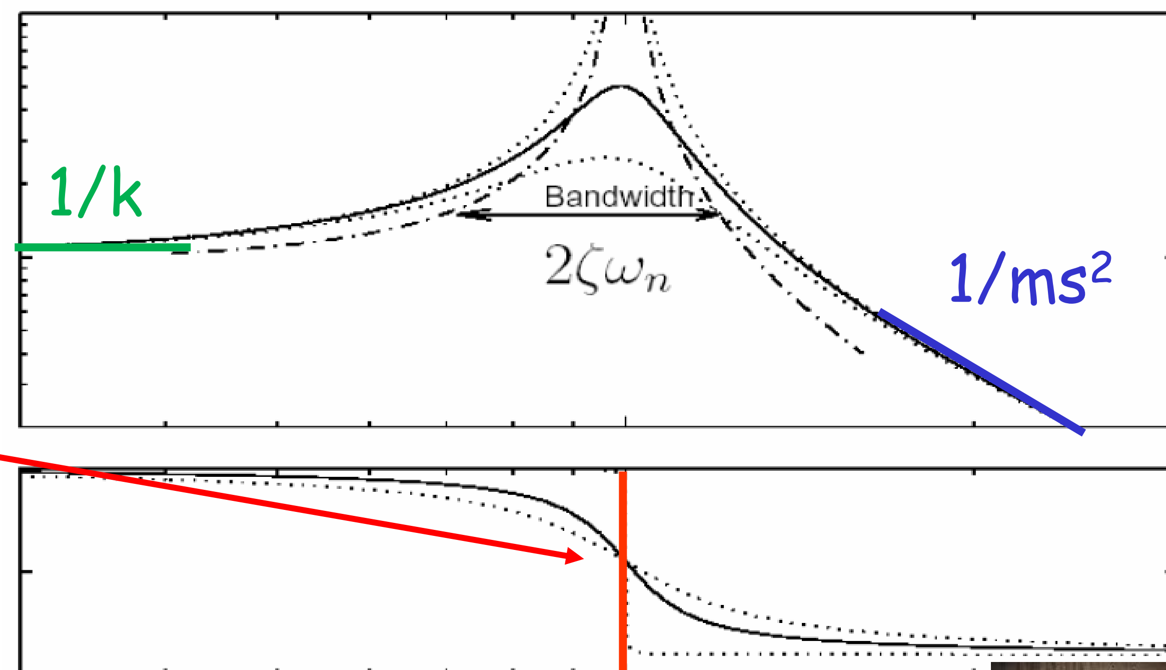
# 1 DOF :Bode plot

$$H(\omega) = \frac{1}{-\omega^2 m + i\omega c + k}$$

Asymptotes :
- Flexibility $1/k$
- Inertia (isolation) $1/ms^2$

Resonance
- amplitude $\propto$ $1/\zeta$
- Phase resonance -90°
- Bandwidth $2\zeta_j\omega_j$

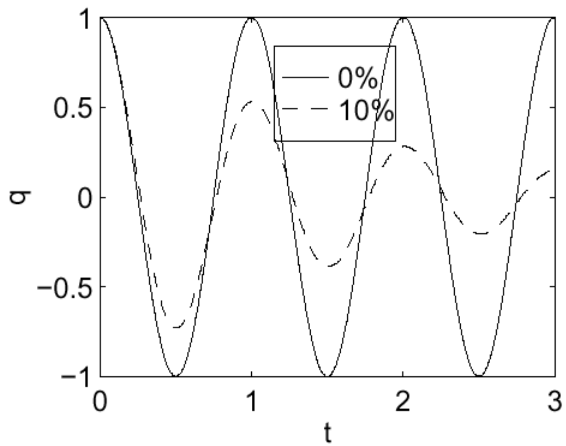
Response at phase resonance
$H(w_n)=1/i2\zeta\omega_n^2$

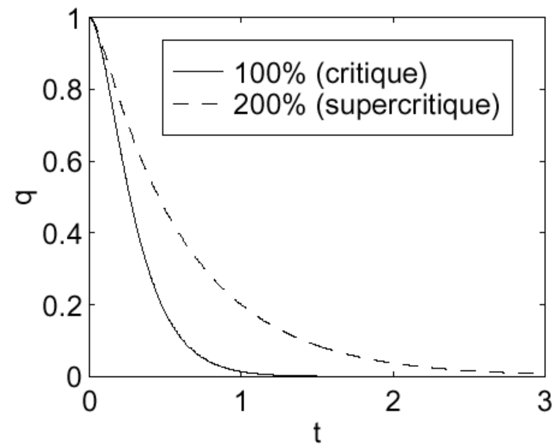



$$\omega_n = \sqrt{k/m},$$

# 1 DOF : time response / poles

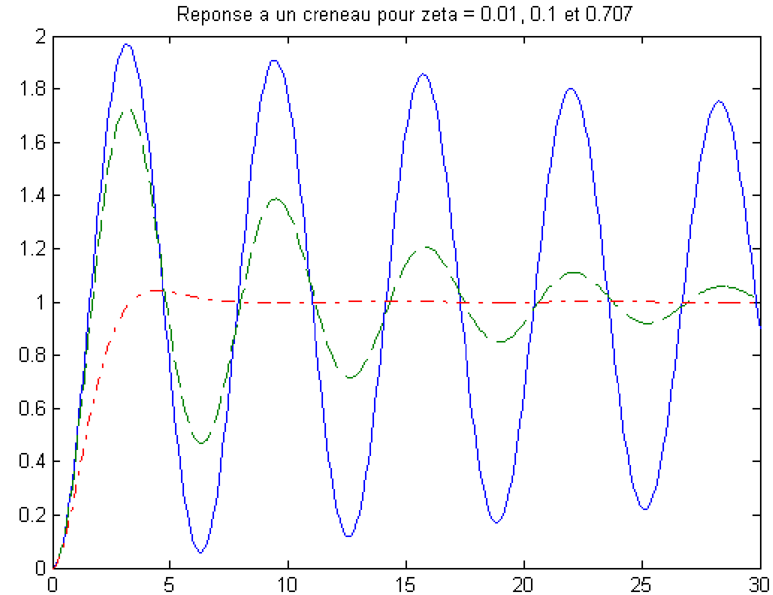
$$q(t) = Re\left(Ae^{\lambda_1 t} + Be^{\lambda_2 t}\right) = A\cos\left(\omega_j\sqrt{1-\zeta_j^2}\,t + \phi\right)e^{-\zeta_j\omega_j t}$$




