Vibrations des structures & acoustique
TP1 : modélisation
- modèles système
- problèmes directs et inverses

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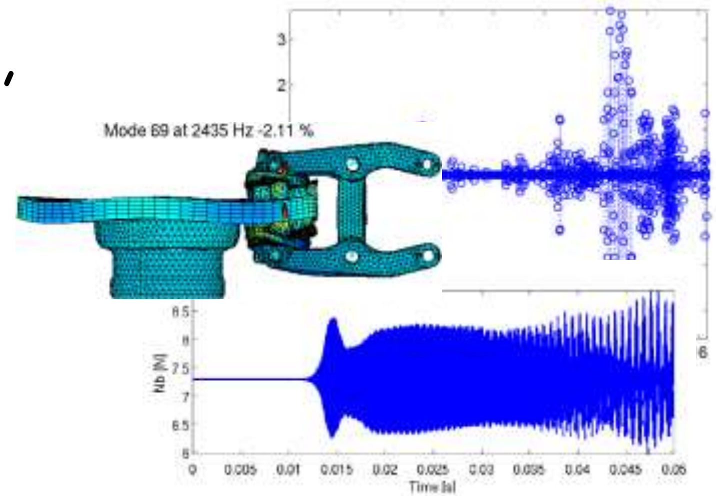
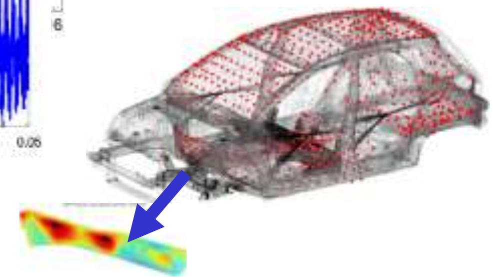
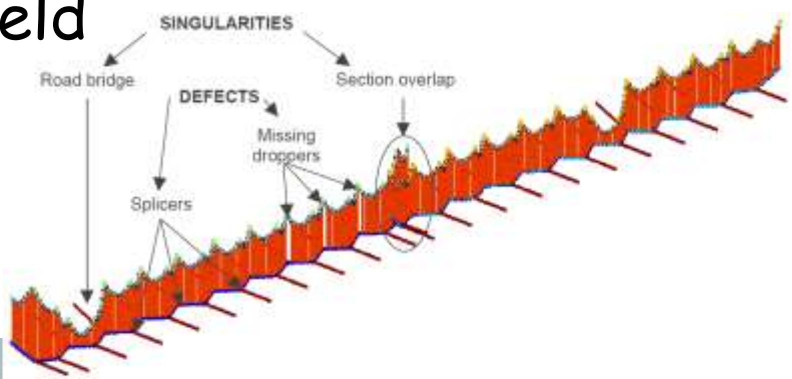
A few vibration problems

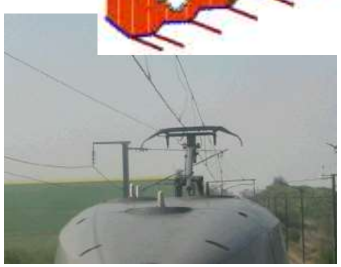
Squeal, automotive acoustics, damping for engine fatigue
Seismic response
Track settling

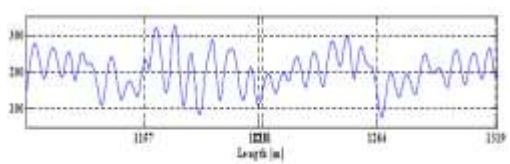


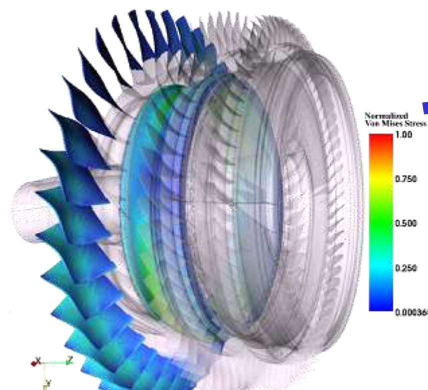


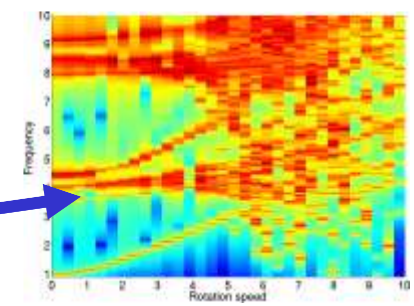
A multi-industry application field

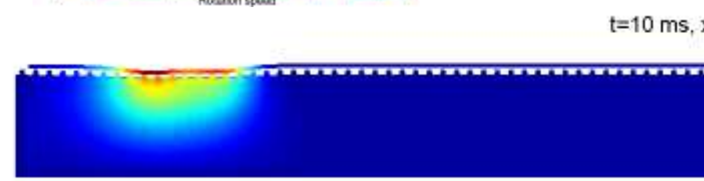






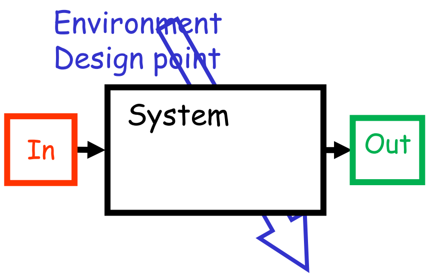






M2 DSMSC (dynamique des structures matériaux et systèmes couplés en commun avec l'ECP)

What is a system ?

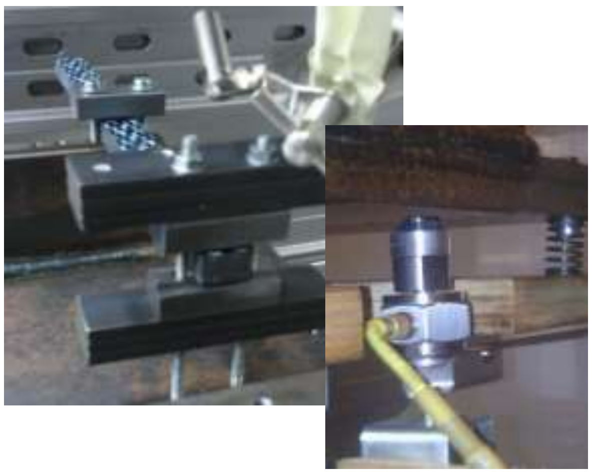


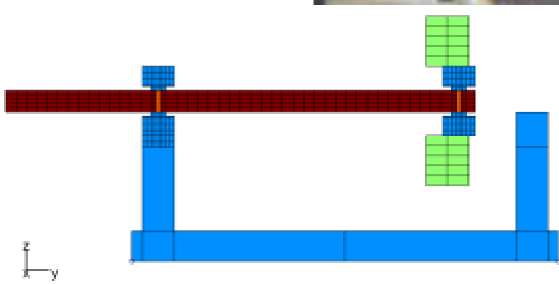
- Inputs $u(t)$: hammer with force measurement
- Outputs $y(t)$
  - Test : acceleration on the testbed
  - Computation : stresses
- State $x(t)$
  - Displacement & velocity field as function of time

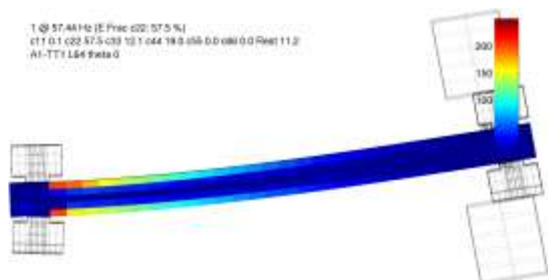
$$\{\dot{x}(t)\} = f(x(t), u(t), p, t) \quad \text{evolution}$$

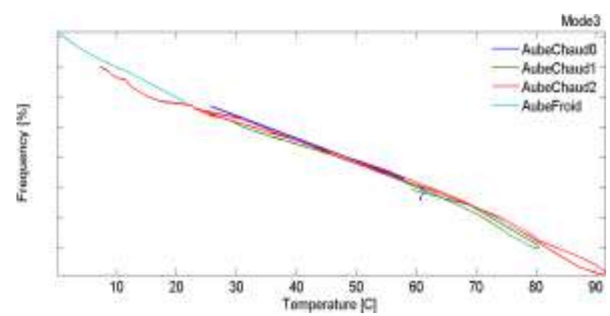
$$\{y(t)\} = g(x(t), u(t), p, t) \quad \text{observation}$$

- Environment variables p
  - Dimensions, test piece (design point)
  - Temperature (value of constitutive law or state of thermo-viscoelastic)
- Feature : ex. modal frequency (function of output)



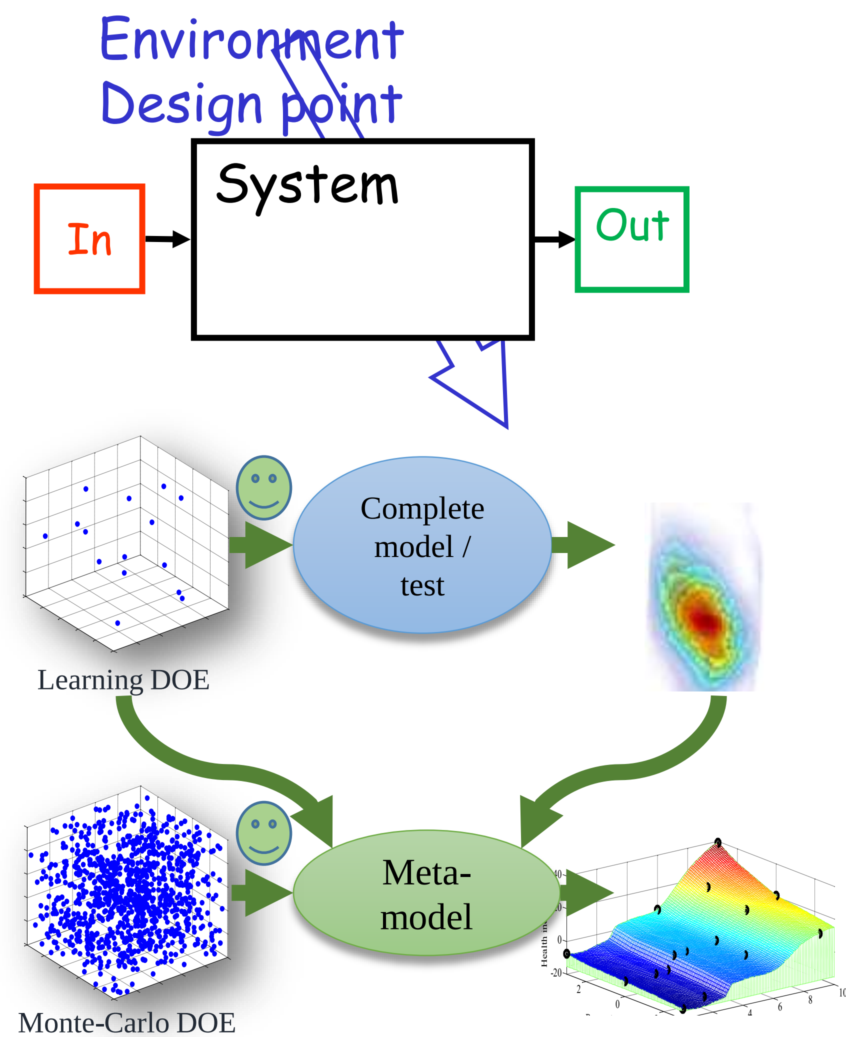






Simple example : modified Oberst test for 3D weaved composite test

System models : nature & objectives?



What is a model

- A function relating input and outputs
- For one or many parametric configurations

Model categories

- Behavior models (meta-models)
  - Test, constitutive laws, Neural networks
  - Difficulties : choice of parametrization, domain of validity
- Knowledge models
  - Physical principles, low level meta-models

Why do we need system models ?

