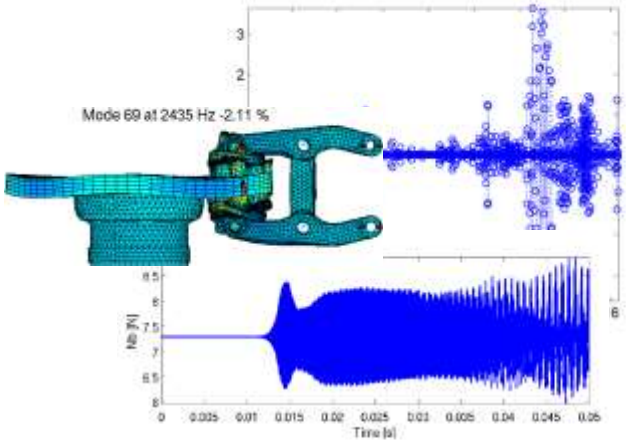
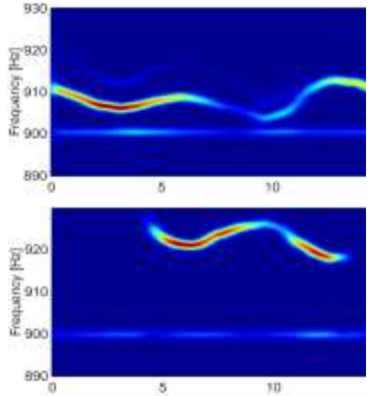
Simulations numériques pour la dynamique
Réduction de modèle

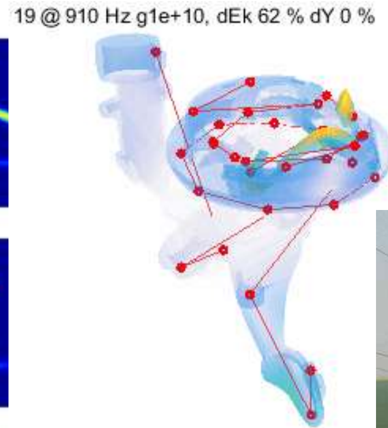
Etienne Balmes Ensam/PIMM, SDTools

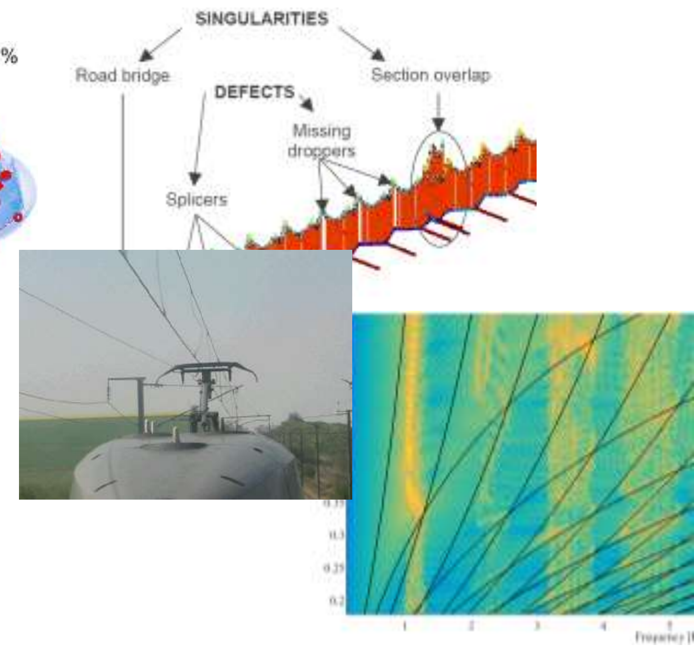
https://savoir.ensam.eu/moodle/course/search.php?search=1874