Initial condition

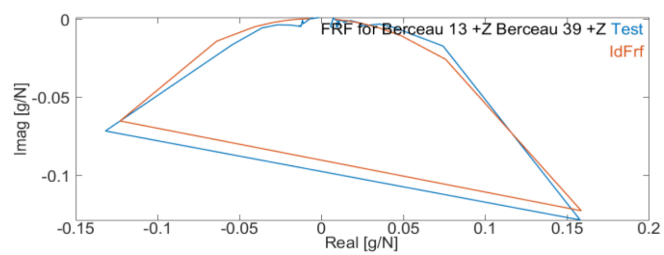
Step input

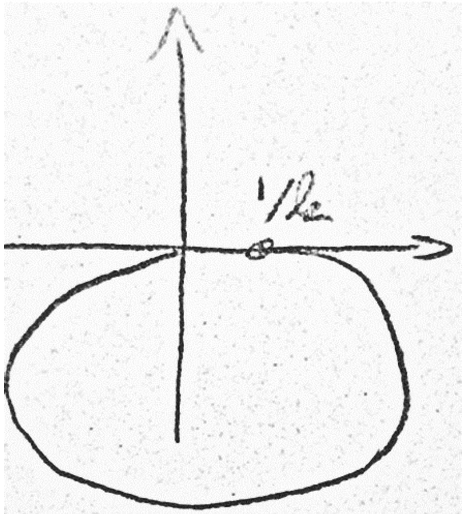


# Measuring damping (historical methods)

Frequency : -3dB Bandwidth $\zeta = \frac{\Delta\omega}{2\omega_{max}}$

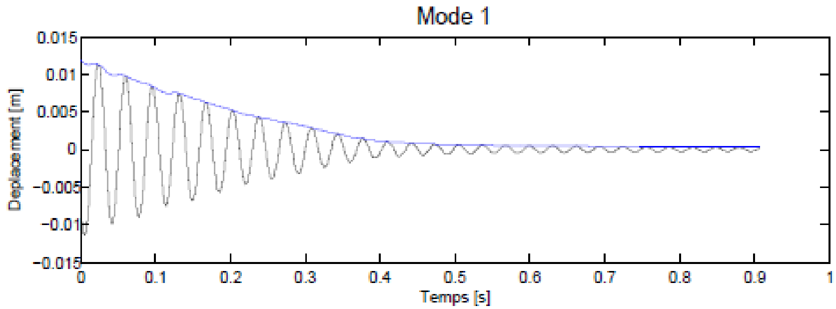
Failures : resolution, noise, multi-mode

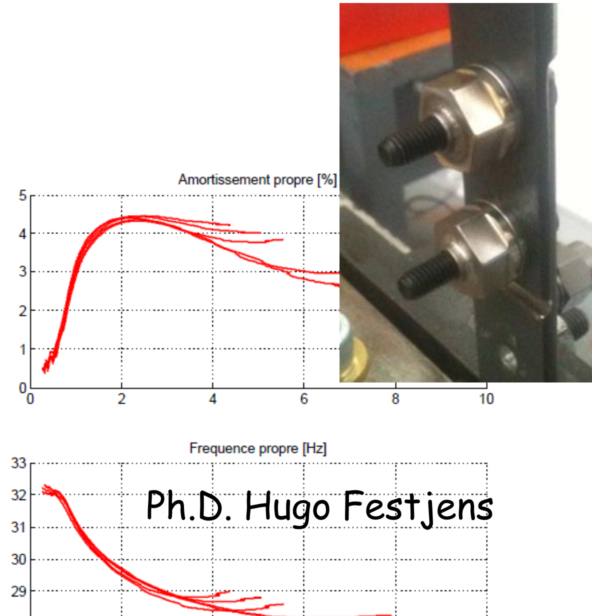


Time : logarithmic decrement $\ln\left(\frac{q_n}{q_{n+1}}\right) = 2\pi\zeta_j \frac{\omega_n}{\omega_d}$

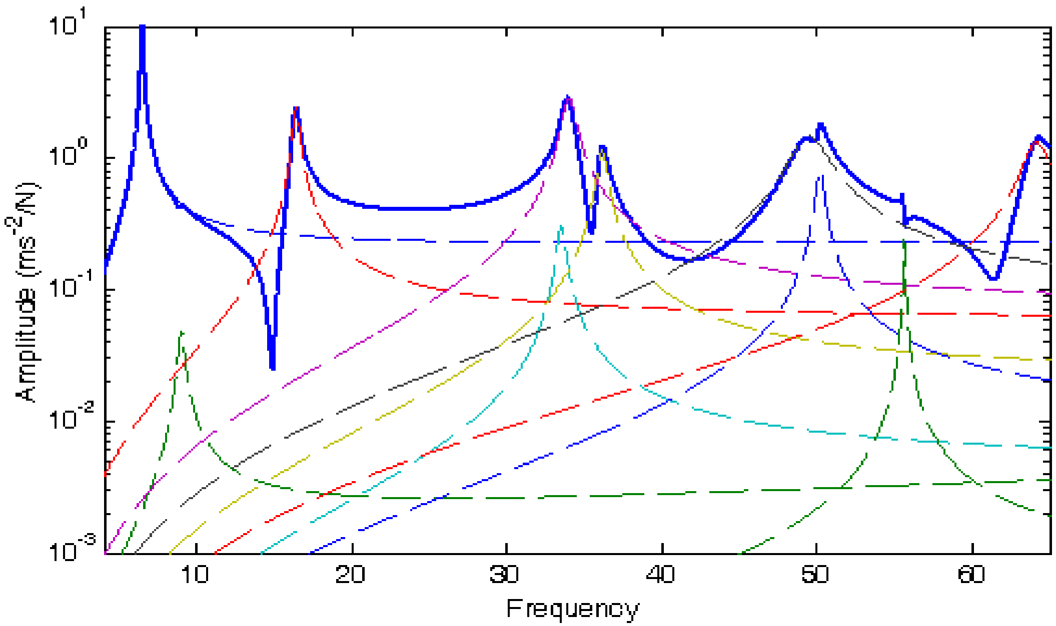
Failures : multi-DOF, amp. dependence




Ph.D. Hugo Festjens

# 1 input, 1 output, many resonances




MDOF multiple degree of freedom
SISO single input single output

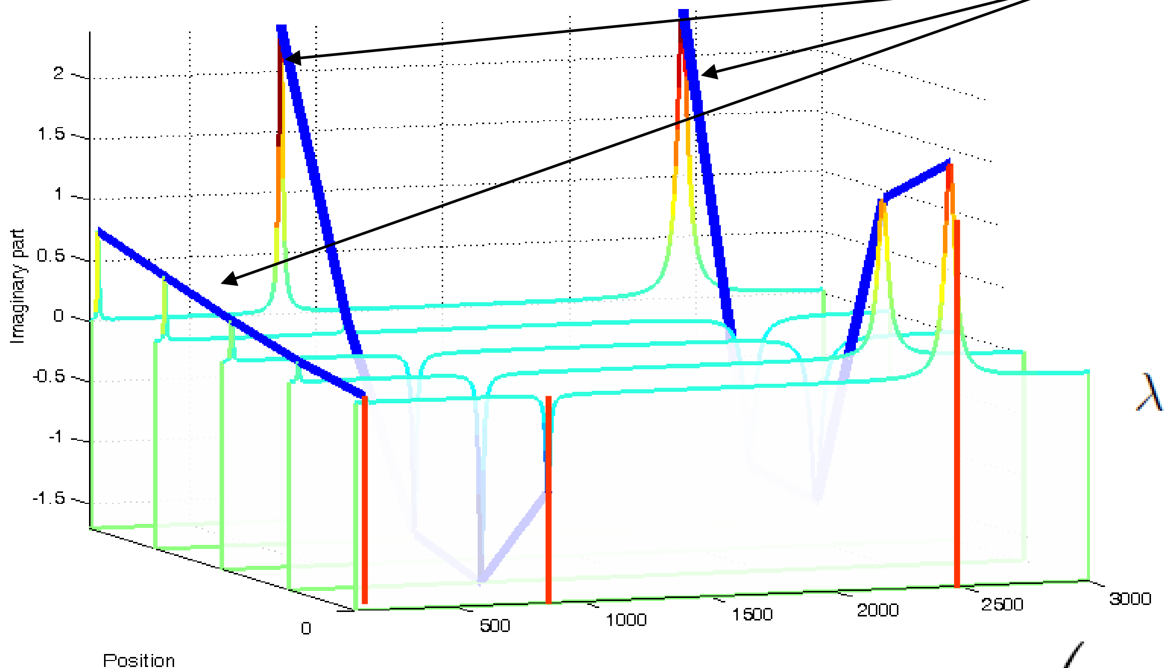
## Spectral decomposition

MDOF (multiple resonances)