Design

- Become predictive : understand, know limitations
- Perform sizing, optimize, deal with robustness

Certify

- Optimize tests : number, conditions
- Understand relation between real conditions and certification
- Account for variability

Maintain during life

- Design full life cycle (plan maintenance)
- Use data for conditional maintenance (SHM)

FEM model as a system

$\sigma(x,t)$
$f(x,t)$

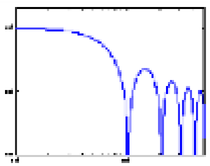
- Geometry
- Constitutive laws
- Discretization / idealization

$u(x,t)$
$\sigma(x,t)$

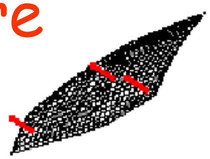
Extensions : coupled, time variant, non-linear, …

System models of structural dynamics

When

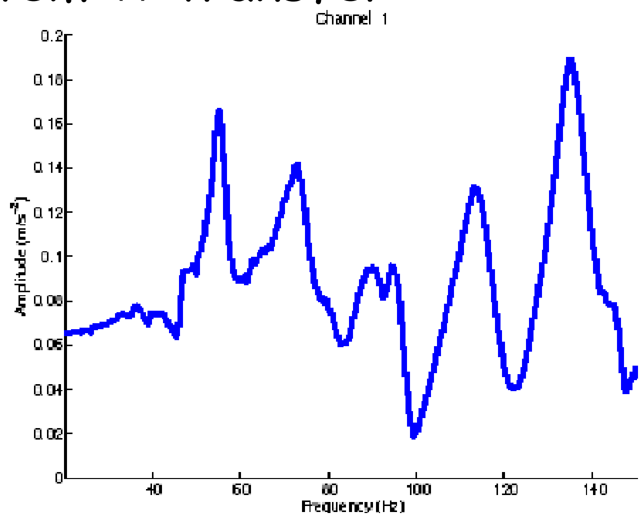


Where

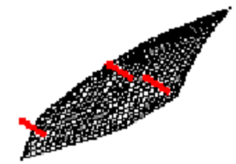


Assumption : linear time invariant

System ⇔ transfer



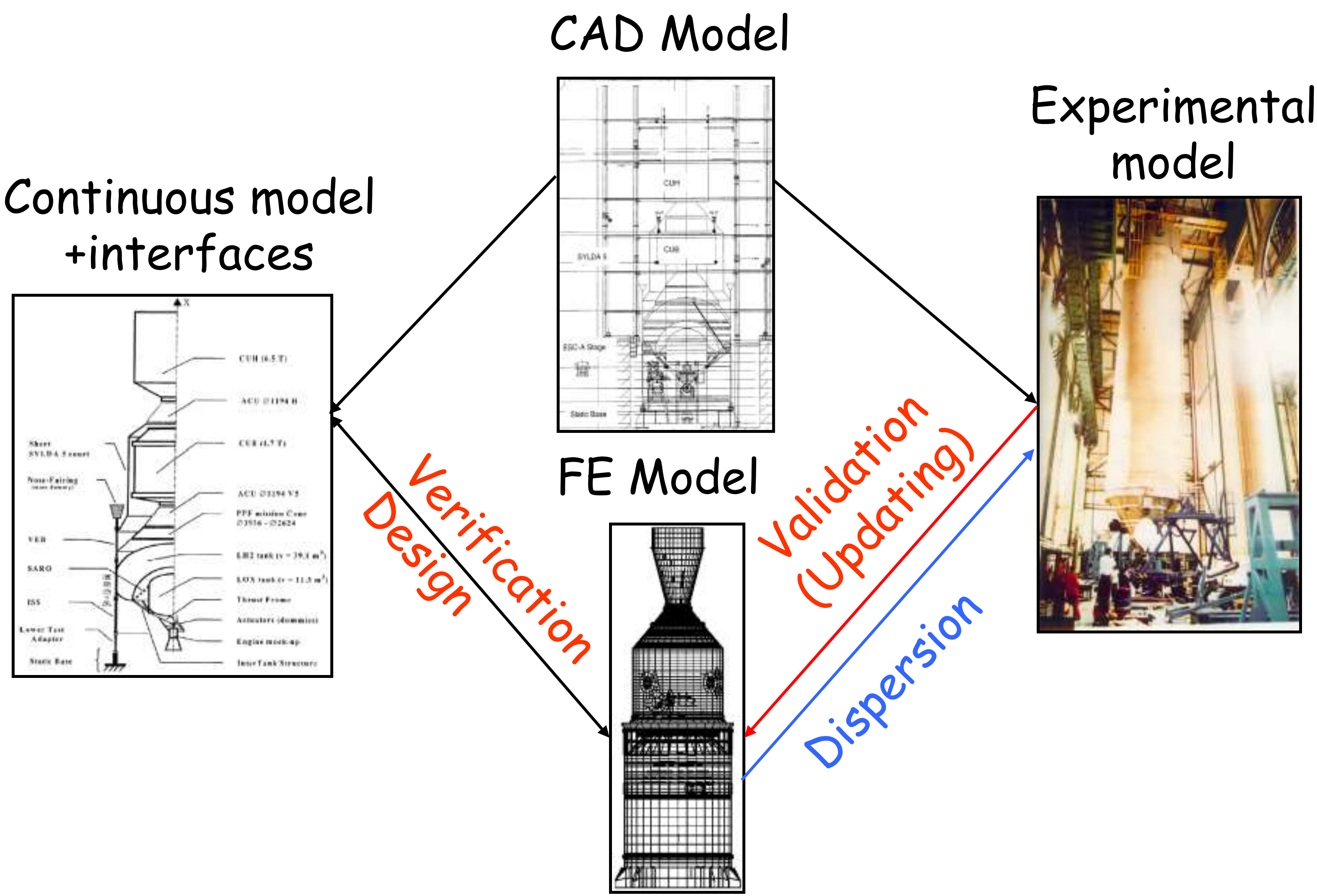
Sensors



Reduction based on restrictions :
- Excitation (space & freq)
- Responses
- Coupling …

Extensions
- Coupling (structure, fluid, control, multi-body, …)
- Optimization, variability, damping, non linearity, …

Model validation and verification



Course outline

- Introduction
- How are modes measured
  - Mode ≈ resonance ≈ 1 DOF (degree of freedom) system
  - Transfer (series of modal contributions)
- How are modes predicted
  - Modes, inputs, outputs, damping
- Test / analysis correlation
  - Identification
  - Topology correlation
  - MAC / Updating
- Vibration design / conclusion

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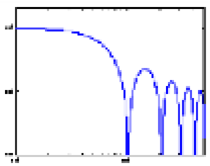
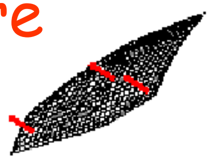
How are modes measured ?

When

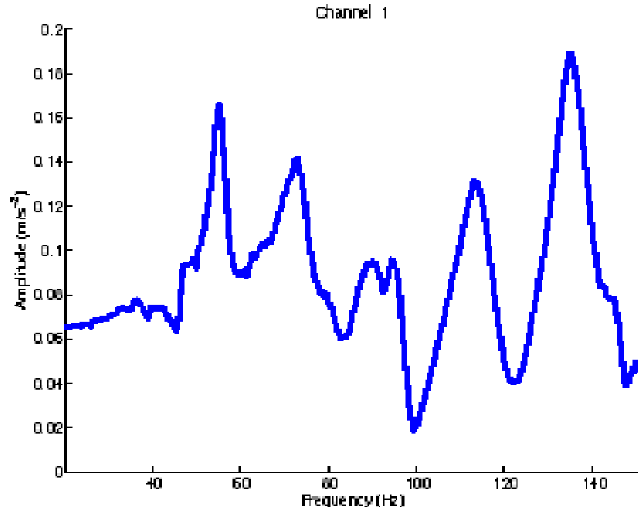


Where

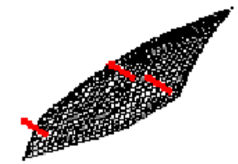


Assumption : linear time invariant

System ⇔ transfer



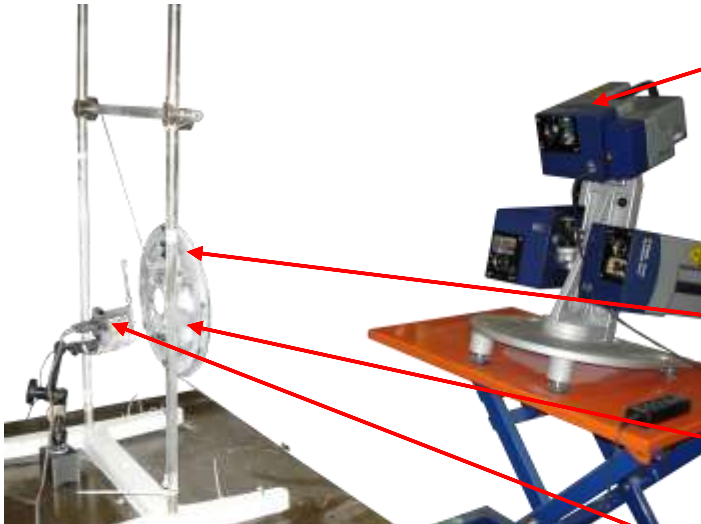
Sensors



Feed-back extensions

- Coupling (structure, fluid, control, multi-body, …)
- Optimization, variability, damping, non linearity, …

Experimental modal analysis : measurements

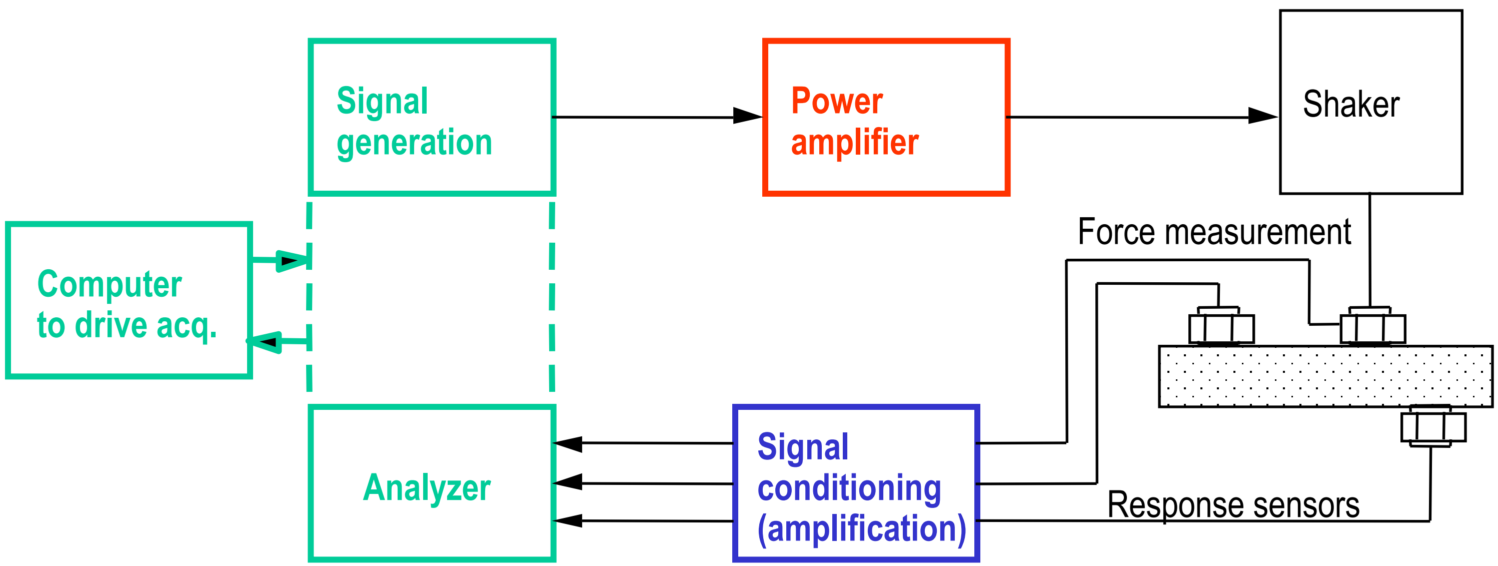


Laser vibrometer (measure)

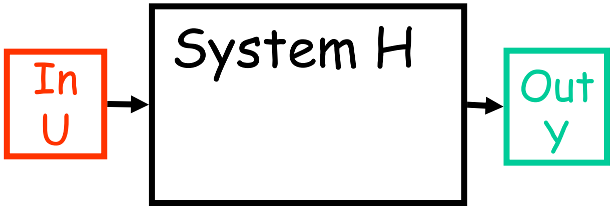
Measurement point

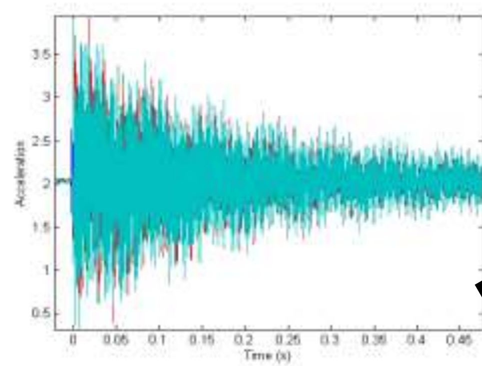
Plate

Shaker (excitation)