A few activities

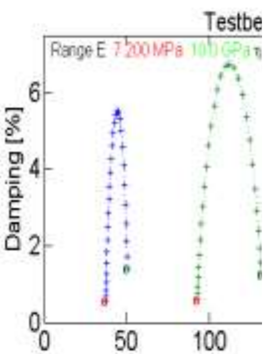


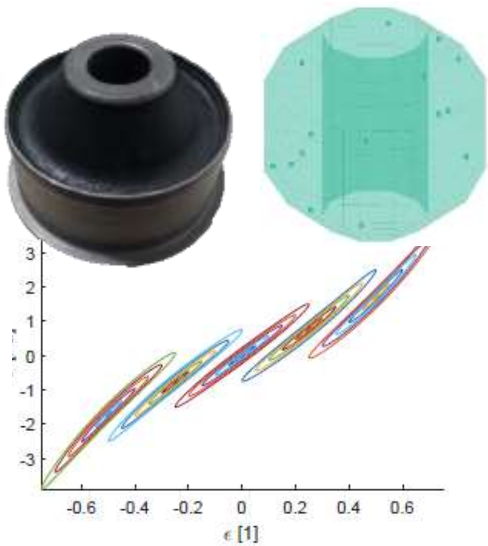


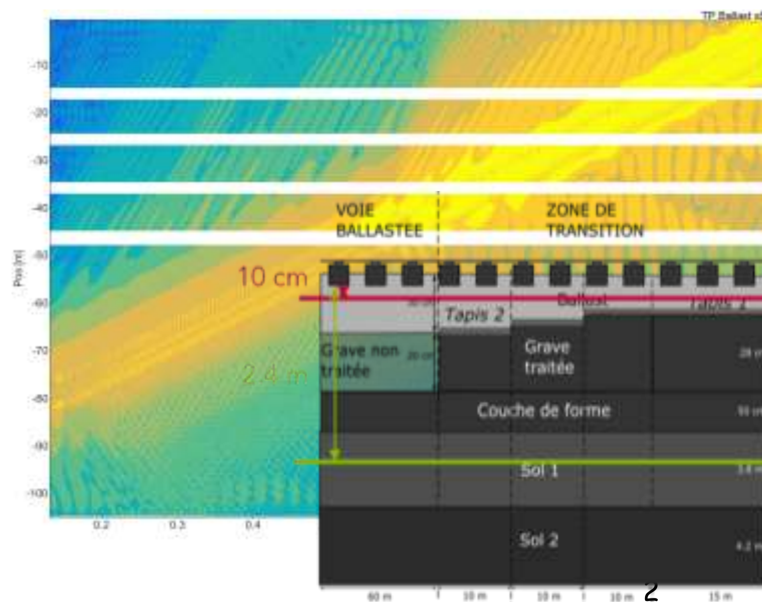




a) G. Vermot des Roches, b) G. Martin, c) J.P. Bianchi
d) F. Conejos, e) R. Penas, f) H. Pinault







Why does SDTools exist ?