SISO Tj is 1x1

$$[\alpha(s)] = \sum_{j \in \mathtt{identified}} \left( \frac{[T_j]}{s^2 + 2\zeta_j \omega_j s + \omega_j^2} \right)$$

# MDOF MIMO system




The shapes

The poles

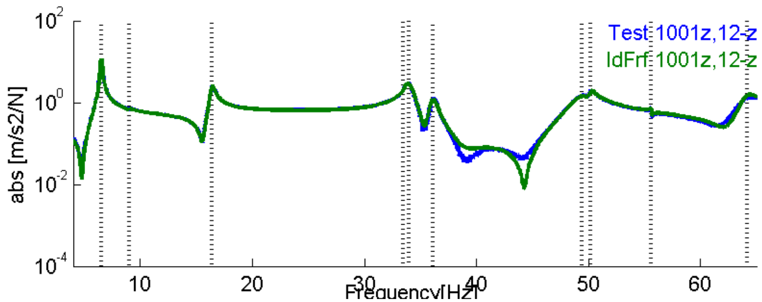
$$\lambda = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$$

$$[\alpha(s)] = \sum_{j \in \text{identified}} \left( \frac{[T_j]}{s^2 + 2\zeta_j\omega_j s + \omega_j^2} \right)$$

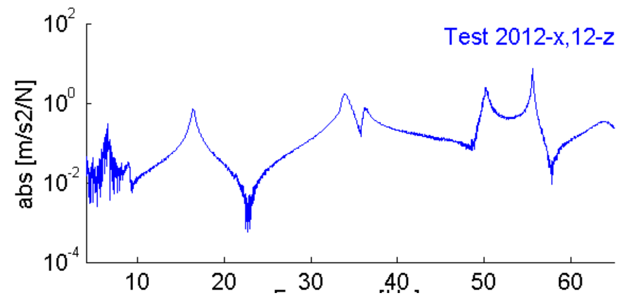
- Poles depend on the system (not the input/output)
- The shape is associated with the input/output locations

# Identification

## Objective $H_{test}$-$H_{id}$




## Data

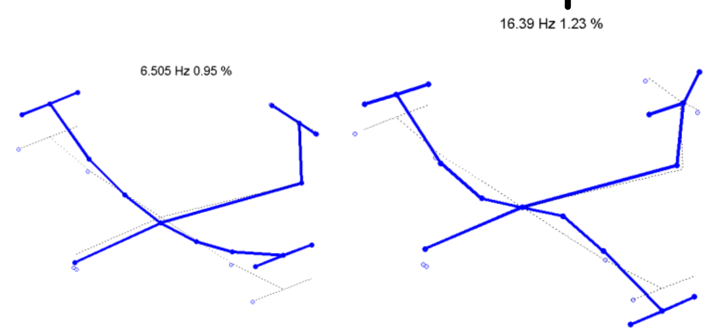



Optimization

## Family of models

$$\sum_{j \in \texttt{identified}} \left( \frac{[T_j]}{s^2 + 2\zeta_j \omega_j s + \omega_j^2} \right)$$

## Result : modes and poles



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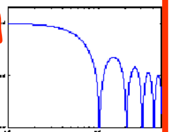
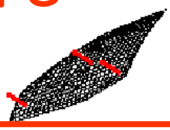
# How are transfers predicted ?

When

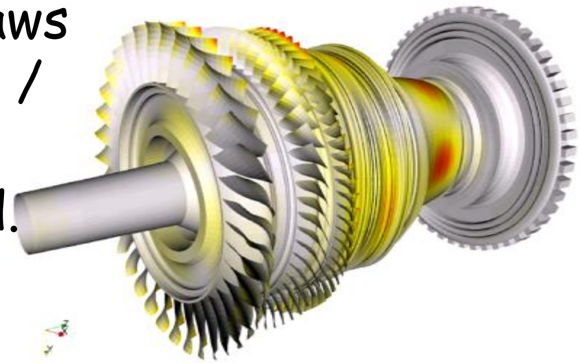


Where



System (FEM)

- Geometry
- Constitutive laws
- Discretization / idealization
- Boundary cond.



- DOFs $\{q\}$
- Mass M and stiffness K matrices

Outputs

$u(x,t)$
$\sigma(x,t)$

Inputs (when)
- Unbalance : harmonic at $\Omega$
- Aerodynamic loads ($n\Omega$)
- Rotor/stator contact

Inputs (where)
- Point mass
- Distributed pressure
- Blade tip

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K]\{q\} = \{F_{Ext}(t)\}$$

# Modes : harmonic solution with no force

$$[Ms^2 + Cs + K]\{q(s)\} - \{F(s)\} = 0$$

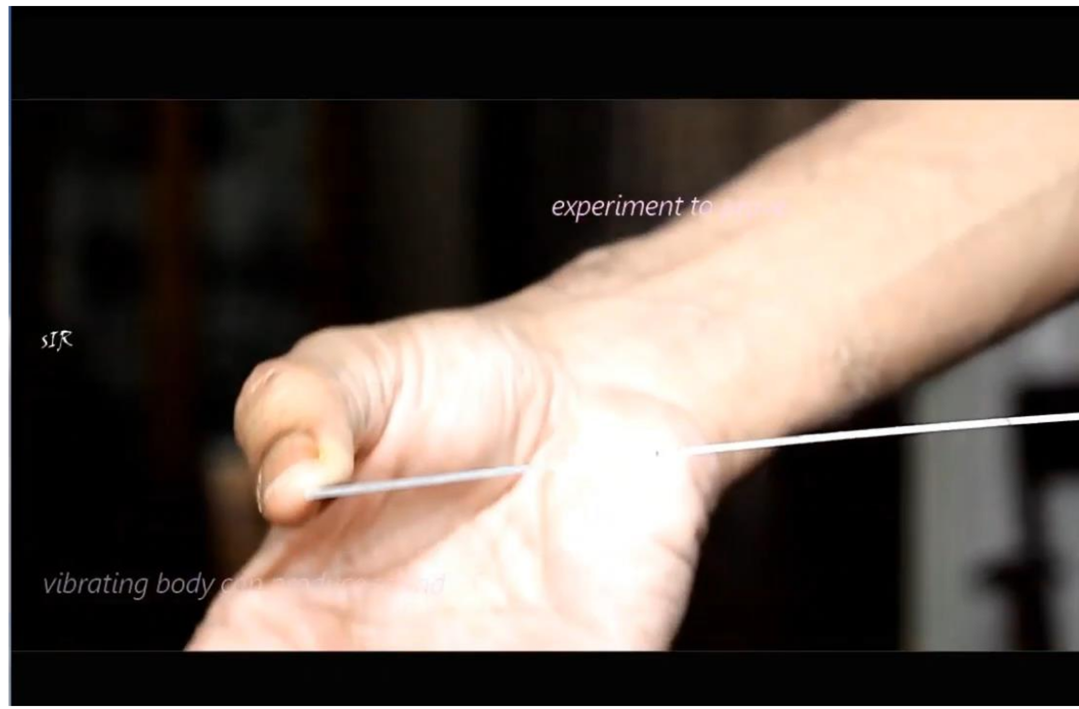
$q(t) = Re\left(\{\phi_j\}e^{i\omega_j t}\right)$ normal mode (no damping)

$$Re\left(\left[-M\omega_j^2 + K\right]\{\phi_j\}e^{i\omega_j t} - \{0\}\right) = 0$$