Modal analysis : transfers



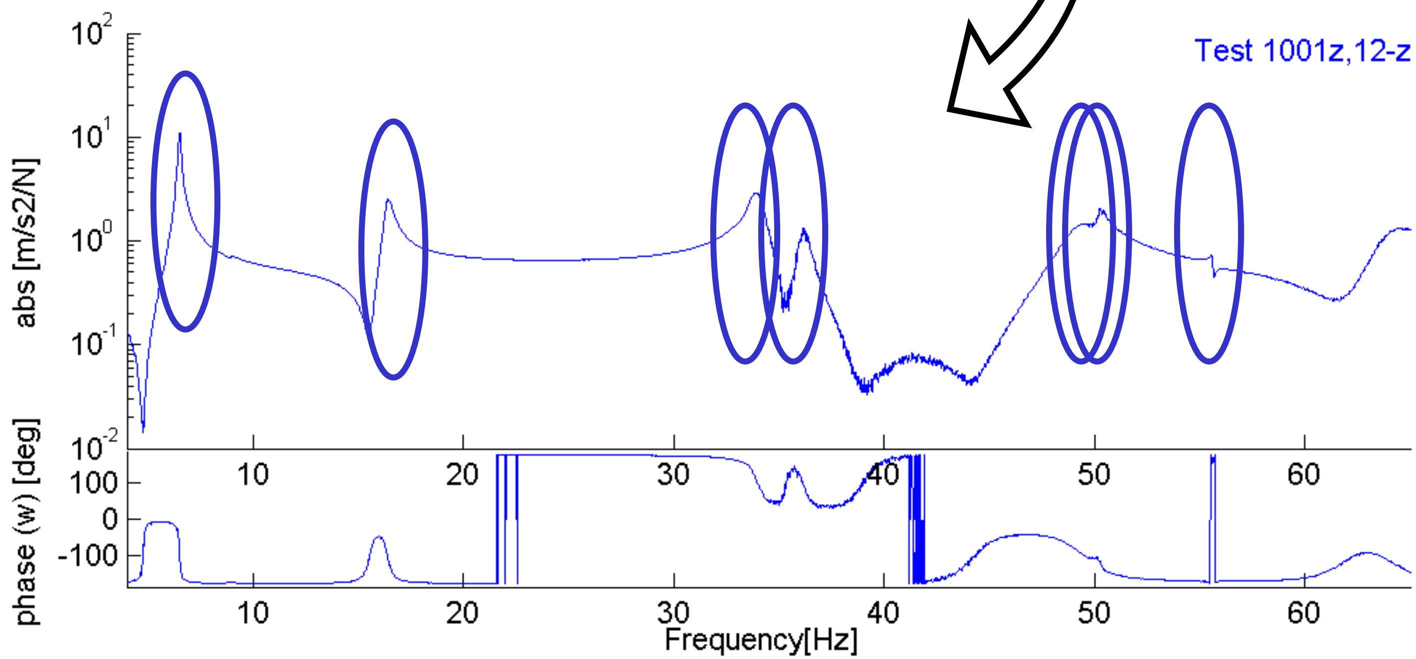


Transfers estimated from time response

ONE input

ONE output

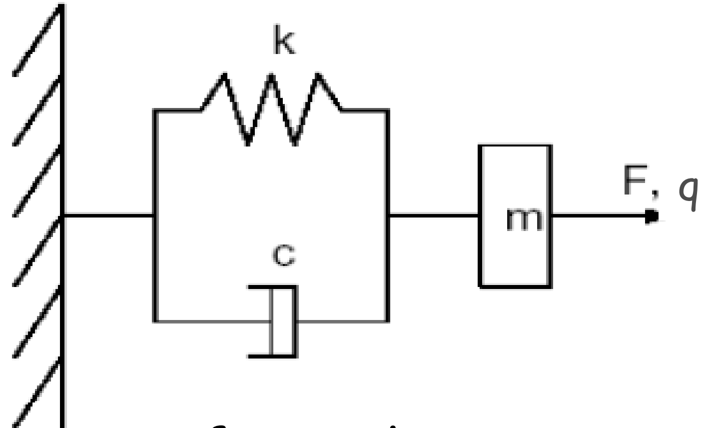
$\{Y(\omega)\}=[H(\omega)]\{U(\omega)\}$



MANY resonances

Bode plot : visualization of transfer function

Resonance (1 DOF oscillator)



Dynamic equation :

$$m\ddot{q}(t) + c\dot{q}(t) + kq(t) = F(t)$$

Harmonic excitation

$$F(t) = \mathrm{Re}\left(F(\omega)e^{i\omega t}\right)$$

Harmonic forced response

$$\mathrm{Re}\left(q(\omega)e^{i\omega t}\right))$$

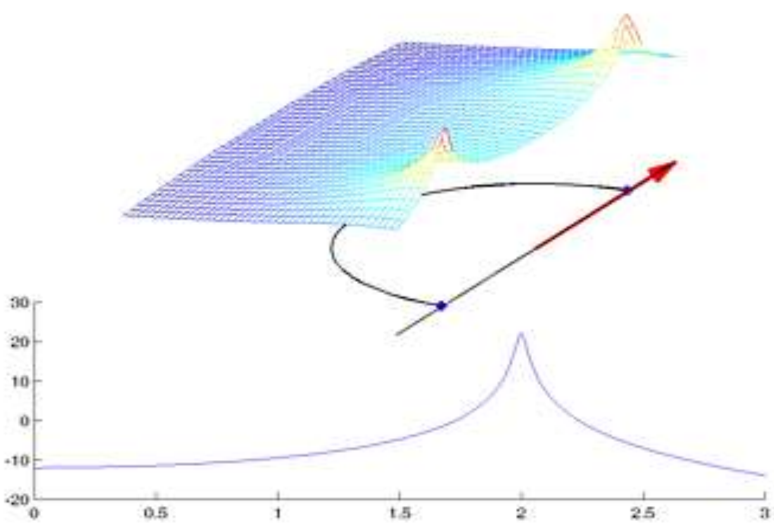
Dynamic equation :

$$\mathrm{Re}\left(\left(-\omega^2 m + i\omega c + k\right) q(\omega)e^{i\omega t} - F(\omega)e^{i\omega t}\right) = 0$$

Tranfer function :

$$H(\omega) = \frac{q(\omega)}{F(\omega)} = \frac{1}{-\omega^2 m + i\omega c + k}$$

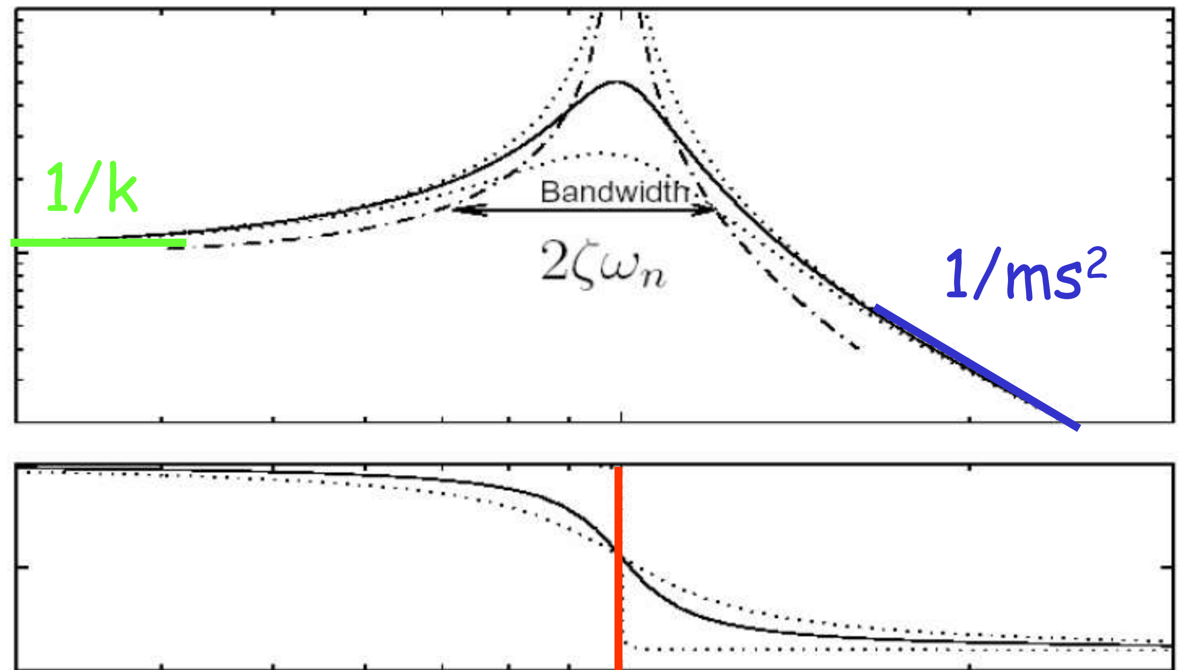
1 DOF (Bode plot)



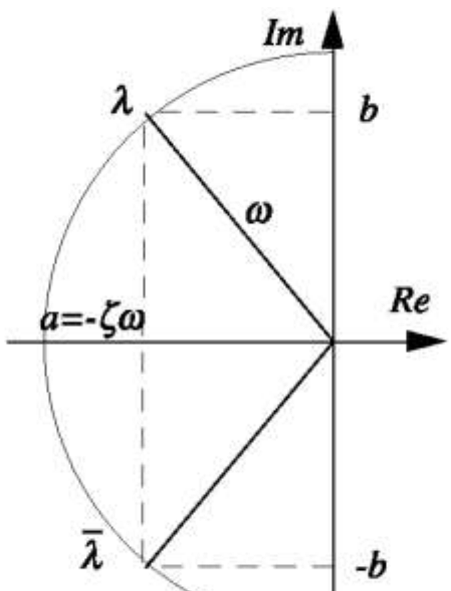
$$H(s) = \frac{1}{s^2 m + cs + k} = \frac{1}{m}\left(\frac{\beta}{s-\lambda} + \frac{\bar{\beta}}{s-\bar{\lambda}}\right)$$

Response at resonance

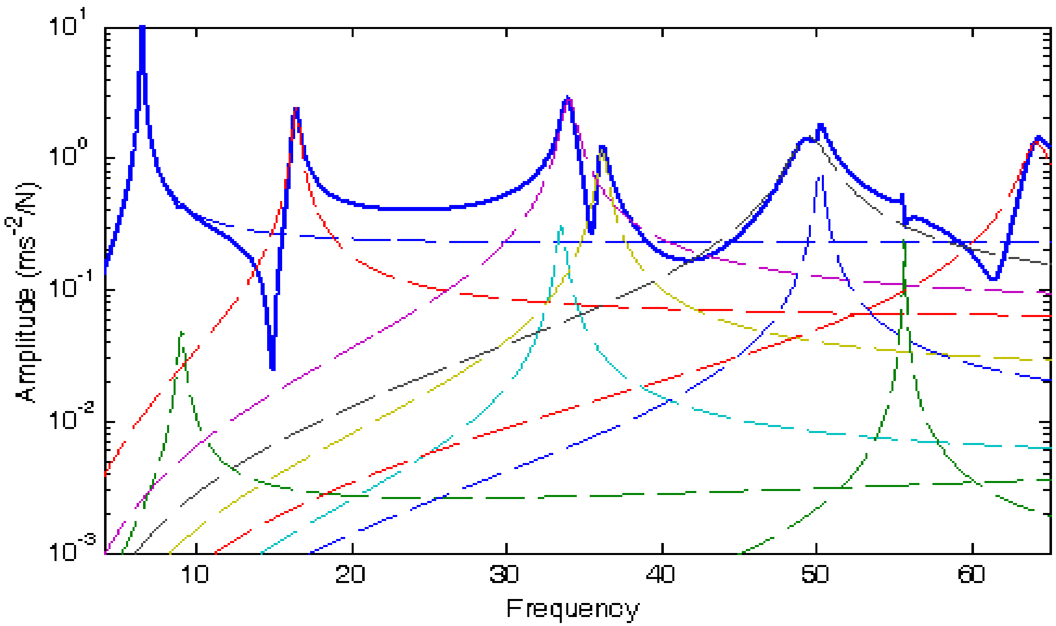
$H(w_n)=1/i2\zeta\omega_n^{2}$



$$\boxed{\omega_n = \sqrt{k/m},}$$



1 input, 1 output, many resonances



MDOF multiple degree of freedom
SISO single input single output

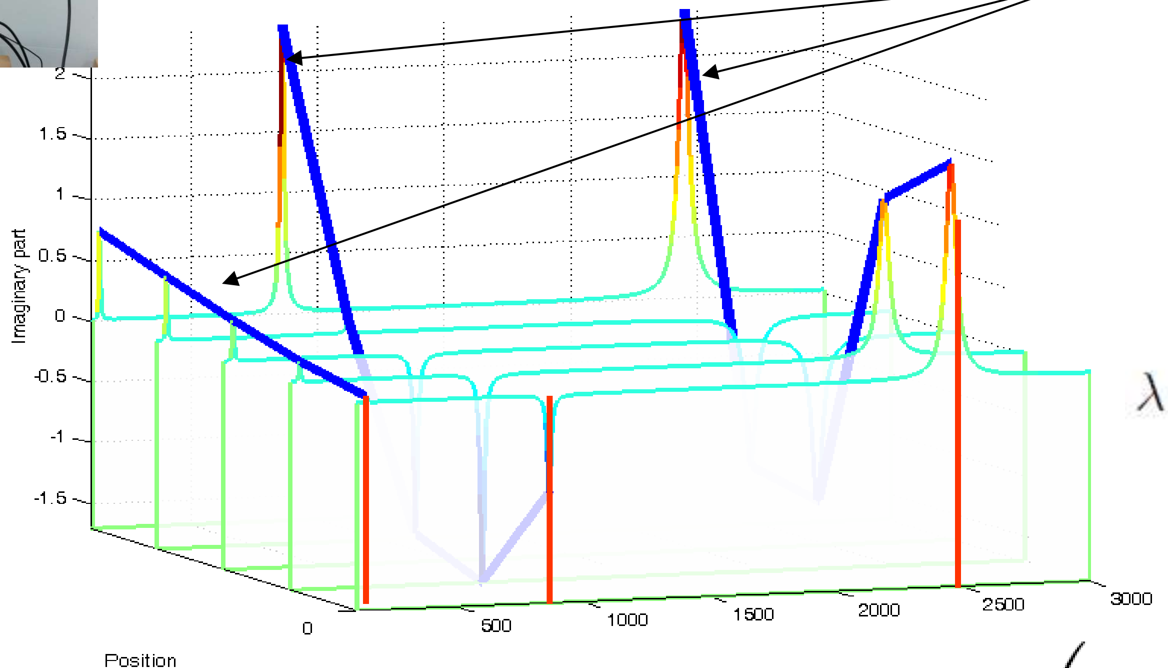
Spectral decomposition

MDOF (multiple resonances)

