A.2 Incompatible meshes

A.2.1 Text

We want to make a displacement connection between two meshes along an interface. The interface as a length $2e$ and is oriented by direction x. The position of the nodes on the interface is parametrized by position x and the origin is in the center of the interface. The mesh situated at le left of the interface is denoted I and the one at the right II.

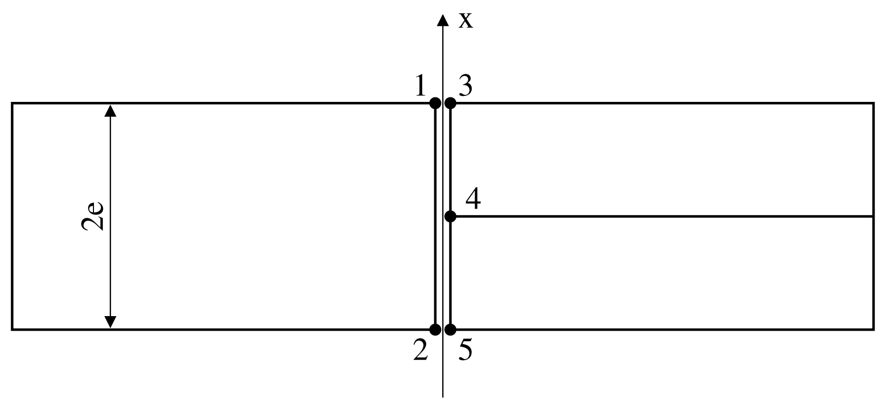

Figure 14: Incompatibles meshes

For simplification, we are only interested in the connection of one term of displacement denoted u. $u^I(x)$ (resp $u^{II}(x)$) is the expression of this term on mesh I (resp II). On the interface, there is just only one linear element on mesh I wich degree of freedom are denoted u_1 are u_2. Mesh II is made of two linear elements which degrees of freedom are u_3, u_4 et u_5 (see figure 14). The analytical expression of the displacements $u^I(x)$ et de $u^{II}(x)$ are then:

$$u^I(x) = \frac{u_1 - u_2}{2e}\,x + \frac{u_1 + u_2}{2}, \quad \forall x \in [-e, e] \qquad ; \qquad u^{II}(x) = \begin{cases} \dfrac{u_3 - u_4}{e}x + u_4 \quad , & \text{if } x \in [0, e] \\ \dfrac{u_4 - u_5}{e}x + u_4 \quad , & \text{if } x \in [-e, 0] \end{cases}$$

1. Explain the notions of master mesh and slave mesh.
2. Explain the notion of *connection at a mean sens*.
3. Gives the conditions that have to be prescribe on degree of freedom u_1, u_2, u_3, u_4 et u_5 depending on the type of connection
   (a) ponctual connection for which, mesh I is the master mesh.
   (b) ponctual connection for which, mesh II is the master mesh.
   (c) connection at a mean sens allowing the transmition of constant forces.
   (d) connection at a mean sens allowing the transmition of linear forces.

A.2.2 Correction

1. See lecture notes
2. See lecture notes
3. Conditions on the degrees of freedom
   (a) ponctual connection for which, mesh I is the master mesh.

$$\begin{cases} u_1 - u_3 & = 0 \\ u_2 - u_5 & = 0 \\ \dfrac{u_1 + u_2}{2} - u_4 & = 0 \end{cases}$$

   (b) ponctual connection for which, mesh II is the master mesh.

$$\begin{cases} u_1 - u_3 & = 0 \\ u_2 - u_5 & = 0 \end{cases}$$

   (c) connection at a mean sens allowing the transmition of constant forces.

$$\int_{-e}^{e} 1.(u^{II} - u^{I})dx = 0$$

   thus

$$\int_{-e}^{0} \frac{u_4 - u_5}{e} x dx + \int_{0}^{e} \frac{u_3 - u_4}{e} x dx + \int_{-e}^{e} \{u_4 - \frac{u_1 - u_2}{2} x - \frac{u_1 - u_2}{2e}\} dx = 0$$

   The condition on the degrees of freedom is then:

$$\frac{u_3 + u_5}{2} + u_4 - u_1 - u_2 = 0$$

   (d) connection at a mean sens allowing the transmission of linear forces: to the conditions obtained at the previous question we had:

$$\int_{-e}^{e} x.(u^{II} - u^{I})dx = 0$$

   thus

$$\int_{-e}^{0} \frac{u_4 - u_5}{e} x^2 dx + \int_{0}^{e} \frac{u_3 - u_4}{e} x^2 dx + \int_{-e}^{e} \{u_4 x - \frac{u_1 - u_2}{2} x^2 - \frac{u_1 - u_2}{2e} x\} dx = 0$$

   thus

$$u_3 - u_5 - u_1 + u_2 = 0$$

   The two conditions are then:

$$\begin{cases} \dfrac{u_3 + u_5}{2} + u_4 - u_1 - u_2 & = 0 \\ u_3 - u_5 - u_1 + u_2 & = 0 \end{cases}$$