Linear time invariant

Eigenvalue problem

- Full solver : scipy.linalg.eig (LAPACK Linear Algebra)
- Partial solvers exist, a few keywords
  - scipy.sparse.linalg.eigs (Matlab eigs)
  - FEM Solvers : Lanczos) AMLS

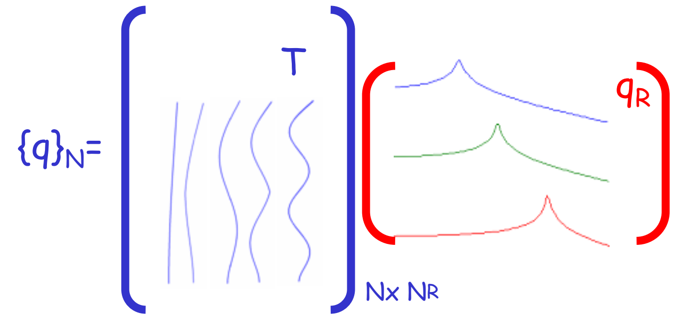


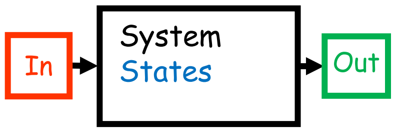

https://www.youtube.com/watch?v=zstmGnaaaCI

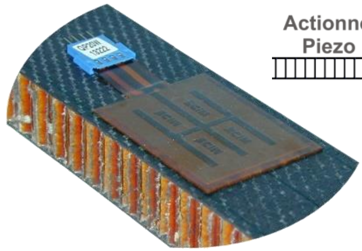
# Kinematics / model reduction

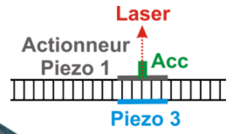
- Displacement u(x,t)

  = shapes (x) x DOF (t)

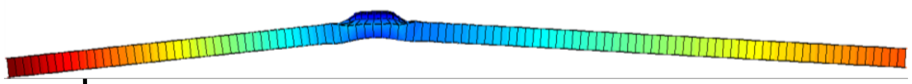


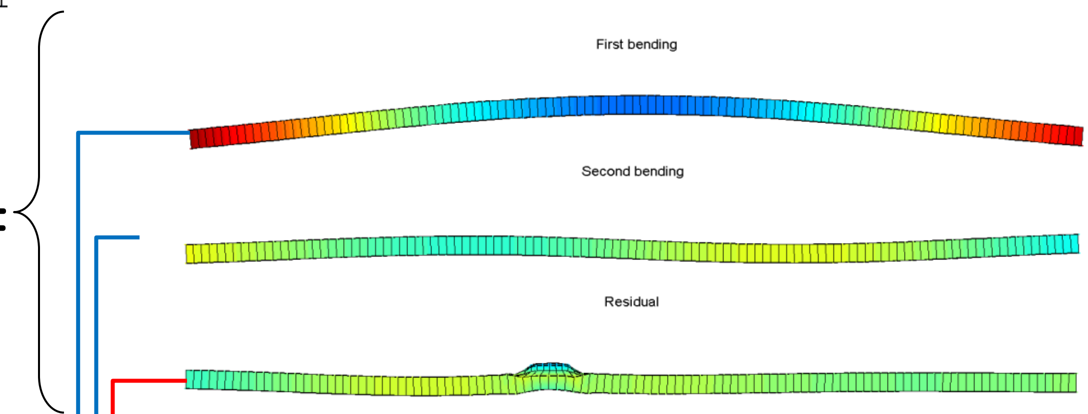



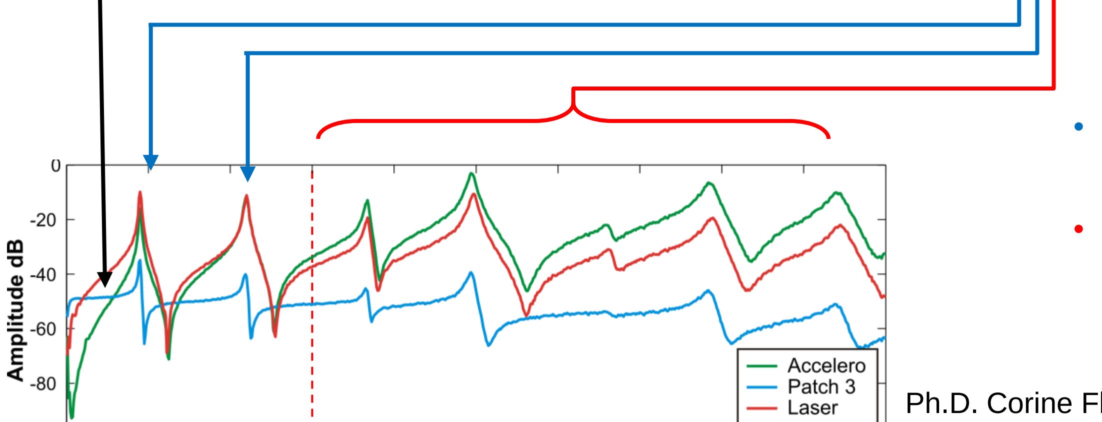






Quasi-static response @ 10Hz





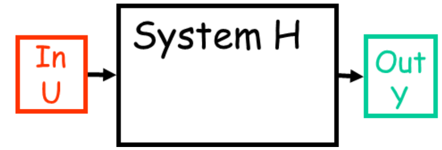




- Modes : high energy, load independent (no blister shape)
- Static response (influence of input=blister), important away from resonance