Simulation

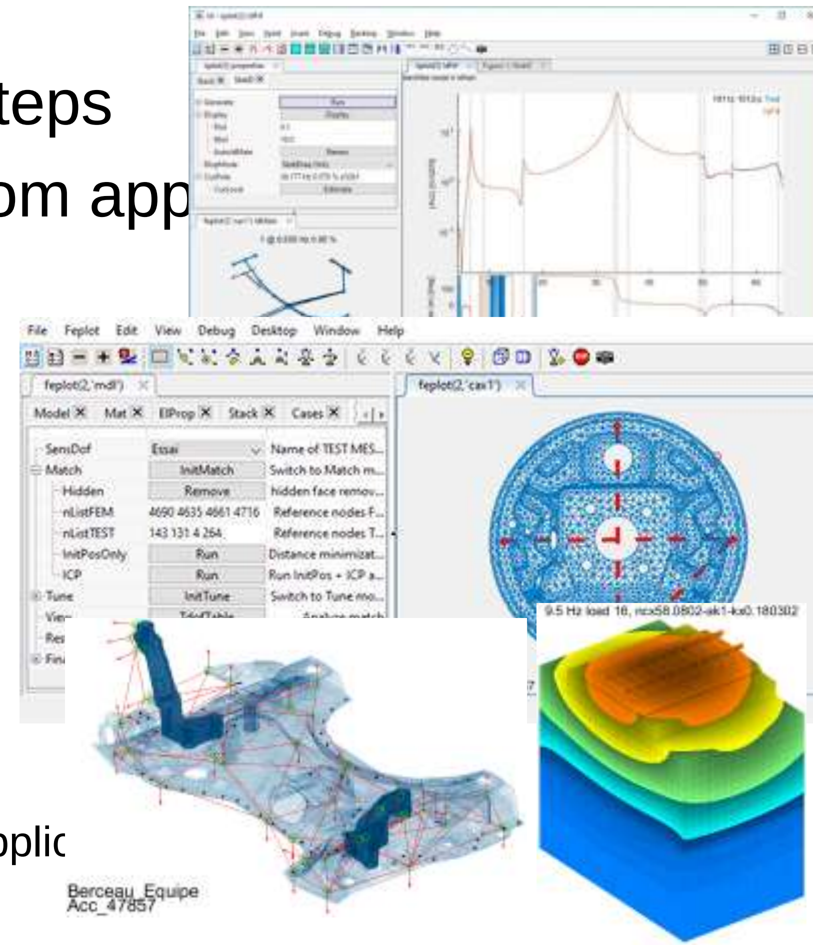
Validation

CAD/Meshing
FEM
Simulation
Testing

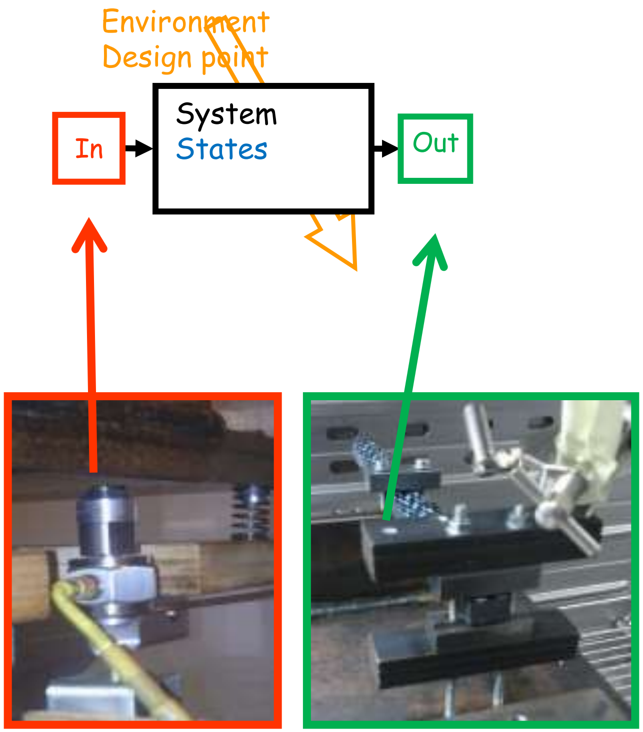
CATIA, Workbench, …
NASTRAN, ABAQUS, ANSYS,...
Adams, Simpack, Simulink,...
Siemens TestLab, ME-Scope, …

- Necessity: programmatic access to all steps
- Chosen solution: flexible toolbox & custom app
  - Experimental Modal Analysis
  - Test / Analysis correlation
  - 3D Finite Element Modeling

- With a modular approach
  - MATLAB environment
  - OpenFEM : Core software for Finite Element Modeling (co-developed with INRIA)
  - FEMlink : import / export industrial modules
  - Runtime SDT : customized and standalone compiled applic



What is a system ?

