Opérateurs Différentiels en Coordonnées Cartésiennes

$$\underline{\textbf{grad}}\, f = \frac{\partial f}{\partial x_1}\underline{e}_1 + \frac{\partial f}{\partial x_2}\underline{e}_2 + \frac{\partial f}{\partial x_3}\underline{e}_3$$

$$\underline{\underline{\textbf{grad}}}\, \underline{U} = \begin{pmatrix} \frac{\partial u_1}{\partial x_1} & \frac{\partial u_1}{\partial x_2} & \frac{\partial u_1}{\partial x_3} \\ \frac{\partial u_2}{\partial x_1} & \frac{\partial u_2}{\partial x_2} & \frac{\partial u_2}{\partial x_3} \\ \frac{\partial u_3}{\partial x_1} & \frac{\partial u_3}{\partial x_2} & \frac{\partial u_3}{\partial x_3} \end{pmatrix}$$

$$\textbf{div}\underline{U} = \frac{\partial u_1}{\partial x_1} + \frac{\partial u_2}{\partial x_2} + \frac{\partial u_3}{\partial x_3}$$

$$\boldsymbol{\Delta} f = \frac{\partial^2 f}{\partial x_1^2} + \frac{\partial^2 f}{\partial x_2^2} + \frac{\partial^2 f}{\partial x_3^2}$$

$$\underline{\textbf{rot}}\, \underline{U} = \left(\frac{\partial u_3}{\partial x_2} - \frac{\partial u_2}{\partial x_3}\right)\underline{e}_1 + \left(\frac{\partial u_1}{\partial x_3} - \frac{\partial u_3}{\partial x_1}\right)\underline{e}_2 + \left(\frac{\partial u_2}{\partial x_1} - \frac{\partial u_1}{\partial x_2}\right)\underline{e}_3$$

$$\underline{\boldsymbol{\Delta}}\, \underline{U} = \boldsymbol{\Delta} u_1 \underline{e}_1 + \boldsymbol{\Delta} u_2 \underline{e}_2 + \boldsymbol{\Delta} u_3 \underline{e}_3$$

$$\underline{\textbf{div}}\, \underline{\underline{\boldsymbol{\sigma}}} = \left(\frac{\partial \sigma_{11}}{\partial x_1} + \frac{\partial \sigma_{12}}{\partial x_2} + \frac{\partial \sigma_{13}}{\partial x_3}\right)\underline{e}_1 + \left(\frac{\partial \sigma_{21}}{\partial x_1} + \frac{\partial \sigma_{22}}{\partial x_2} + \frac{\partial \sigma_{23}}{\partial x_3}\right)\underline{e}_2 + \left(\frac{\partial \sigma_{31}}{\partial x_1} + \frac{\partial \sigma_{32}}{\partial x_2} + \frac{\partial \sigma_{33}}{\partial x_3}\right)\underline{e}_3$$

Opérateurs Différentiels en Coordonnées Cylindriques

$$\underline{\textbf{grad}}\, f = \frac{\partial f}{\partial r}\underline{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\underline{e}_\theta + \frac{\partial f}{\partial z}\underline{e}_z$$

$$\underline{\underline{\textbf{grad}}}\, \underline{U} = \begin{pmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) & \frac{\partial u_r}{\partial z} \\ \frac{\partial u_\theta}{\partial r} & \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) & \frac{\partial u_\theta}{\partial z} \\ \frac{\partial u_z}{\partial r} & \frac{1}{r}\frac{\partial u_z}{\partial \theta} & \frac{\partial u_z}{\partial z} \end{pmatrix}$$

$$\textbf{div}\, \underline{U} = \frac{\partial u_r}{\partial r} + \frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{\partial u_z}{\partial z}$$

$$\boldsymbol{\Delta} f = \frac{\partial^2 f}{\partial r^2} + + \frac{1}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{\partial^2 f}{\partial z^2}$$

$$\underline{\textbf{rot}}\, \underline{U} = \left(\frac{1}{r}\frac{\partial u_z}{\partial \theta} - \frac{\partial u_\theta}{\partial z}\right)\underline{e}_r + \left(\frac{\partial u_r}{\partial z} - \frac{\partial u_z}{\partial r}\right)\underline{e}_\theta + \left(\frac{\partial u_\theta}{\partial r} + \frac{u_\theta}{r} - \frac{1}{r}\frac{\partial u_r}{\partial \theta}\right)\underline{e}_z$$

$$\underline{\boldsymbol{\Delta}}\, \underline{U} = \left(\boldsymbol{\Delta} u_r - \frac{2}{r^2}\frac{\partial u_\theta}{\partial \theta} - \frac{u_r}{r^2}\right)\underline{e}_r + \left(\boldsymbol{\Delta} u_\theta + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2}\right)\underline{e}_\theta + \boldsymbol{\Delta} u_z \underline{e}_z$$

$$\begin{aligned} \underline{\textbf{div}}\, \underline{\underline{\sigma}} = & \left(\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{\partial \sigma_{rz}}{\partial z} + \frac{\sigma_{rr} - \sigma_{\theta\theta}}{r}\right)\underline{e}_r + \\ & \left(\frac{\partial \sigma_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{\partial \sigma_{\theta z}}{\partial z} + 2\frac{\sigma_{r\theta}}{r}\right)\underline{e}_\theta + \\ & \left(\frac{\partial \sigma_{zr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{z\theta}}{\partial \theta} + \frac{\partial \sigma_{zz}}{\partial z} + \frac{\sigma_{zr}}{r}\right)\underline{e}_z \end{aligned}$$