# Observation

$$\left[Ms^2 + Cs + K\right]\{q\} = \{F(s)\}$$

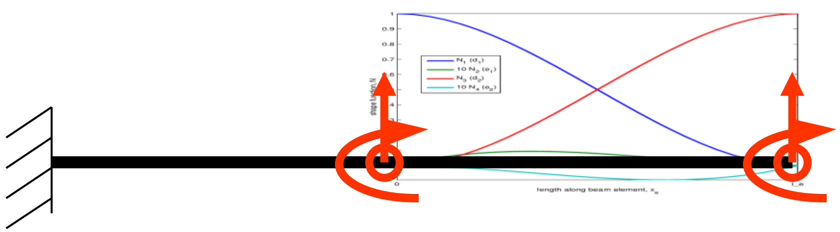
$$\{y(s)\} = [c]\{q\}$$



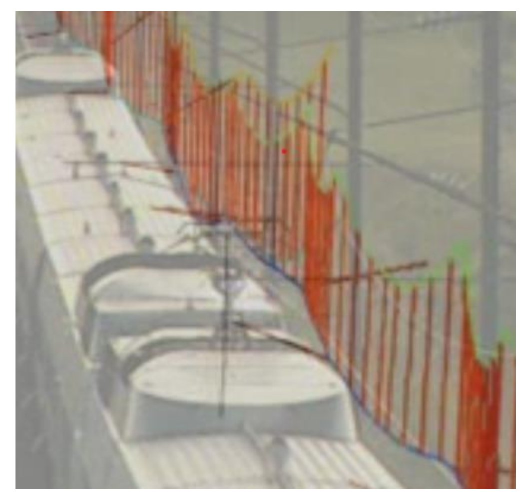

- {y} outputs are linearly related to DOFs {q} using an observation equation

$$\{y\} = [c]\{q(t)\}$$

- Simple case : extraction $\{w_2\} = [0\ \ 0\ \ 1\ 0]\begin{Bmatrix} w_1 \\ \theta_1 \\ w_2 \\ \theta_2 \end{Bmatrix}$

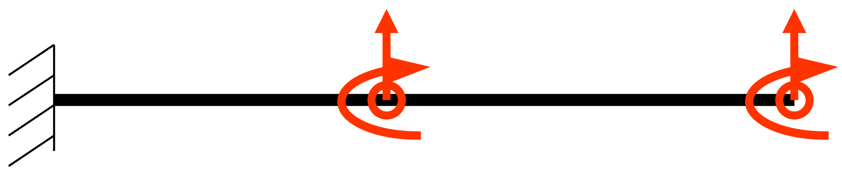


- More general : intermediate points, reactions, strains, stresses, …



# Command

$$[Ms^2 + Cs + K]\,\{q\} = \{F(x,s)\} = [b(x)]\{u(s)\}$$



- Loads decomposed as spatially unit loads and inputs

  {F(t)} = [b] {u(t)}

Abaqus : *CLOAD + *AMPLITUDE, ...

NASTRAN : FORCE-MOMENT + RLOAD

ANSYS, CODE Aster, ...

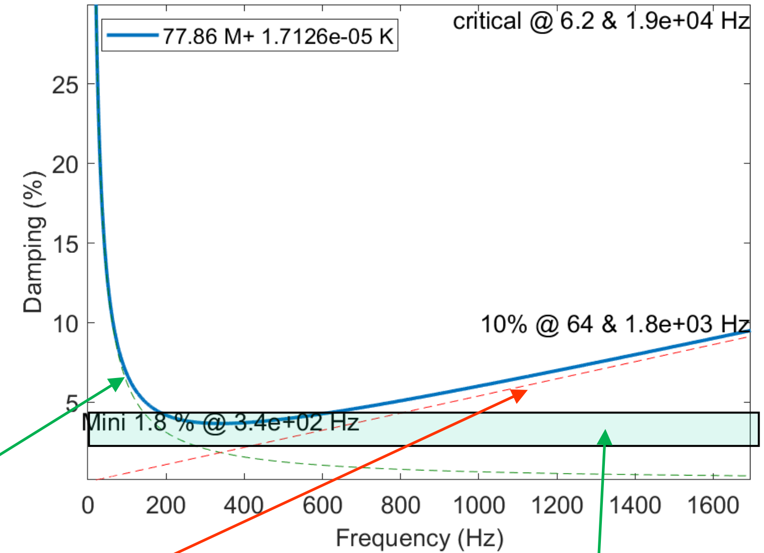
# Modal damping

## Rayleigh damping

- Physical domain $[C] = \alpha[M] + \beta[K]$

- Modal $\phi^T[C]\phi = \left[{}^{\backslash}2\zeta_j\omega_{j\,\backslash}\right] = \alpha[I] + \beta\left[{}^{\backslash}\omega_j^2{}_{\backslash}\right]$

$$\zeta_j = \frac{\alpha}{2\omega_j} + \frac{\beta\omega_j}{2}$$



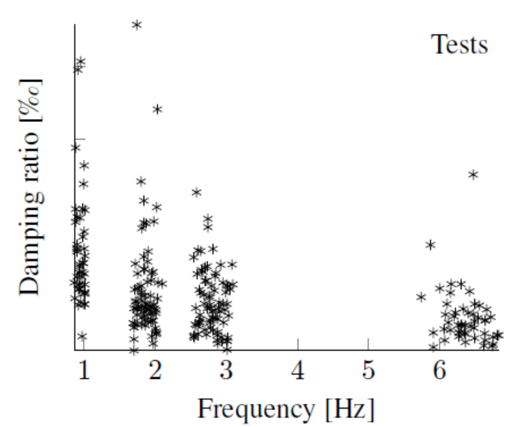

Mass

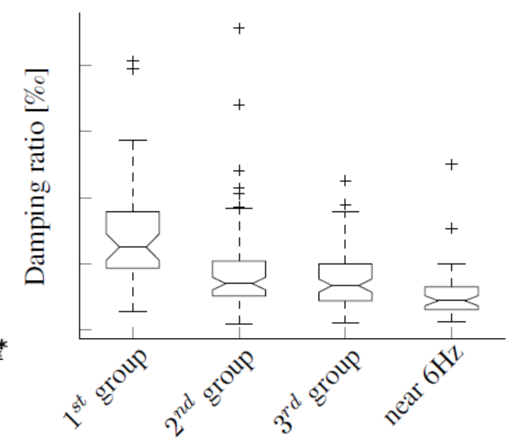
Stiffness
can be > 100%

Reality

## Modal damping $\zeta_j$ derived from test

Physical domain $[C] = [M\phi]\left[{}^{\backslash}2\zeta_j\omega_{j\,\backslash}\right][\phi^T M]$



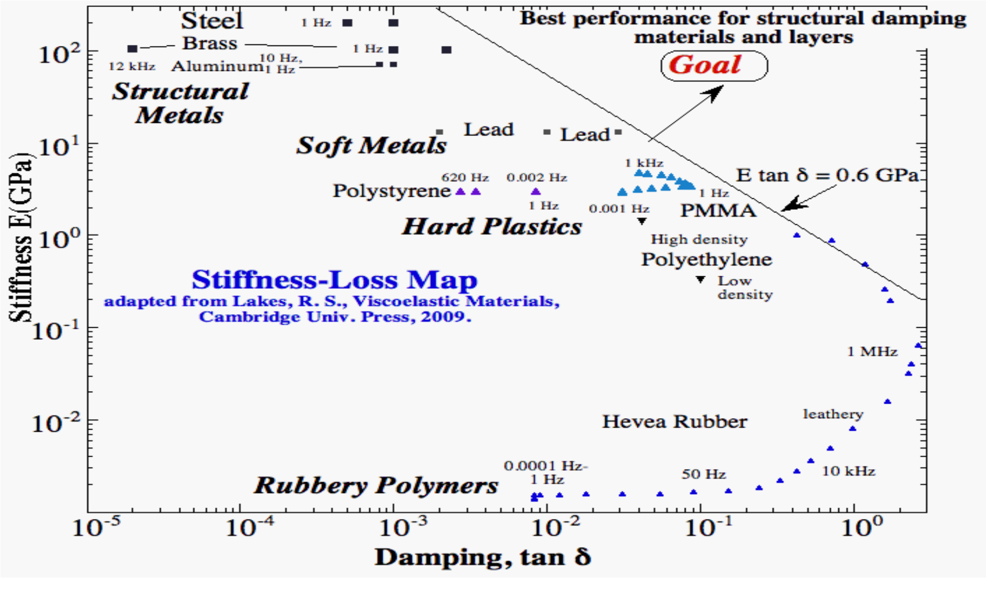

O. Vo Van, E. Balmes, et X. Lorang, « Damping characterization of a high speed train catenary », IAVSD, Graz, 2015, http://sam.ensam.eu/handle/10985/10918.