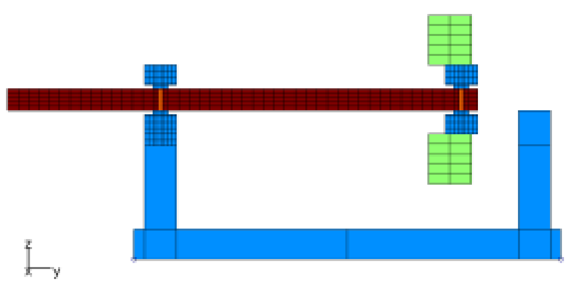


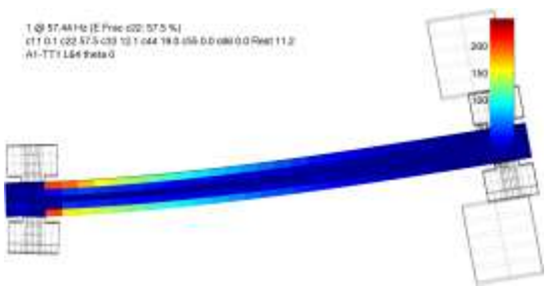
- **Inputs $u(t)$** : hammer with force measurement
- **Outputs $y(t)$**
  - Test : vibrometer on testbed
  - Computation : stresses
- **State $x(t)$**
  - Displacement & velocity field as function of time

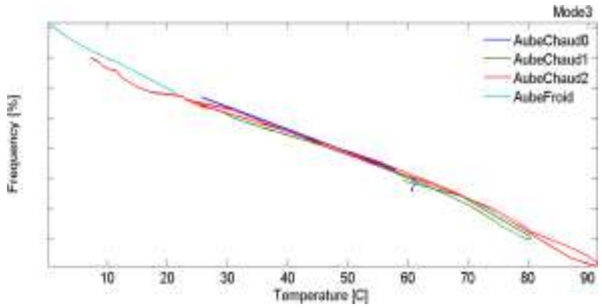
    $\{\dot{x}(t)\} = f(x(t), u(t), p, t)$ evolution

    $\{y(t)\} = g(x(t), u(t), p, t)$ observation
- **Environment variables p**
  - Dimensions, test piece (design point)
  - Temperature (value of constitutive law or state of thermo-viscoelastic)

- Feature : function of output (example modal frequency)