SISO Tj is 1x1

$$[\alpha(s)] = \sum_{\boxed{j \in \mathtt{identified}}} \left(\frac{\boxed{[T_j]}}{s^2 + 2\zeta_j \omega_j s + \omega_j^2} \right)$$

MDOF MIMO system



The shapes

The poles

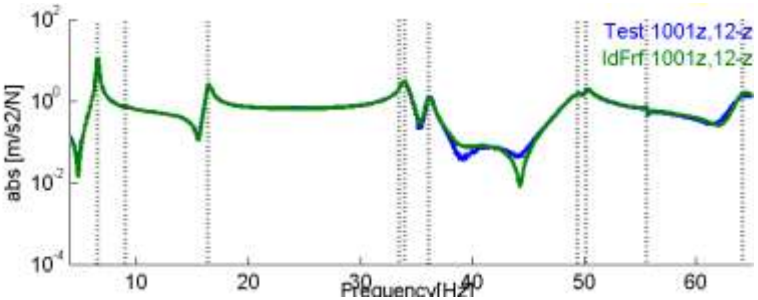
$$\lambda = -\zeta\omega_n \pm i\omega_n\sqrt{1-\zeta^2}$$

$$[\alpha(s)] = \sum_{\boxed{j\in\texttt{identified}}} \left(\frac{\boxed{[T_j]}}{s^2 + 2\zeta_j\omega_j s + \omega_j^2} \right)$$

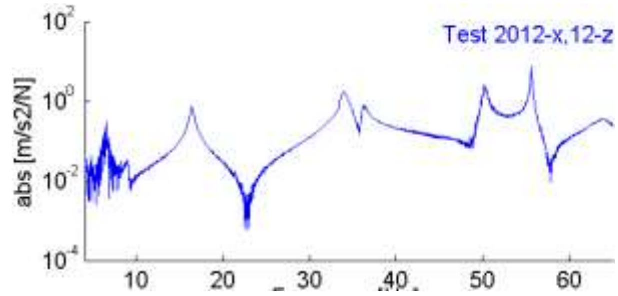
- Poles depend on the system (not the input/output)
- The shape is associated with the input/output locations

Identification

Objective H_{test}-H_{id}



Data

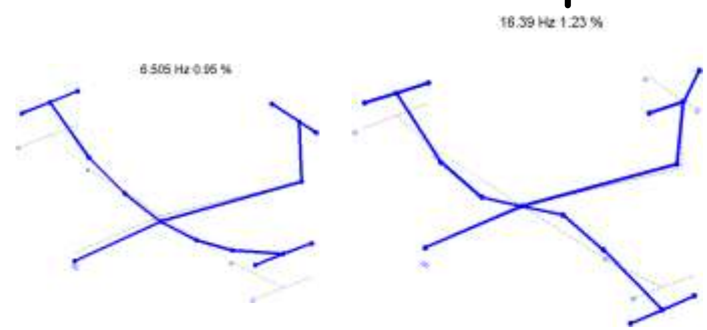


Optimization

Family of models

$$\sum_{j \in \texttt{identified}} \left(\frac{[T_j]}{s^2 + 2\zeta_j \omega_j s + \omega_j^2} \right)$$

Result : modes and poles



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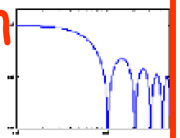
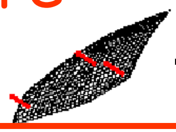
How are transfers predicted ?

When

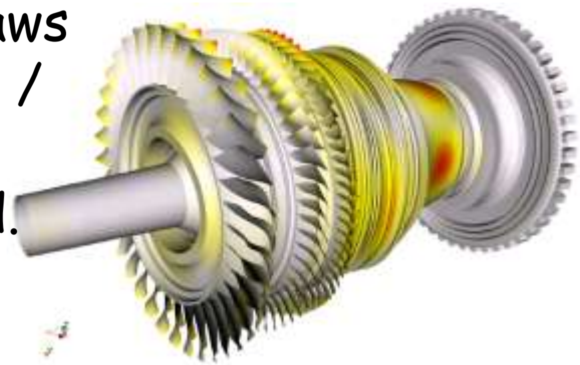


Where



System (FEM)

- Geometry
- Constitutive laws
- Discretization / idealization
- Boundary cond.



- DOFs {q}
- Mass M and stiffness K matrices

Outputs

$u(x,t)$
$\sigma(x,t)$

Inputs (when)
- Unbalance : harmonic at Ω
- Aerodynamic loads ($n\Omega$)
- Rotor/stator contact

Inputs (where)
- Point mass
- Distributed pressure
- Blade tip

Normal modes of elastic structure

- Nominal model (elastic + viscous damping)

$$\left[Ms^2 + Cs + K\right]\{q(s)\} = [b]\{u(s)\}$$
$$\{y(s)\} = [c]\{q(s)\}$$

- Conservative eigenvalue problem

$$-[M]\{\phi_j\}\omega_j^2 + [K]_{N\times N}\{\phi_j\}_{N\times 1} = \{0\}_{N\times 1}$$

- M>0 & K≥0 $\Rightarrow$ ϕ real
- Partial solvers exist

Normal modes of elastic structure

- Orthogonality

$$[\phi]^T[M][\phi] = \left[\diagdown \mu_j \diagdown\right]$$

$$[\phi]^T[K][\phi] = \left[\diagdown \mu_j \omega_j^2 \diagdown\right]$$

- Scaling conditions
  - Unit mass

$$\boxed{\{\phi_j\}^T[M]\{\phi_j\} = 1}$$

  - Unit amplitude

$$[c_s]\{\tilde{\phi}_j\} = 1 \quad \mu_j(c_s) = ([c_i]\{\phi_j\})^{-2}$$

- Principal coordinates

$$\left[[I]s^2 + \boxed{[\Gamma]s} + \left[\diagdown \omega_j^2 \diagdown\right]\right]\{p(s)\} = \left[\phi^T b\right]\{u(s)\}$$

$$\{y(s)\} = [c\phi]\{p(s)\}$$

Command and observation

$$\left[Ms^2 + Cs + K\right]\{q(s)\} = [b]\{u(s)\}$$
$$\{y(s)\} = [c]\{q(s)\}$$

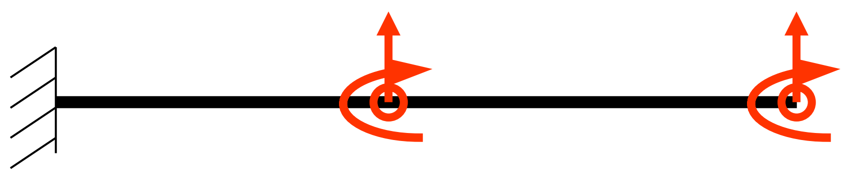
- Loads decomposed as spatially unit loads and inputs

  $\{F(t)\} = [b]\, \{u(t)\}$

- $\{y\}$ outputs are linearly related to DOFs $\{q\}$ using an observation equation

  $\{y(t)\} = [c]\, \{q(t)\}$

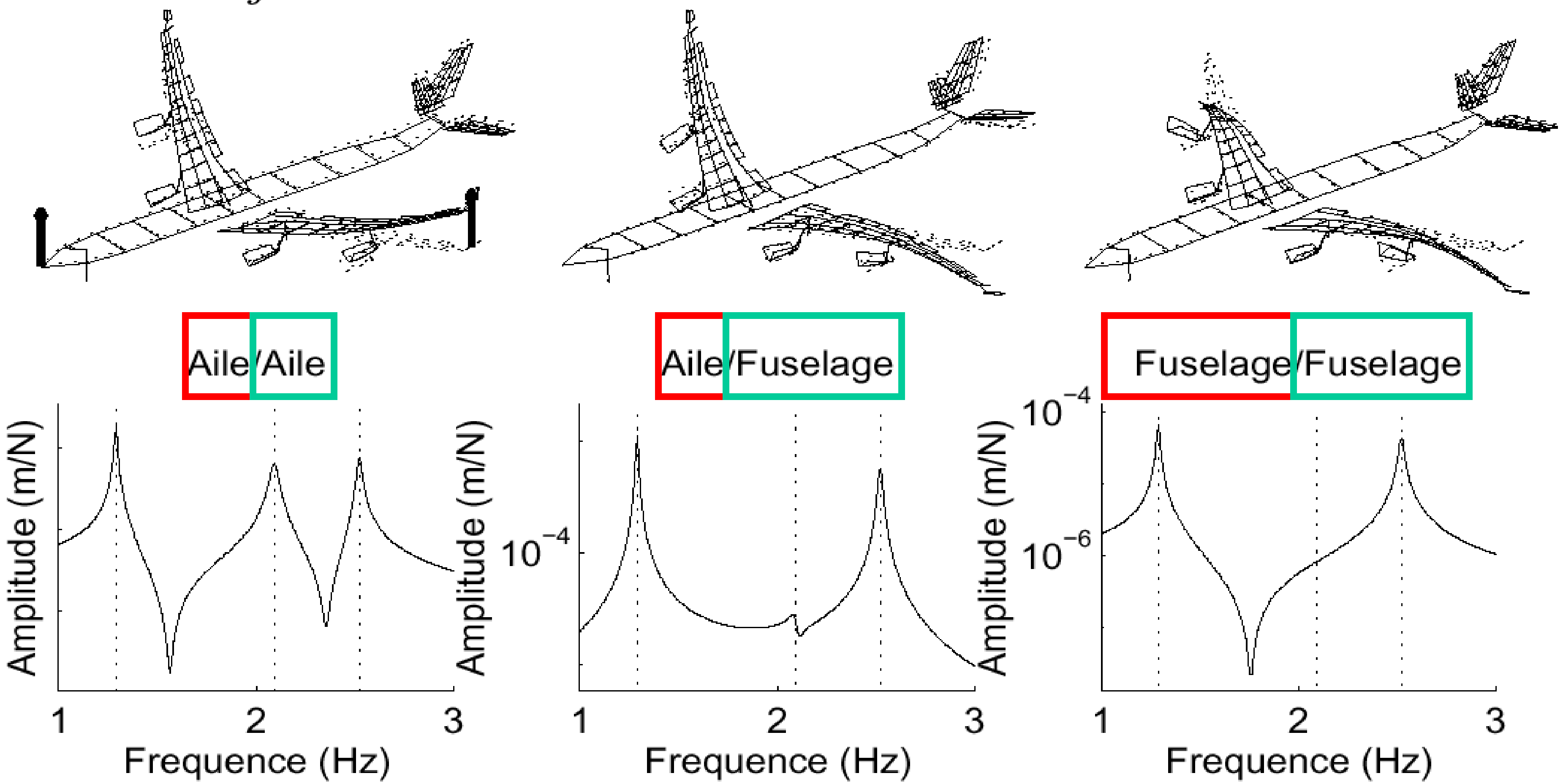
- Simple case : extraction $\{w_2\}=[0\ 0\ 1\ 0]\{q\}$



- More general : intermediate points, reactions, strains, stresses, …

Observability/controlability

$$H(s) = \sum_{j=1}^{N} \frac{\boxed{[c]}\{\phi_j\}\{\phi_j\}^T\boxed{[b]}}{s^2 + 2\zeta_j\omega_j s + \omega_j^2}$$



Modal damping assumption

- Assume Γ diagonal

$$[\Gamma] = [\phi^T C \phi] = [\backslash \, 2\boxed{\zeta_j}\omega_j \, \backslash]$$