# Classical values for modal damping

- Damping ratio $\zeta$ measured or design parameter

- Pure metal 0.05 %
- Assembled structure ≈ 1%
- Full car ≈ 2-4%
- Soil radiation up to 10 %

Material loss $\eta = tan\delta \approx 2\zeta_j$



# Physical / modal & spectral decomposition

- Physical

$$[Ms^2 + Cs + K]\,\{q(s)\}_{Nq} = [b]\{u(s)\}$$
$$\{y(s)\} = [c]\{q(s)\}$$

- Modal coordinate $\{q(s)\} = [T]\{q_R(s)\} = [\phi_1 \ \dots \phi_{NM}]\{q_R(s)\}$
- Modal equations (modal damping)

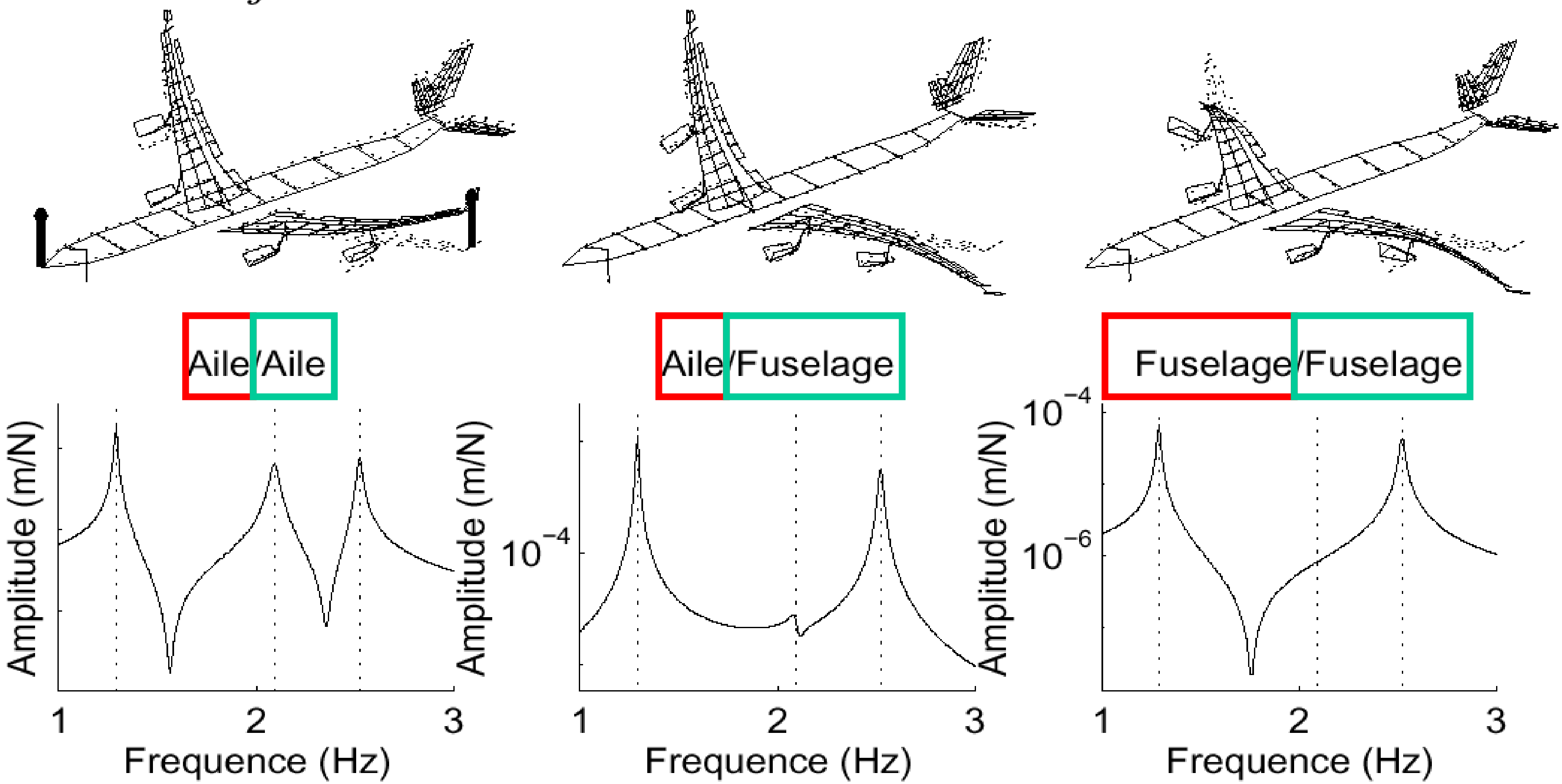
$$\left[Is^2 + \left[\ ^{\backslash}2\zeta_j\omega_{j_{\backslash}}\right]s + \left[\ ^{\backslash}\omega_j^2{}_{\backslash}\right]\right]\{q_R(s)\}_{Nqr} = [\phi_j^T b]\{u(s)\}$$
$$\{y(s)\} = [c\phi_j]\{q_R(s)\}$$

- Reduced matrices = diagonal
  Modal observability/commandability
- Spectral equations (inverse of diagonal matrix)

$$H(s) = [c][Ms^2 + Cs + K]^{-1}[b] = \sum_j \frac{\{c\phi_j\}\{\phi_j^T b\}}{s^2 + 2\zeta_j\omega_j s + \omega_j^2}$$

# Observability/controlability

$$H(s) = \sum_{j=1}^{N} \frac{[c]\{\phi_j\}\{\phi_j\}^T[b]}{s^2 + 2\zeta_j\omega_j s + \omega_j^2}$$



# Course outline

- Introduction
- How are modes measured
  - Mode ≈ resonance ≈ 1 DOF (degree of freedom) system
  - Transfer (series of modal contributions)
- How are modes predicted
  - Modes, inputs, outputs, damping
- Test / analysis correlation
  - Identification
  - Topology correlation
  - MAC / Updating
- Vibration design / conclusion