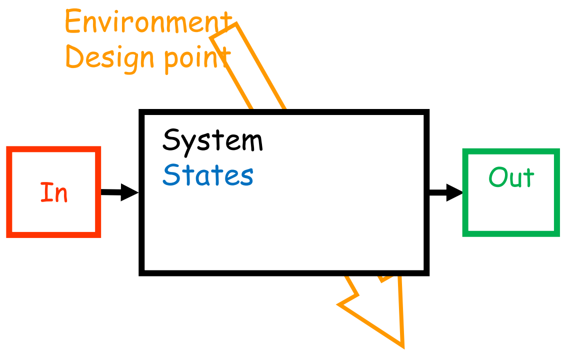


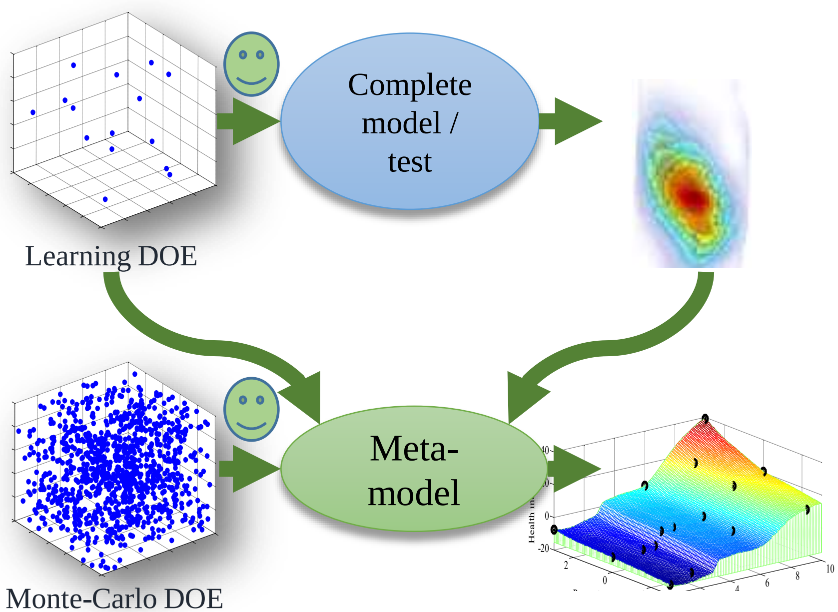




 Simple example : modified Oberst test for 3D weaved composite test

System models : nature & objectives





What is a model

- A function relating input and outputs
- For one or many parametric configurations

Model categories

- Behavior models (meta-models)
  - Test, constitutive laws, Neural networks
  - Difficulties : choice of parametrization, domain of validity
- Knowledge models
  - Physical principles, low level meta-models

Why do we need system models ?

Design

- Become predictive : understand, know limitations
- Perform sizing, optimize, deal with robustness

Certify

- Optimize tests : number, conditions
- Understand relation between real conditions and certification
- Account for variability

Maintain during life

- Design full life cycle (plan maintenance)
- Use data for conditional maintenance (SHM)

Equations of motion

- Nominal model (elastic + viscous damping)

$$[M]\{\ddot{q}\} + [C]\{\dot{q}\} + [K(q)]\{q\} = [b(q)]\{u(t)\}$$

$\{q\}$ DOF, M mass, K stiffness

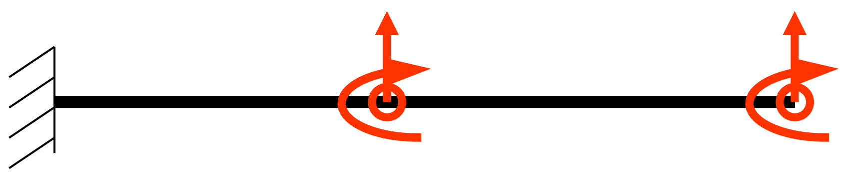
- Loads decomposed as spatially unit loads and inputs

$$\{F(t)\} = [b]\ \{u(t)\}$$

- {y} outputs are linearly related to DOFs {q} using an observation equation

$$\{y(t)\} = [c]\ \{q(t)\}$$

- Simple case : extraction $\{w_2\}=[0\ 0\ 1\ 0]\{q\}$



- More general : intermediate points, reactions, strains, stresses, …

Equations of motion

FEM ⇔ Reduction

	Finite elements Continuous → discrete full	Reduction Full → reduced
Support	Element: line, tria, tetra, …	FE mesh
Variable separ. Shape functions	$w(x,t) = N_i(x)q_i(t)$ $\epsilon(x,t) = B_i(x)q_i(t)$	$\{q(t)\} = \{T_i\}q_i(t)$ T_i simple FE solutions
Matrix comp. Weak form	$K_{ij} = \int_\Omega B_i^T \Lambda B_j = \sum_g B_i^T(g)\Lambda B_j w_g J_g$ numerical integration	$K_{ijR} = T_i^T K T_j$ FEM matrix projection
Assembly	Localization matrix	Boundary continuity, CMS
Validity	Fine mesh for solution gradients	Good basis for considered loading

[1] O. C. Zienkiewicz et R. L. Taylor, *The Finite Element Method*. MacGraw-Hill, 1989
[2] J. L. Batoz et G. Dhatt, *Modélisation des Structures par Éléments Finis*. Hermès, Paris, 1990
[3] K. J. Bathe, *Finite Element Procedures in Engineering Analysis*. Prentice-Hall Inc., Englewood Cliffs, NJ, 1982