Opérateurs Différentiels en Coordonnées Sphériques

$$\underline{\mathbf{grad}}\, f = \frac{\partial f}{\partial r}\underline{e}_r + \frac{1}{r}\frac{\partial f}{\partial \theta}\underline{e}_\theta + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \varphi}\underline{e}_\varphi$$

$$\underline{\underline{\mathbf{grad}}}\, \underline{U} = \begin{pmatrix} \frac{\partial u_r}{\partial r} & \frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right) & \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial u_r}{\partial \varphi} - u_\varphi\right) \\ \frac{\partial u_\theta}{\partial r} & \frac{1}{r}\left(\frac{\partial u_\theta}{\partial \theta} + u_r\right) & \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial \varphi} - \frac{u\varphi}{\tan\theta}\right) \\ \frac{\partial u_\varphi}{\partial r} & \frac{1}{r}\frac{\partial u_\varphi}{\partial \theta} & \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\theta}{\tan\theta} + u_r\right) \end{pmatrix}$$

$$\mathbf{div}\, \underline{U} = \frac{\partial u_r}{\partial r} + 2\frac{u_r}{r} + \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial u_\varphi}{\partial \varphi} + \frac{u_\theta}{r\tan\theta}$$

$$\boldsymbol{\Delta} f = \frac{\partial^2 f}{\partial r^2} + + \frac{2}{r}\frac{\partial f}{\partial r} + \frac{1}{r^2}\frac{\partial^2 f}{\partial \theta^2} + \frac{1}{r^2\tan\theta}\frac{\partial f}{\partial \theta} + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \varphi^2}$$

$$\begin{aligned} \underline{\mathbf{rot}}\, \underline{U} = & \left(\frac{1}{r}\frac{\partial u_\varphi}{\partial \theta} - \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial \varphi} - \frac{u\varphi}{\tan\theta}\right)\right)\underline{e}_r + \\ & \left(\frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial u_r}{\partial \varphi} - u_\varphi\right) - \frac{\partial u_\varphi}{\partial r}\right)\underline{e}_\theta + \\ & \left(\frac{\partial u_\theta}{\partial r} - \frac{1}{r}\left(\frac{\partial u_r}{\partial \theta} - u_\theta\right)\right)\underline{e}_\varphi \end{aligned}$$

$$\begin{aligned} \underline{\boldsymbol{\Delta}}\, \underline{U} = & \left(\boldsymbol{\Delta} u_r - 2\frac{u_r}{r^2} - \frac{2}{r^2\sin\theta}\frac{\partial(u_\theta\sin\theta)}{\partial \theta} - \frac{2}{r^2\sin\theta}\frac{\partial u_\varphi}{\partial \varphi}\right)\underline{e}_r + \\ & \left(\boldsymbol{\Delta} u_\theta + \frac{2}{r^2}\frac{\partial u_r}{\partial \theta} - \frac{u_\theta}{r^2\sin^2\theta} - \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\varphi}{\partial \varphi}\right)\underline{e}_\theta + \\ & \left(\boldsymbol{\Delta} u_\varphi + \frac{2}{r^2\sin^2\theta}\frac{\partial u_r}{\partial \varphi} + \frac{2\cos\theta}{r^2\sin^2\theta}\frac{\partial u_\theta}{\partial \varphi} - \frac{u_\varphi}{r^2\sin^2\theta}\right)\underline{e}_\varphi \end{aligned}$$

$$\begin{aligned} \underline{\mathbf{div}}\, \underline{\underline{\sigma}} = & \left(\frac{\partial \sigma_{rr}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{r\theta}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{r\varphi}}{\partial \varphi} + \frac{1}{r}\left(2\sigma_{rr} - \sigma_{\theta\theta} - \sigma_{\varphi\varphi} + \frac{\sigma_{r\theta}}{\tan\theta}\right)\right)\underline{e}_r + \\ & \left(\frac{\partial \sigma_{\theta r}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\theta\theta}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\theta\varphi}}{\partial \varphi} + \frac{1}{r\tan\theta}\left(\sigma_{\theta\theta} - \sigma_{\varphi\varphi}\right) + \frac{3\sigma_{r\theta}}{r}\right)\underline{e}_\theta + \\ & \left(\frac{\partial \sigma_{\varphi r}}{\partial r} + \frac{1}{r}\frac{\partial \sigma_{\varphi\theta}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial \sigma_{\varphi\varphi}}{\partial \varphi} + \frac{1}{r}\left(3\sigma_{r\varphi} + 2\frac{\sigma_{\theta\varphi}}{\tan\theta}\right)\right)\underline{e}_\varphi \end{aligned}$$