Damping ratio ζ measured or design parameter

Pure metal 0.05 %, assembled structure ≈ 1%

Full car ≈ 2-4%, with soil radiation up to 10 %

- Leads to second order spectral decomposition

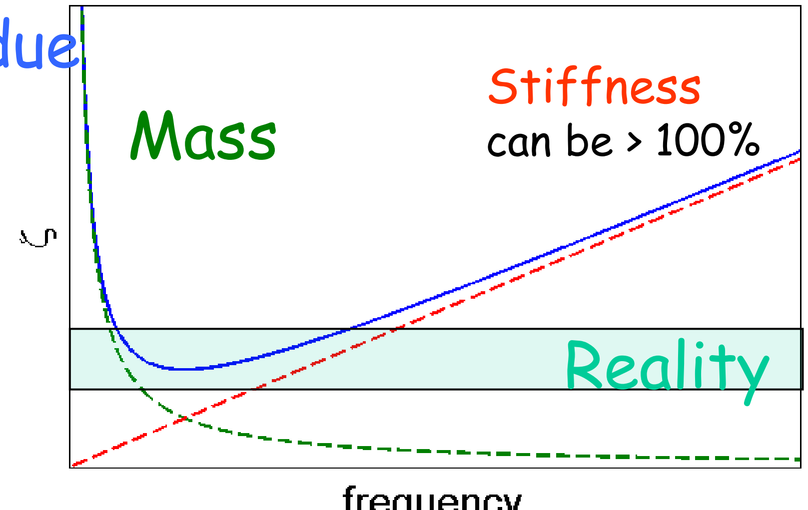
$$H(s) = \sum_{j=1}^{N} \frac{\boxed{[c]\{\phi_j\}}\boxed{\{\phi_j\}^T [b]}}{s^2 + 2\zeta_j\omega_j s + \omega_j^2} = \sum_{j=1}^{N} \frac{\boxed{[T_j]}}{s^2 + 2\zeta_j\omega_j s + \omega_j^2}$$

Mode shape Participation factor Residue

- Rayleigh
  - Physical domain
  - Modal domain

$$C = \alpha[M] + \beta[K]$$

$$\boxed{\zeta_j} = \boxed{\frac{\alpha}{2}\frac{1}{\omega_j}} + \boxed{\frac{\beta}{2}\omega_j}$$



TP ultérieurs (éléments)

Acoustic finite elements

- Unknown pressure, gradient = particle velocity

$$\begin{Bmatrix} p,x \\ p,y \\ p,z \end{Bmatrix} = \begin{bmatrix} N,x \\ N,y \\ N,z \end{bmatrix} \{\ p\ \}$$

- Mass and stiffness matrices

$$M_{ij} = \int_\Omega \frac{1}{\rho_0 C^2} \{N_i\}\{N_j\} \quad K_{ij} = \int_\Omega \frac{1}{\rho_0} \{N_{i,k}\}\{N_{j,k}\}$$

- "Load" ≈ fluid source

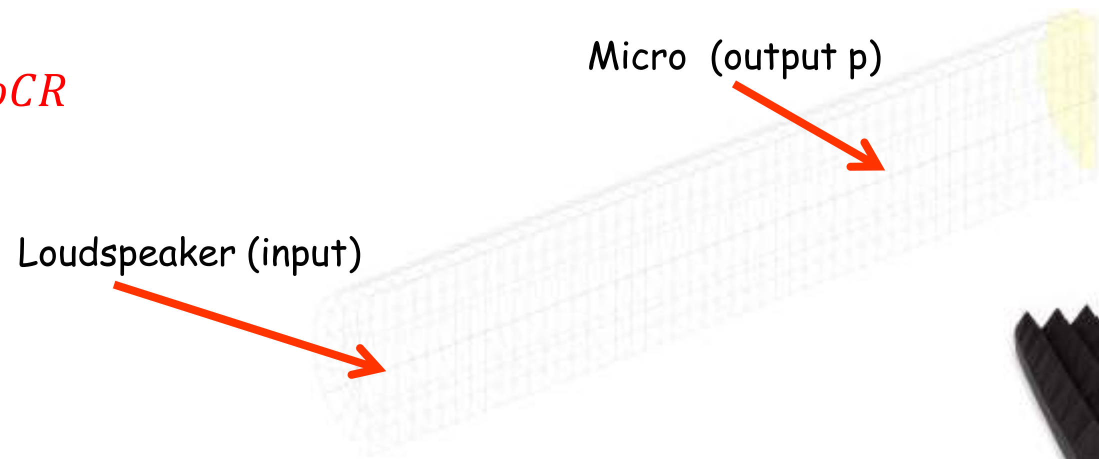
$$B_i = \int_{\partial\Omega} \{N_i\}\{V_e\}$$

- Wall impedance $Z = \rho C R$ ≈ viscous damping

$$C_{ij} = \int_{\partial\Omega_Z^e} \frac{1}{Z} \{N_i\}\{N_j\}$$

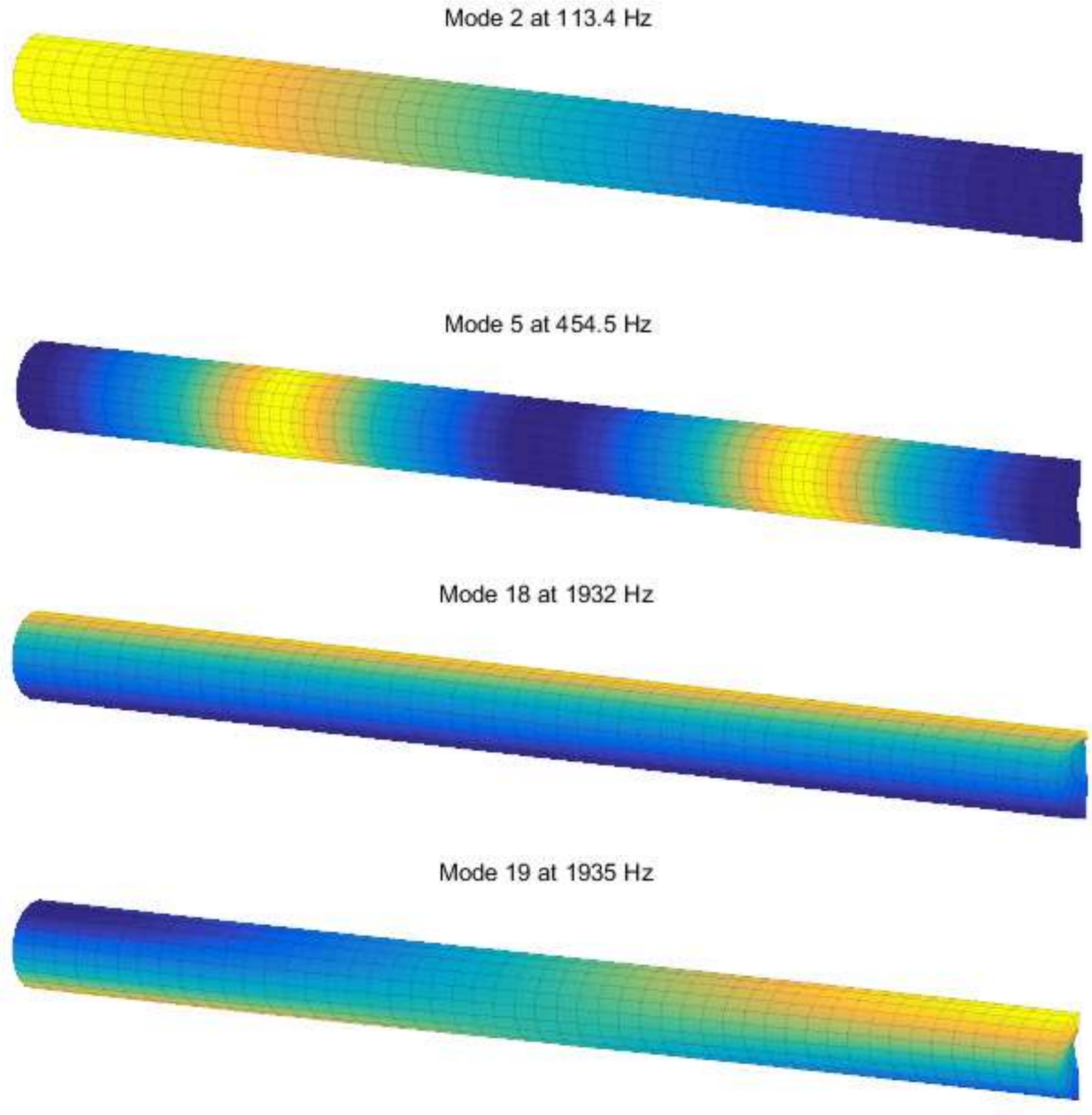
Kundt tube experiment

- Identify impedance $Z = \rho C R$
- Verify sound velocity C



Elastic modes of closed tube

$$[K - \omega_j^2 M]\{\phi_j\} = 0$$

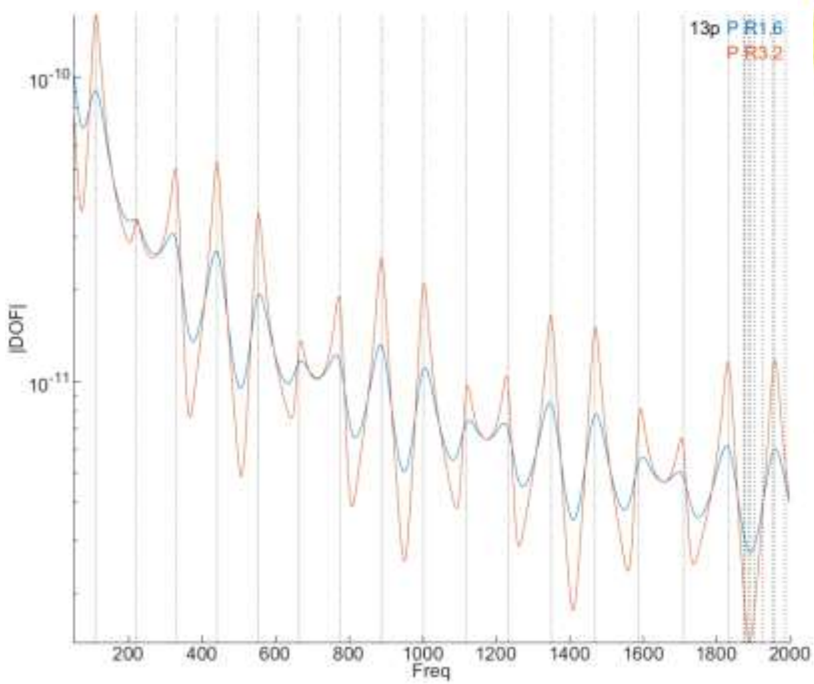


Forced response with wall impedance

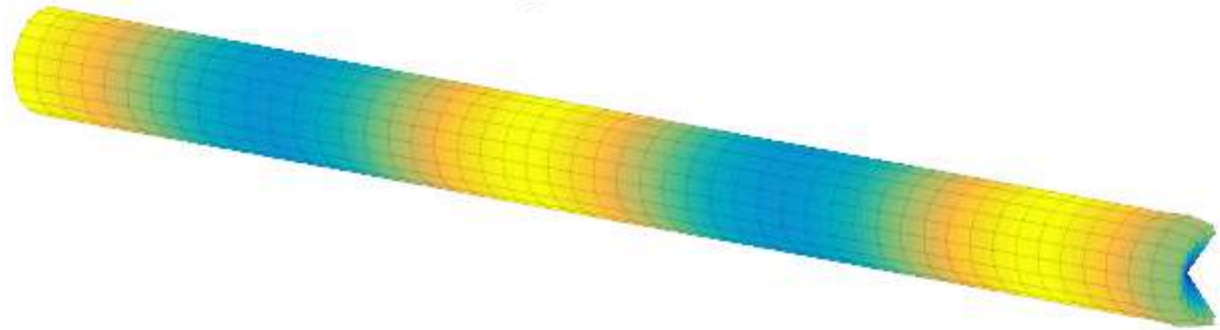
$$\{p(\omega)\} = \left[-\omega_j^2 M + i\omega C_Z + K\right]^{-1}\{b\}$$