Ritz/Galerkin reduction from full

- Basis building steps
  - FEM : cinematically admissible subspace, virtual work principle
  - Reduction : 1) learn, 2) generate basis 3) choose DOF

$$\{q(p,t)\}_N \approx [T]_{N\times NR}\{q_R(p,t)\}_{NR}$$

- Virtual work principle / reduction / Ritz-Galerkin

  Matrices $[M_R(p)] = T^T M(p) T, K_R(p) = T^T K(p) T$

  Loads $\{\mathrm{f}(p,t)\} = [b_R(p)]\{u(t)\} = [T^T b]\{u\}$

  Observations $\{\mathrm{y}(p,t)\} = [c_R(p)]\{q_R(p,t)\} = [cT]\{q_R\}$

- Solve time/freq (same model form)

$$\begin{aligned} [M_R]\{\ddot{q}_R\} + [C_R]\{\dot{q}_R\} + [K_R]\{q_R\} &= [b]\{u(t)\} \\ \{y(t,p)\} &= [c_R]\{q_R\} \end{aligned}$$

Outline : solvers for dynamics

Continuous/discrete/reduced models (a brief reminder)

Full order model solvers

- Direct frequency resolution
- Direct time integration (implicit/explicit, first/second order, Newmark, … Gaël Chevallier)

Reduced order model + time/frequency resolution

- Basic reduction : modal superposition, static correction, Guyan, Craig-Bampton, …
- Modern vision of reduction: learning phase, basis building, DOF choice
- Substructuring
- Parametric model reduction, error control

When does reduction become useful ?

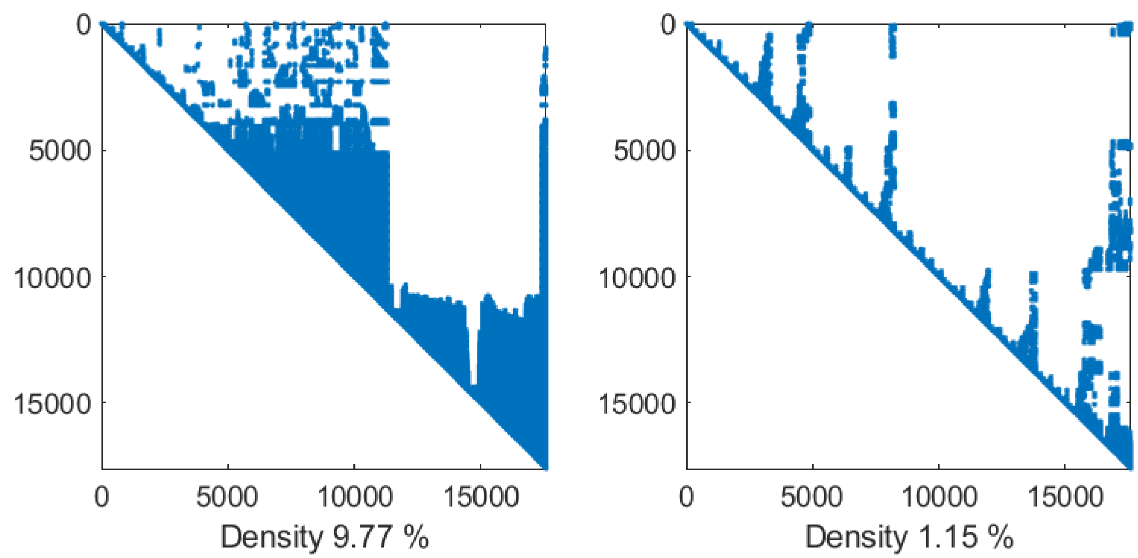
Basic building blocks ?