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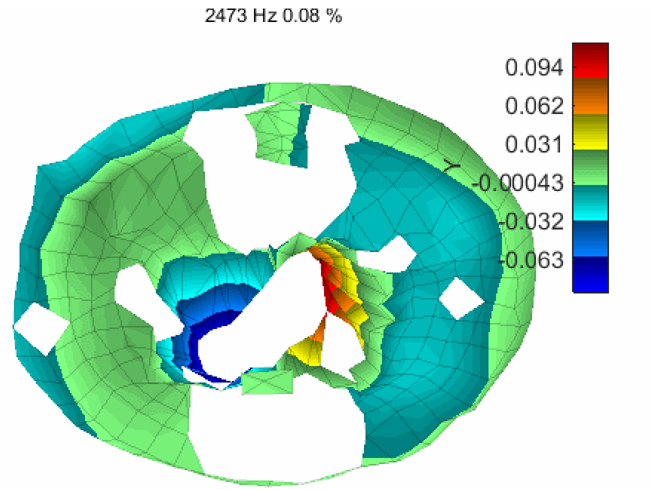
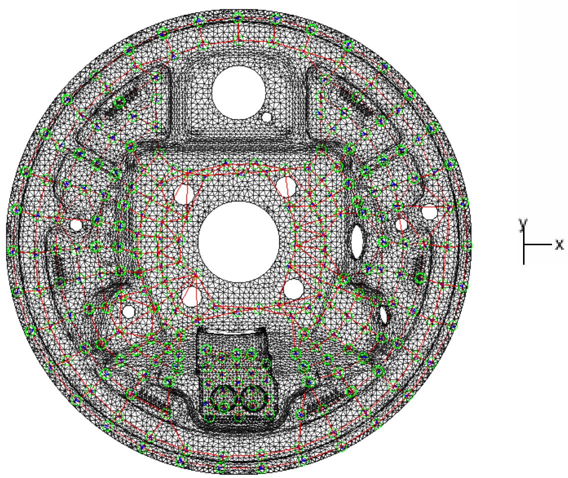
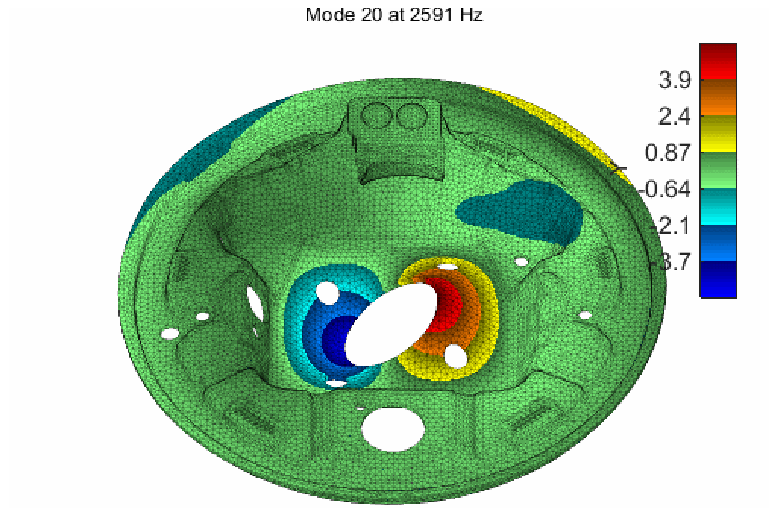
# Comparing test & FEM






Identification
known @ sensors

FEM known @ nodes

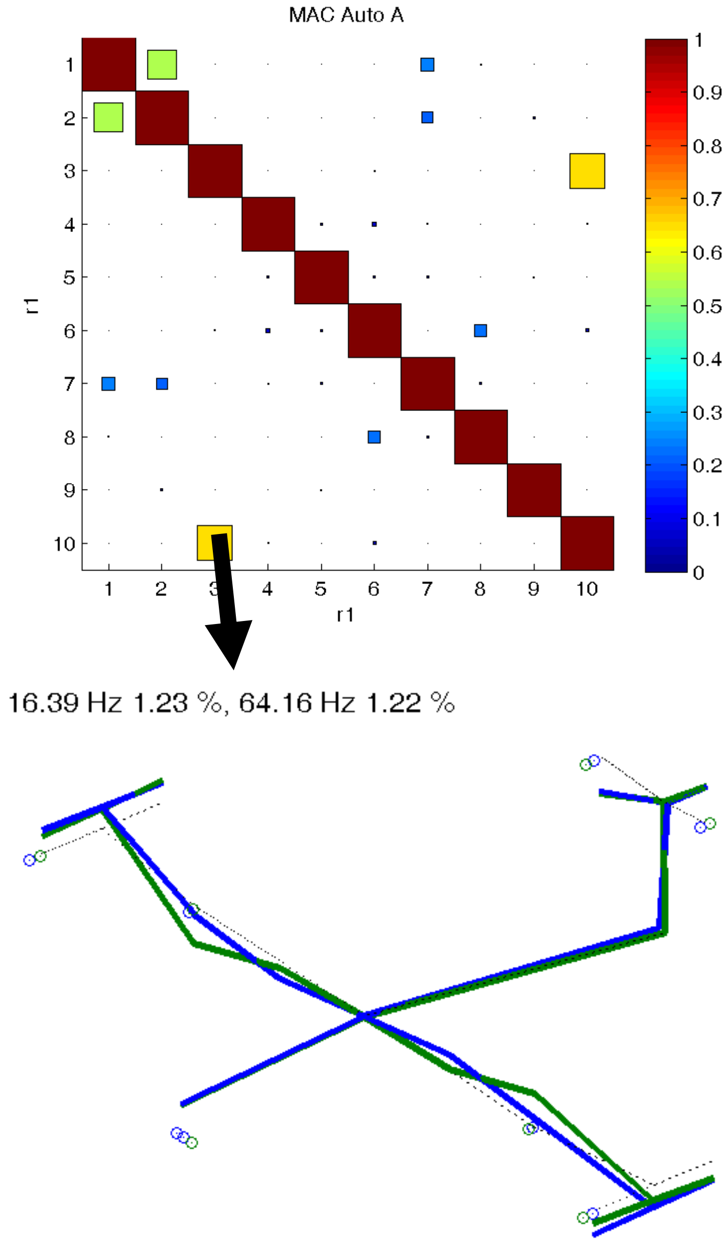
Topology correlation
= observe FEM @ sensors

{y(t)} = [c] {q(t)}

# MAC : comparing shapes

Shapes are compared through correlation coefficient (Modal Assurance Criterion)

$$\mathrm{MAC}(U,V) = \frac{|\{U\}^H \ \{V\}|^2}{|\{U\}^H \{U\}||\{V\}^H \{V\}|}$$




Next step : modal updating (*recalage*) = use correlation to correct model parameters

# Course outline

- Introduction
- How are modes measured
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# Stiffness perturbation in modal coord.

- Stiffness perturbation

$$[M]\{\ddot{q}\} + [K + \Delta K]\{q\} = [b]\{u\}$$

- Modal coordinates reanalysis $\{q\} = [\phi]\{q_R\}$

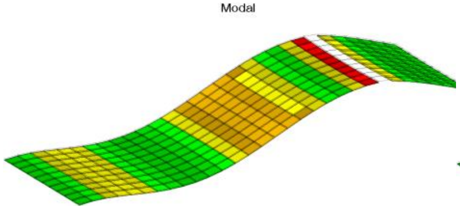
$$[I]\{\ddot{q}_R\} + \left[\left[\backslash \omega_j^2 \backslash\right] + \phi^T \Delta K \phi\right]\{q_R\} = [\phi^T b]\{u\}$$

- Sensitivity on frequency

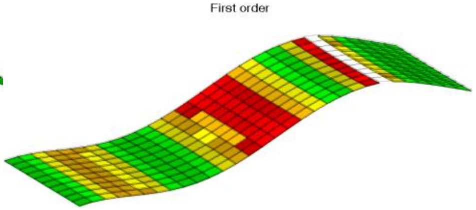
$$\frac{\partial \omega_j^2}{\partial p} = \{\phi_j\}^T \left[\frac{\partial K}{\partial p} - \omega_j^2 \frac{\partial M}{\partial p}\right]\{\phi_j\}$$