View space

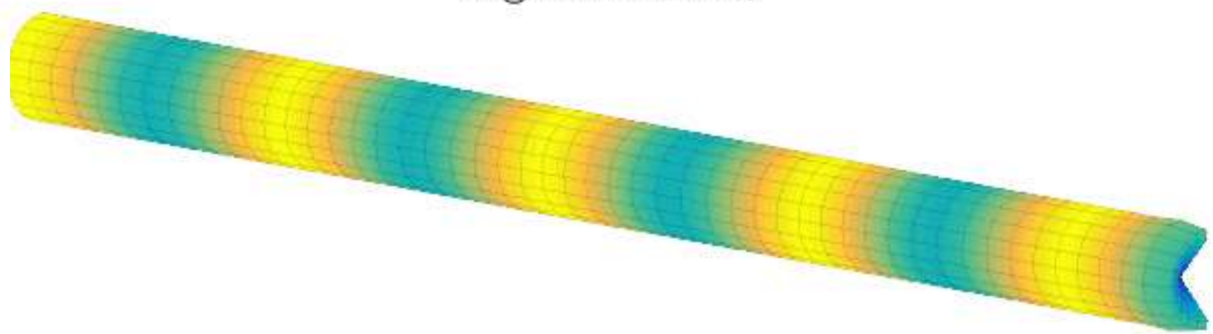
View frequency



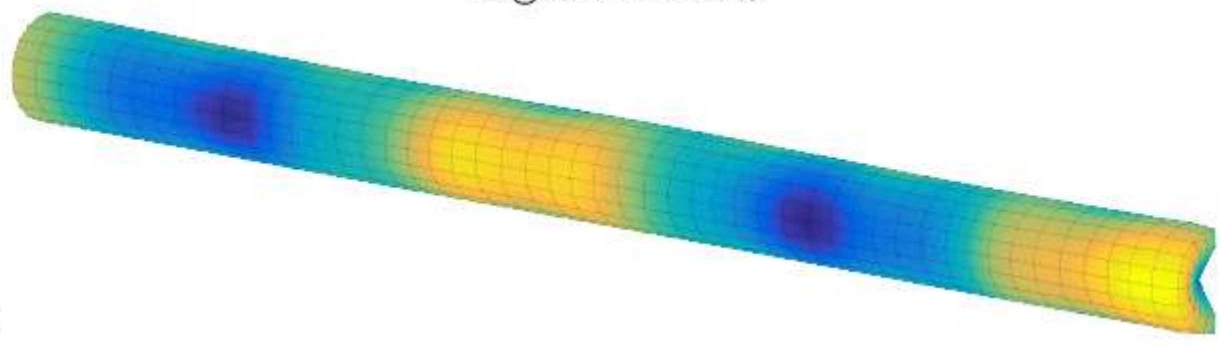
24 @ 503 Hz 100.00 %



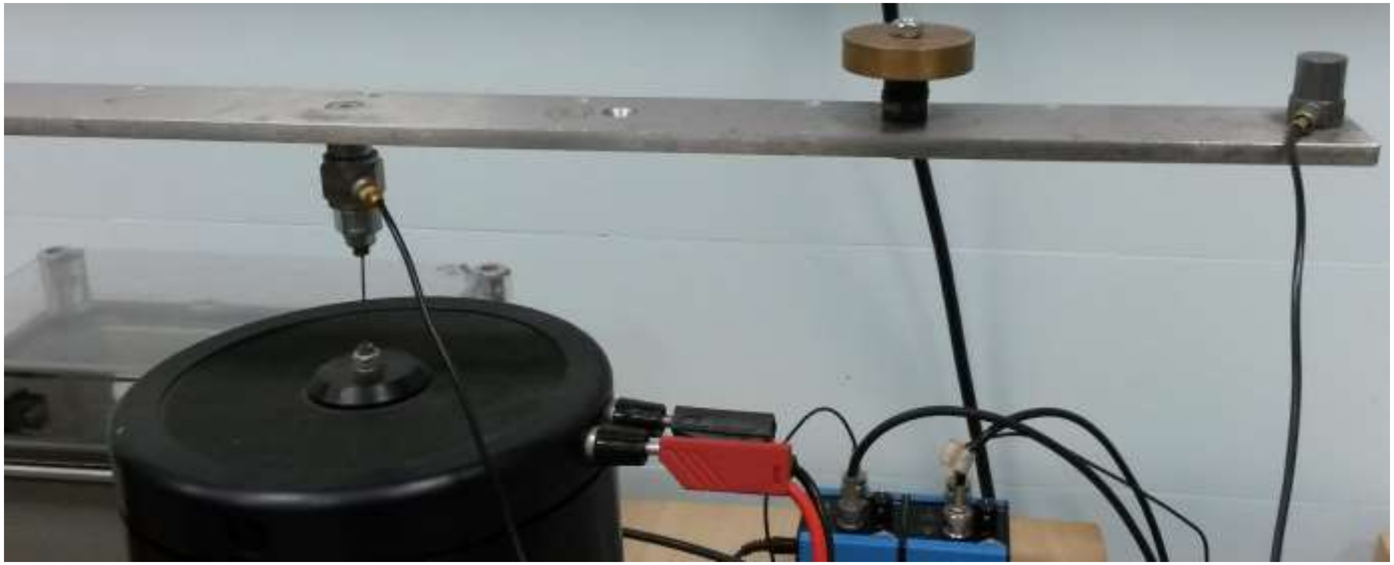
49 @ 995.5 Hz 100.00 %



96 @ 1921 Hz 100.00 %

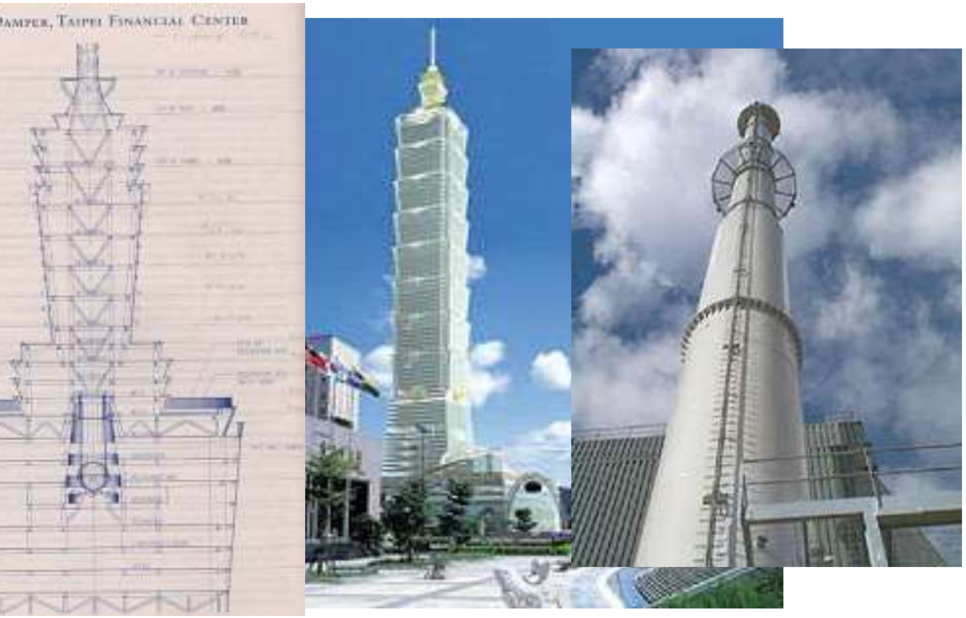


Beam vibration

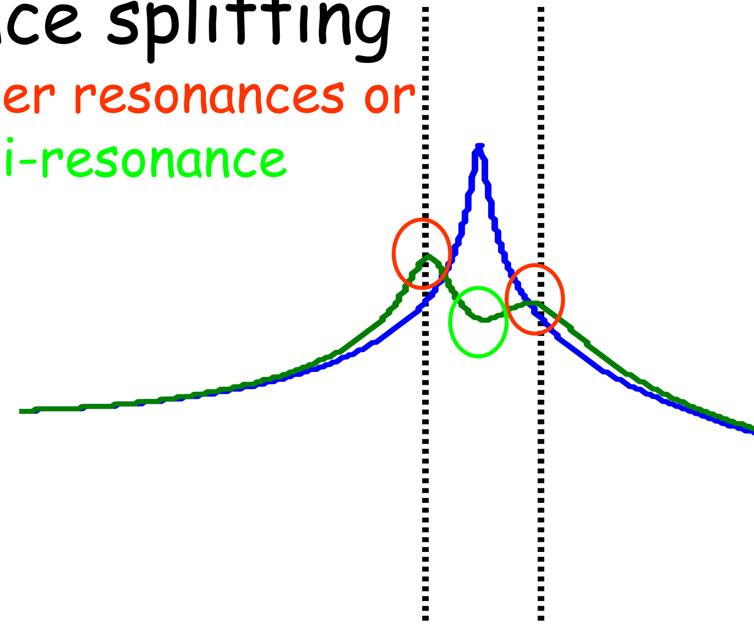


- Transfer measurement
- Find resonances, see influence of shape
- Analyze damping

Tuned mass dampers / vibration absorber



- Resonance splitting
  - Two lower resonances or
  - One anti-resonance



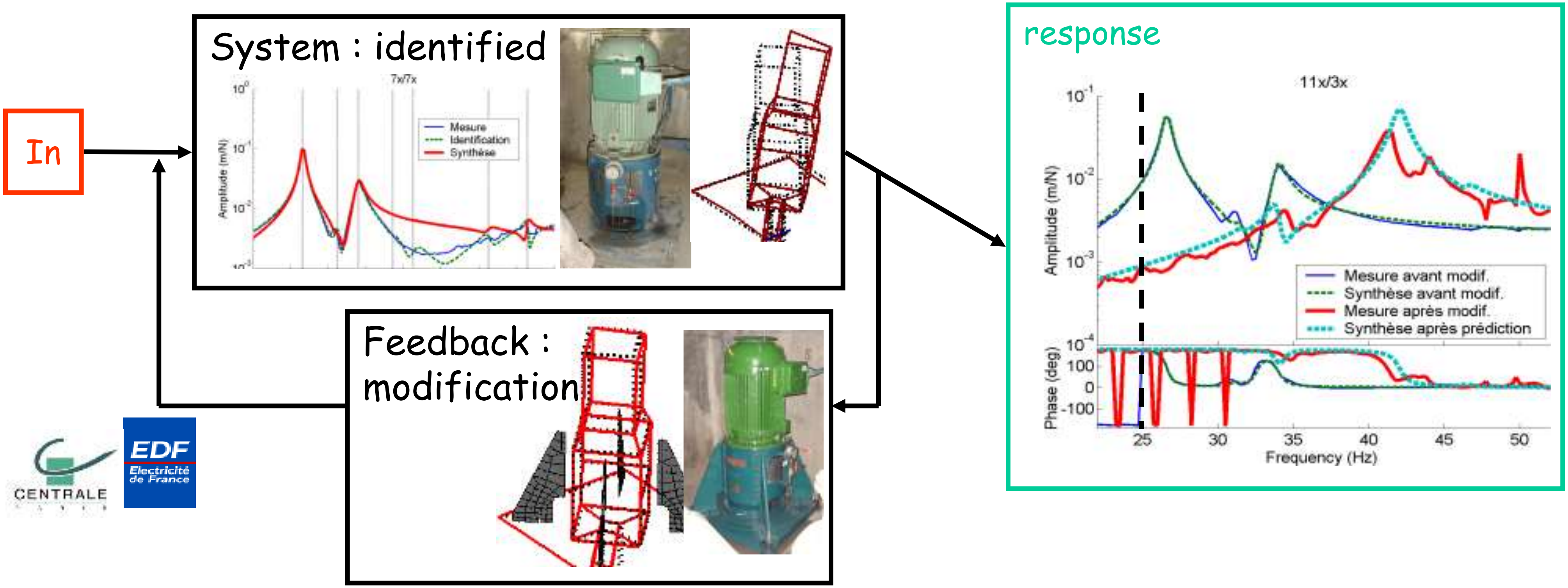
- Countless applications : helicopters, buildings, lamp posts, cars, …
- Current trend : non-linear absorber (self tuning)

Sample dampers

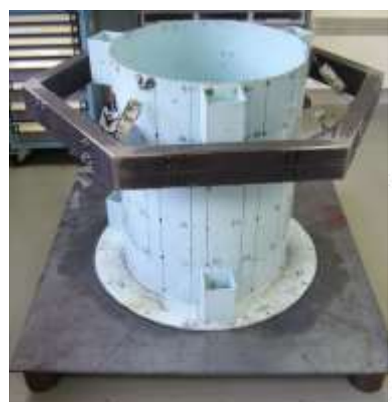
- Fluid sloshing : No moving parts
- Rings, cables, …

