

After a Laplace transformation, the system (3.36) becomes

$$\frac{d^2 \theta}{dz^2} = \frac{p}{a} \theta \quad (3.38)$$

where p is the Laplace parameter. The solution of this equation is

$$\theta = K_1 \sinh(\alpha z) + K_2 \cosh(\alpha z) \quad \text{with } \alpha^2 = p/a \quad (3.39)$$

The flux Φ at abscissa z is associated with the temperature T at the same location:

$$\Phi = -\lambda S \frac{\partial T}{\partial z} \quad (3.40)$$

where λ is the thermal conductivity of the wall and S is the area of the plane isothermal surface that is considered for the z -transfer. After a Laplace transformation of this equation, and taking into account equation (3.39), we obtain

$$\phi = -\lambda S \frac{d\theta}{dz} = -\lambda S \alpha (K_1 \cosh(\alpha z) + K_2 \sinh(\alpha z)) \quad (3.41)$$

Finally, the constants K_1 and K_2 can be easily eliminated from equations (3.39) and (3.41), written for $z = 0$ and for $z = e$, to provide the following input ($z = 0$)/output ($z = e$) equations:

$$\begin{cases} \theta_{\text{in}} = \cosh(\alpha e) \theta_{\text{out}} + \frac{1}{\lambda S \alpha} \sinh(\alpha e) \phi_{\text{out}} \\ \phi_{\text{in}} = \lambda S \alpha \sinh(\alpha e) \theta_{\text{out}} + \cosh(\alpha e) \phi_{\text{out}} \end{cases} \quad (3.42)$$

We can give the above pair of equations in the following form:

$$\begin{bmatrix} \theta_{\text{in}} \\ \phi_{\text{in}} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \theta_{\text{out}} \\ \phi_{\text{out}} \end{bmatrix} \quad \text{with} \quad \begin{cases} A = D = \cosh(\alpha e) \\ B = \frac{1}{\lambda S \alpha} \sinh(\alpha e) \\ C = \lambda S \alpha \sinh(\alpha e) \end{cases} \quad (3.43)$$

Remarks regarding the symmetry of the system and the unit determinant of a quadrupole matrix

The reader should note that two relations exist between the coefficients of the quadrupole matrix:

$$A = D \quad (\text{symmetrical system}) \quad (3.44)$$

$$AD - BC = 1 \quad (3.45)$$