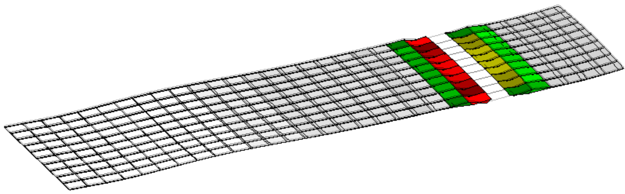
- Need to know :
  may be significantly wrong without residual terms/static correction

Modal



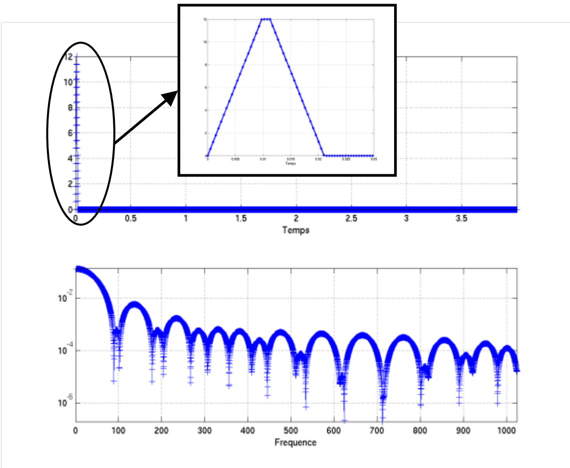
First order



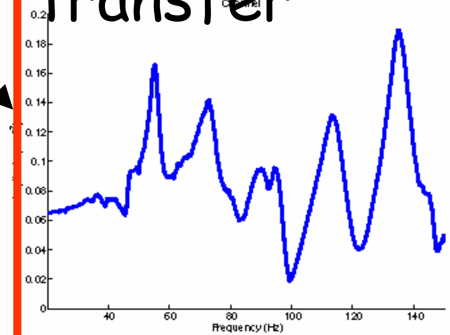


# Design process

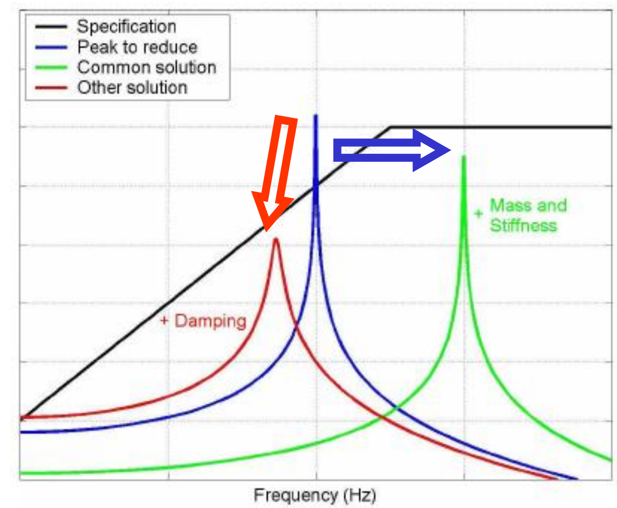
## Define inputs




## Compute/test transfer




## Verify specification




Adjust mass/stiffness

$$\omega_j = \sqrt{\frac{\sum_{elt} \{\phi_j\}^T K^{(e)} \{\phi_j\}}{\sum_{elt} \{\phi_j\}^T M^{(e)} \{\phi_j\}}}$$

Adjust damping

# Software selection

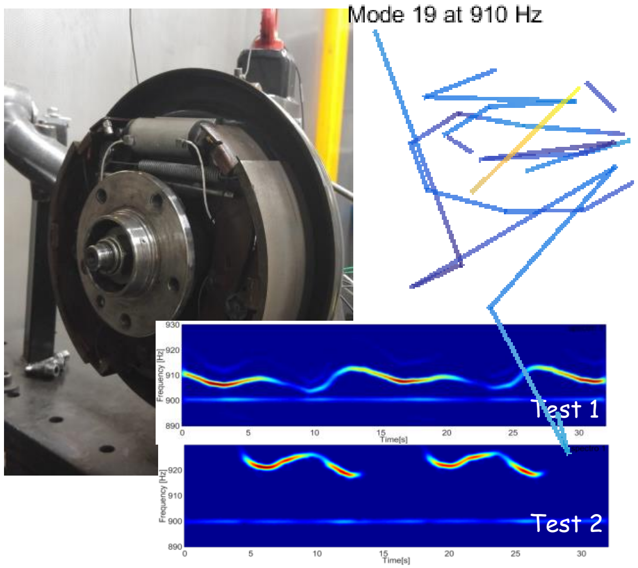
Simulation

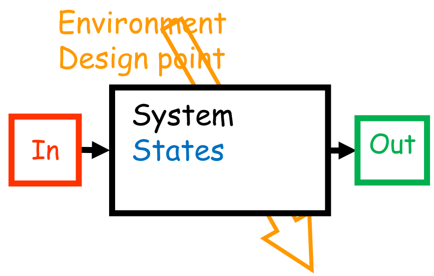
- Major players : NASTRAN, ANSYS, ABAQUS
- Here : SDT for MATLAB www.sdtools.com (FEM core is open source : OpenFEM)

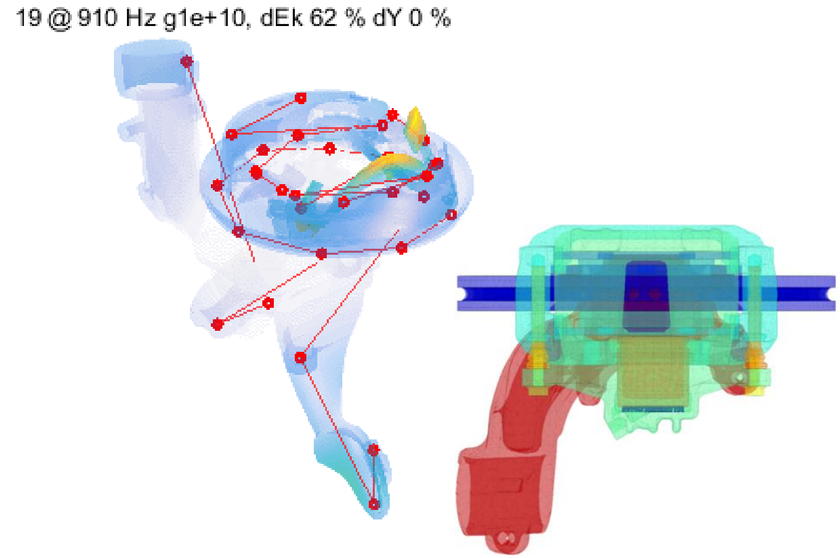
Test

- Major players : Siemens-TestLab, ME-Scope
- Here : SDT

# A system = I/O representation




**Prototype**

- ☺ all physics (no risk on validity)
- ☺ in operation response
- ☹ limited test inputs
- ☹ measurements only
- ☹ few designs

- 😐 Cost : build and operate

**Virtual prototype**

- ☹ limited physics (unknown & long CPU)
- ☹ design loads
- ☺ user chosen loads
- ☺ all states known
- ☺ multiple (but 1 hour, 1 night, several days, … thresholds)
- 😐 Cost : setup, manipulate