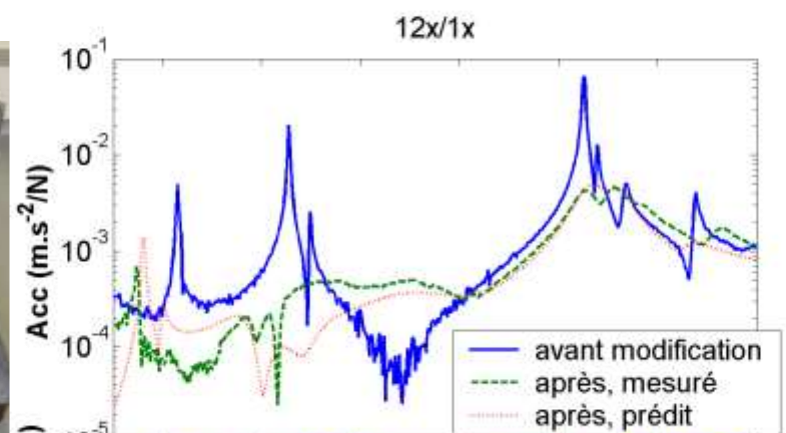
www.multitech-fr.com

Example : structural dynamics modification



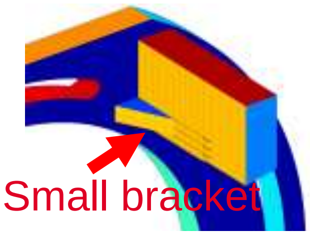
Coincidence problem
Modification : mass, stiffness or damping modifications

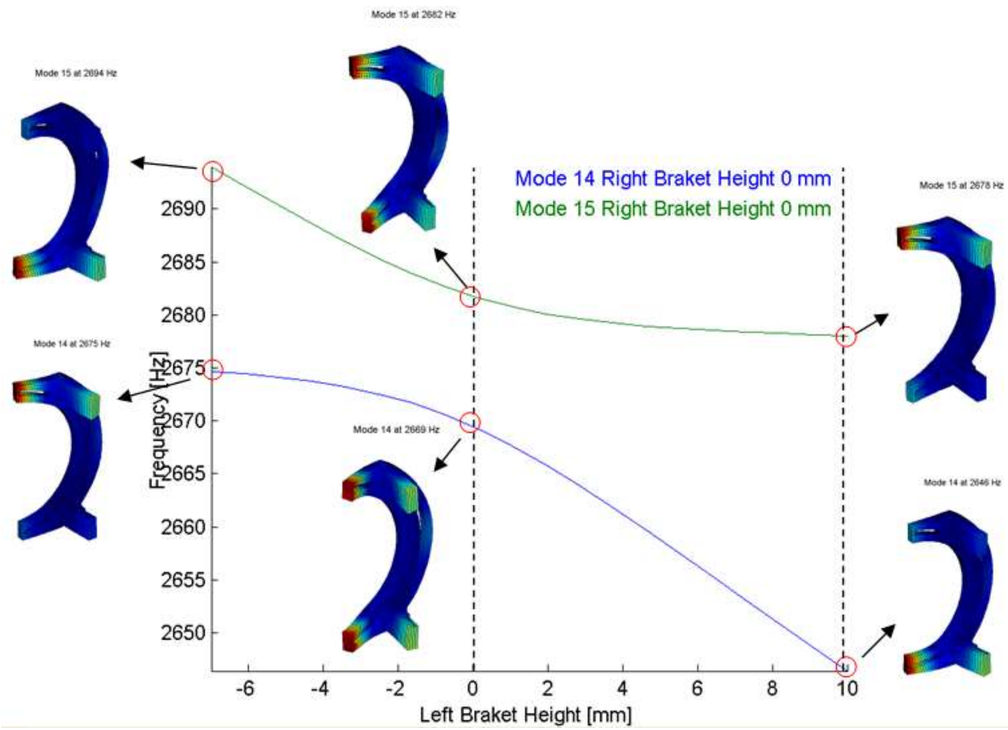




Mode crossing

- High sensitivity for close modes associated with mode crossing





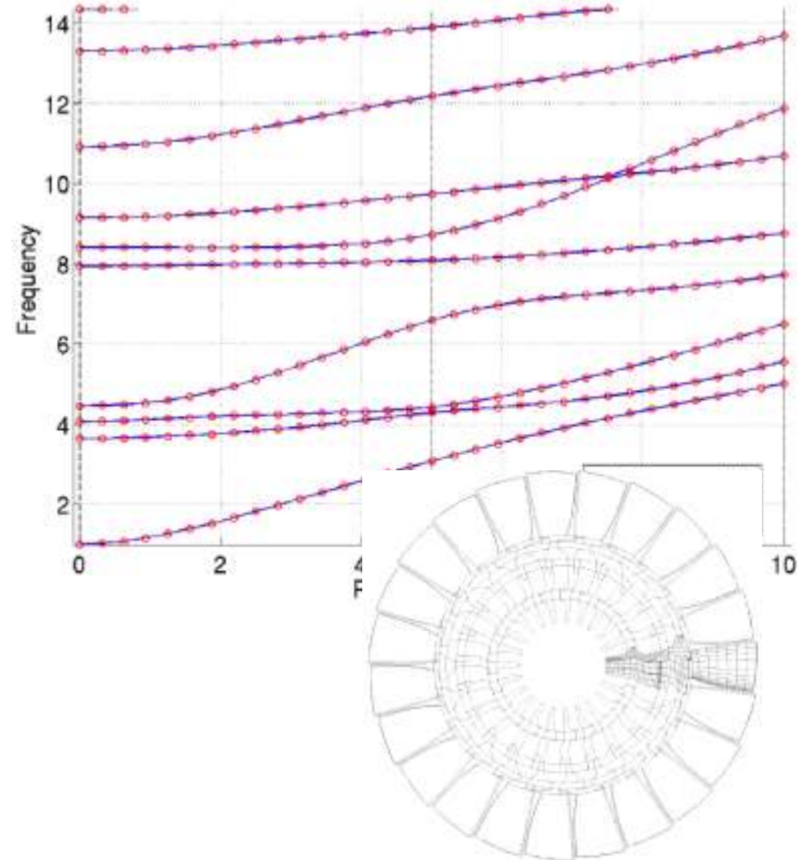
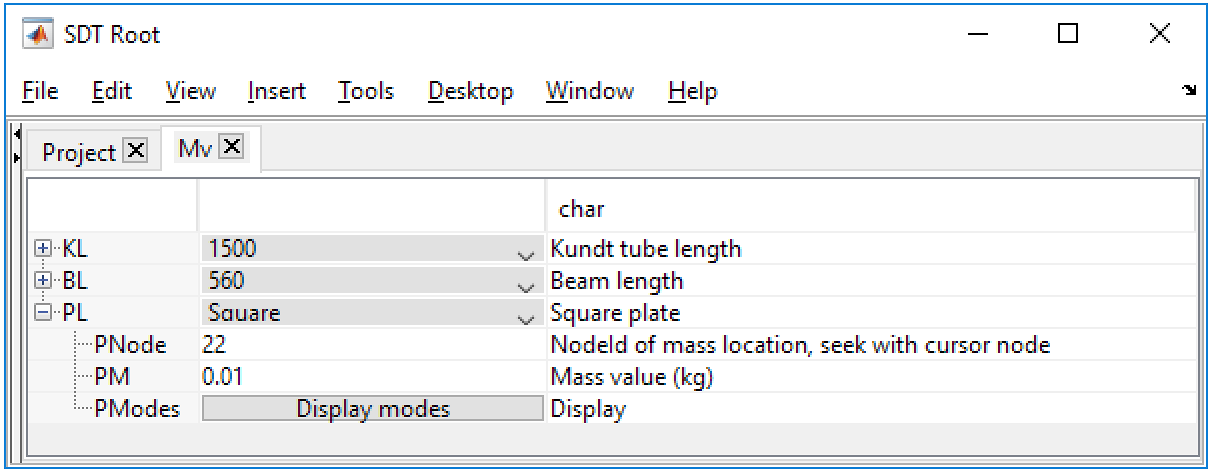


Plate /structural modification

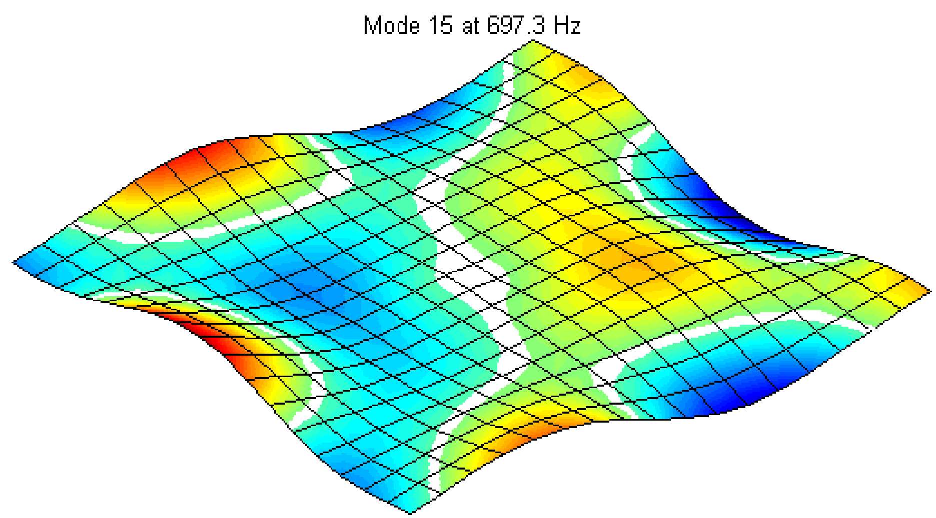


mevib16('initmv') % Initialize

PNode : right click "cursor node" to find node number

PM : Adjust mass value (kg)

```
mevib16('SetMv', ...
   struct('PNode',9,'PM',.001,'PModes','do'));
```

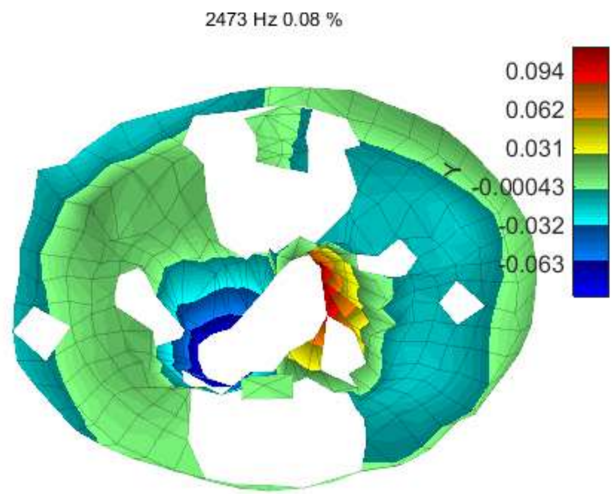


Course outline

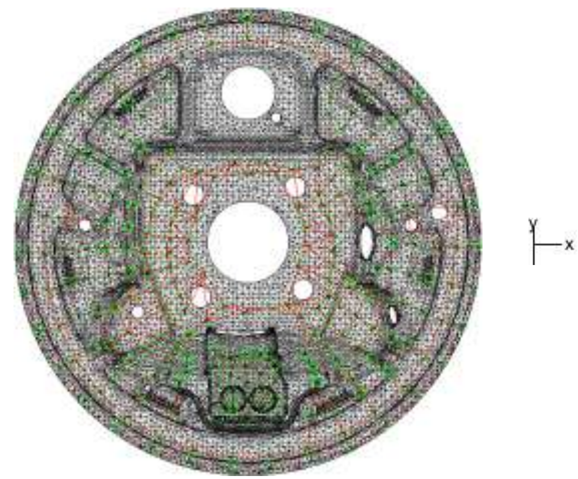
- Introduction
- How are modes measured
  - Mode ≈ resonance ≈ 1 DOF (degree of freedom) system
  - Transfer (series of modal contributions)
- How are modes predicted
  - Modes, inputs, outputs, damping
- Test / analysis correlation
  - Identification
  - Topology correlation
  - MAC / Updating
- Vibration design / conclusion

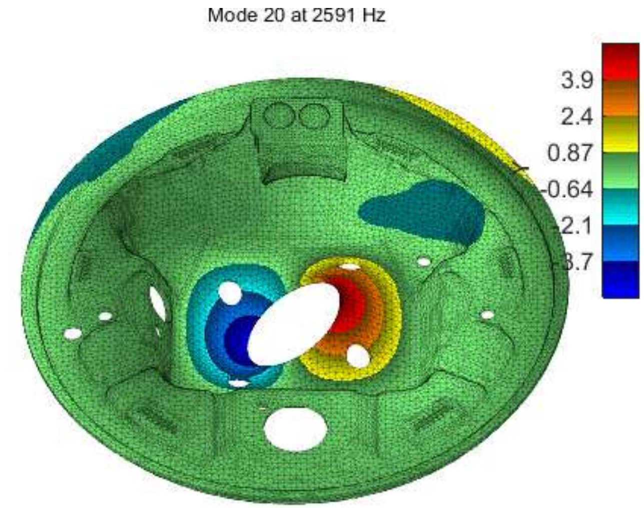
Copie : X:\Enseignants\balmes\Mevib\Ensam_Modal_S2.pdf

Comparing test & FEM



Identification
known @ sensors



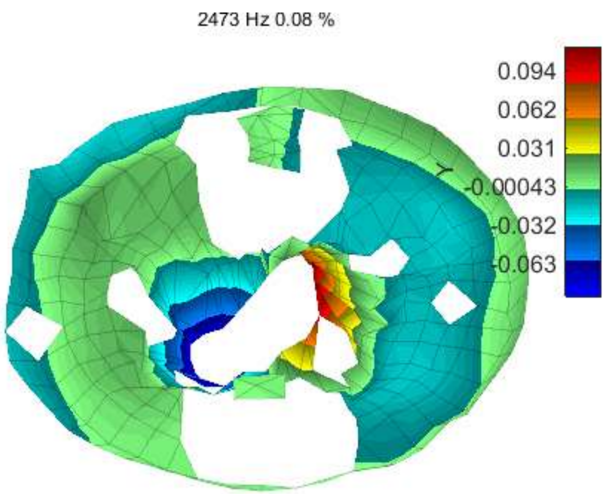


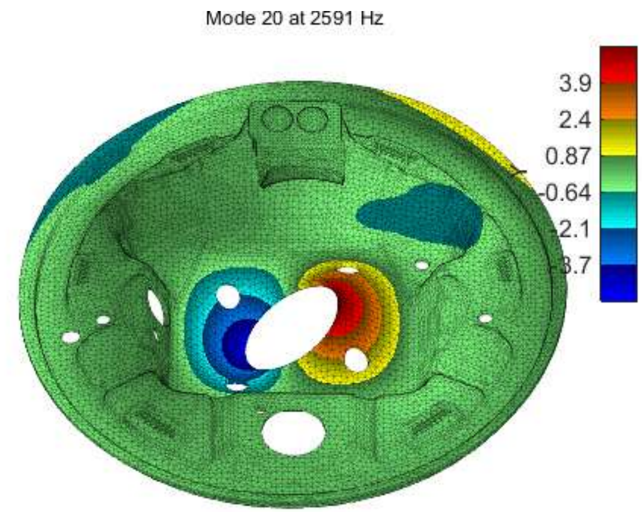
FEM known @ nodes

Topology correlation
= observe FEM @ sensors

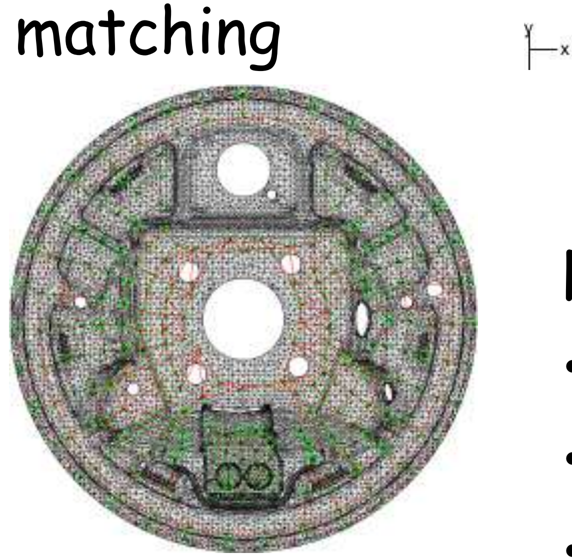
$\{y(t)\} = [c] \{q(t)\}$

Where is the error?





Topology errors
- sensor/act position
- matching

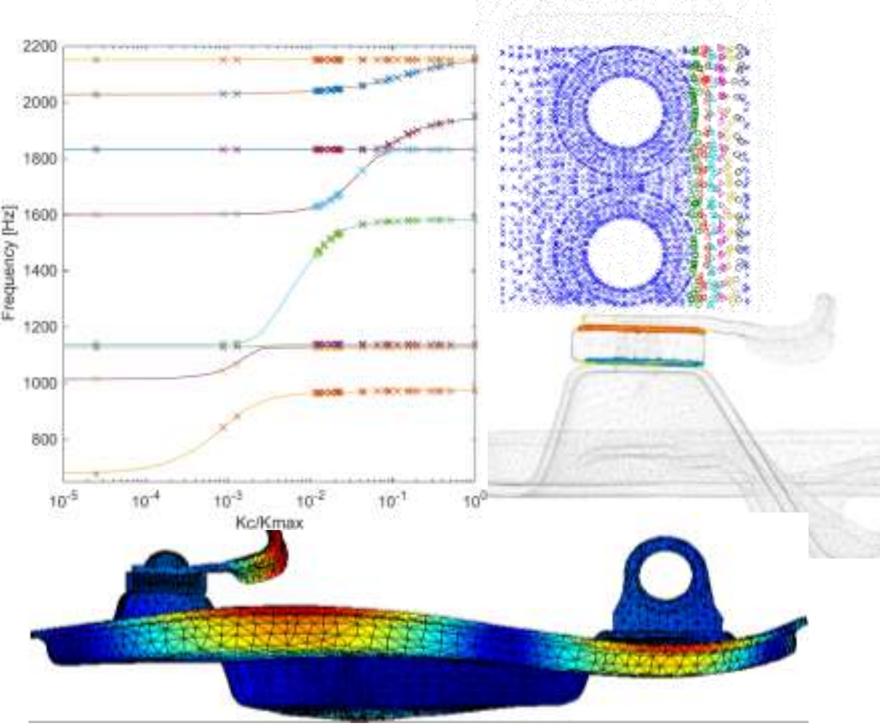


Identification error
- Noisy measurements
- Identification bias
- NL, time varying, …

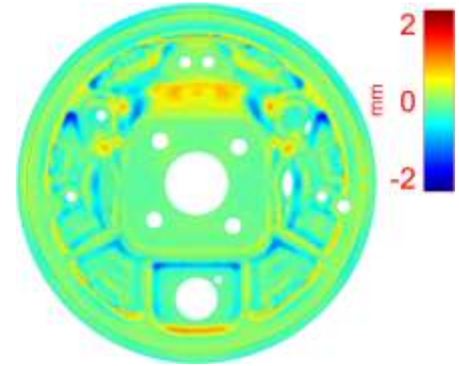
FEM error
- Geometry
- Material parameters
- Contact properties

Parametrization

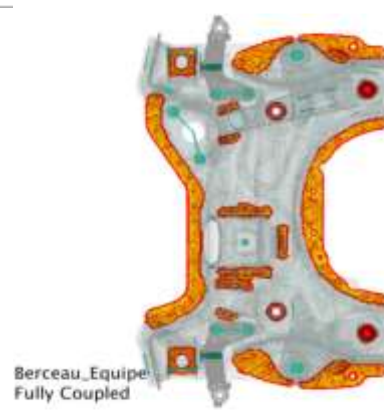
- Variable contact surface, contact, sliding

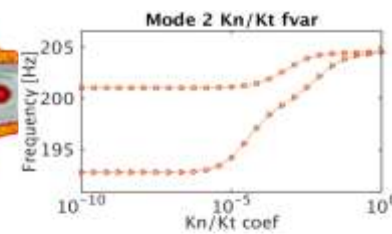


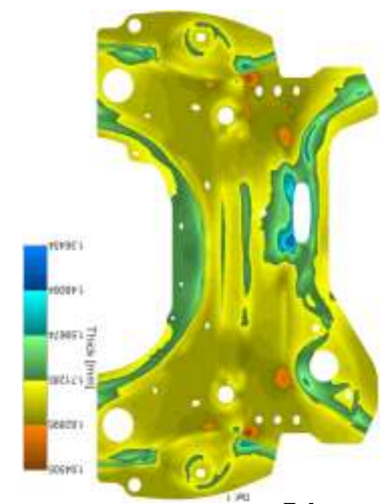
Chassis Brakes International
Eurobrake 2014



Variable geometry

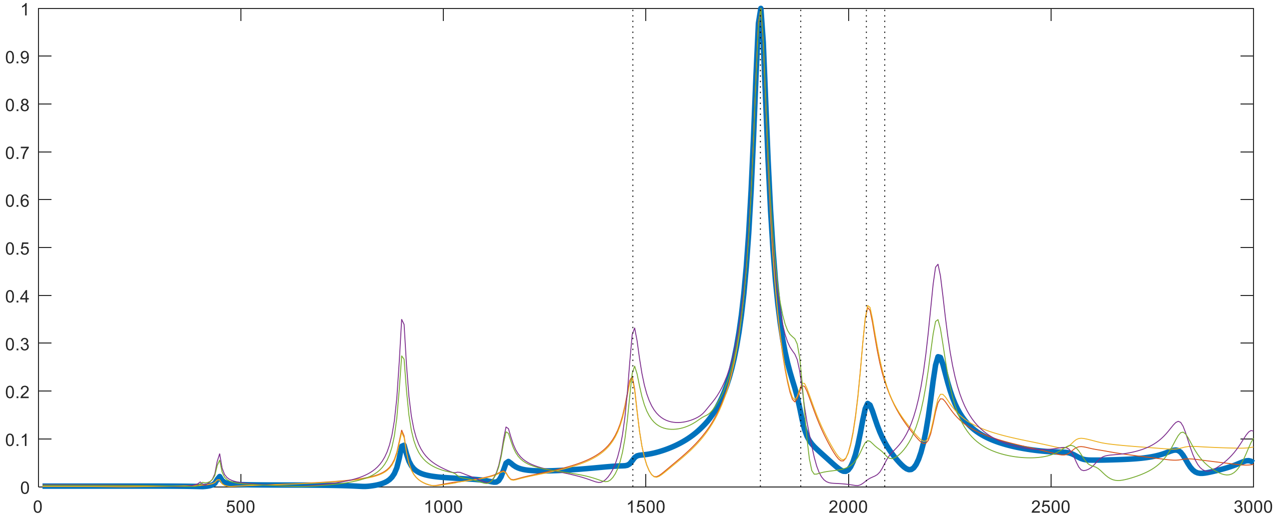






Balmes, ISMA 2016

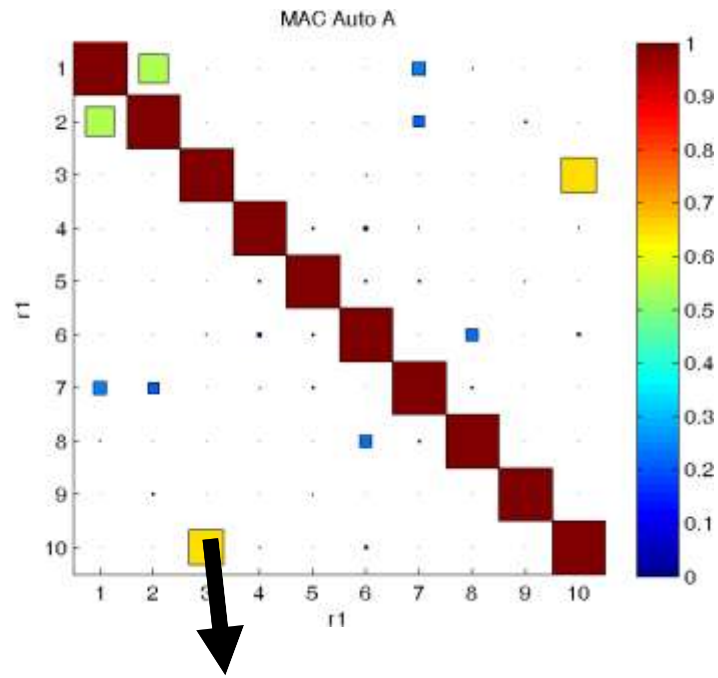
Plateau frein / filtre modal



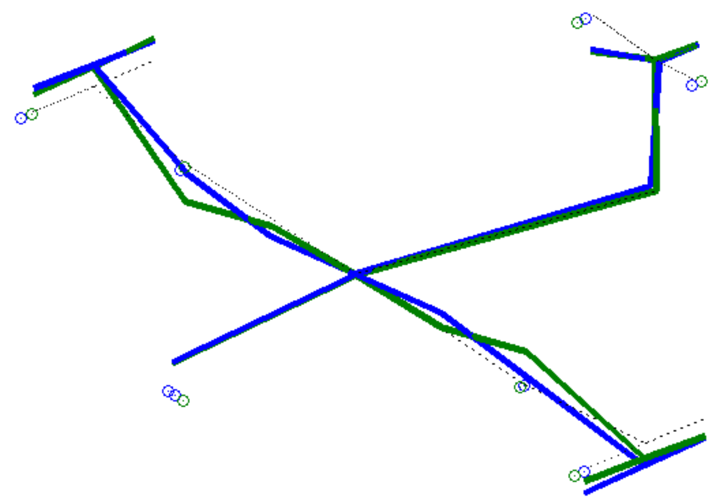
MAC : comparing shapes

Shapes are compared through
correlation coefficient
(Modal Assurance Criterion)

$$\mathrm{MAC}(U,V) = \frac{|\{U\}^H \ \{V\}|^2}{|\{U\}^H \{U\}||\{V\}^H \{V\}|}$$



16.39 Hz 1.23 %, 64.16 Hz 1.22 %



Next step : modal updating (*recalage*) = use correlation to correct model parameters

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- TP ultérieurs

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