MATLAB Tutorial : direct frequency response issues

- Step1 : assembly, sparse matrices
- Step 2 : point load, collocated displacement, factorization strategies
- Step 3 : subspace around resonance, phase collinearity, SVD
- Step 4 : Rayleigh-Ritz, reduced FRF

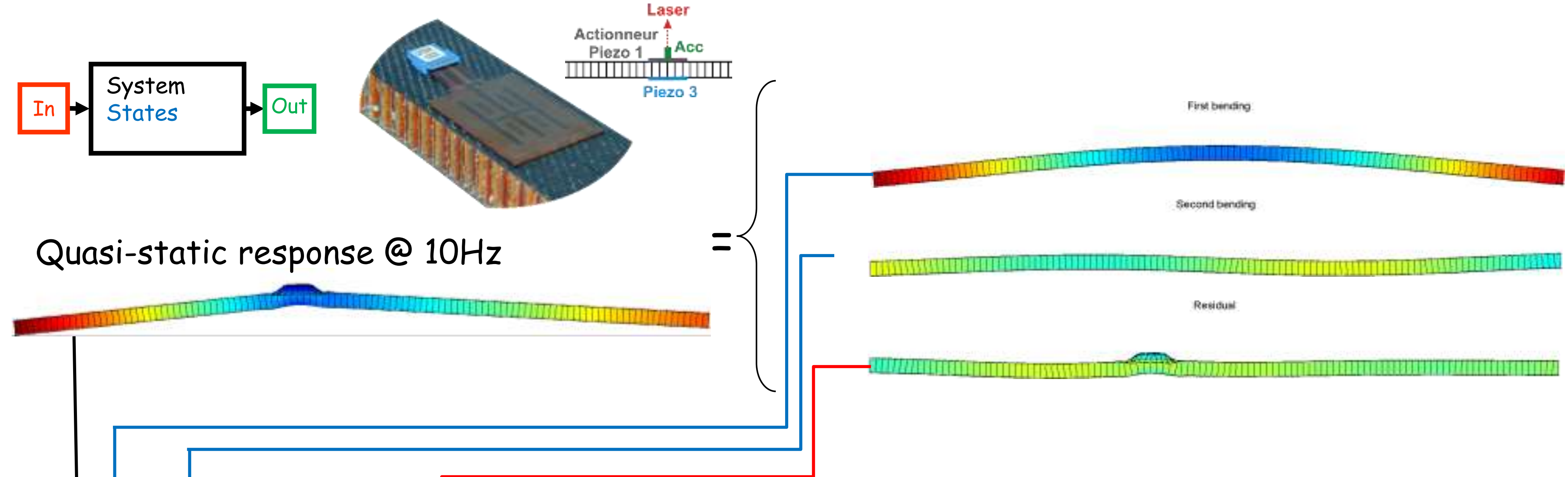
Direct frequency response : Zq=F (step2)

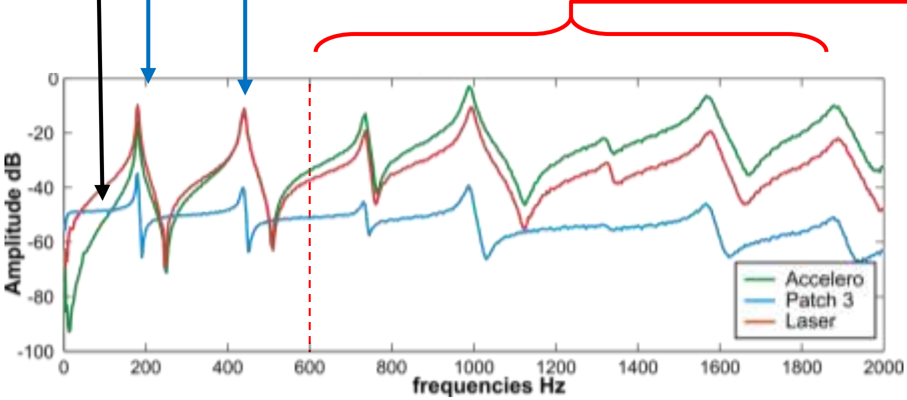
1. Renumbering (fill in reduction, symbolic factorization, METIS, symrcm, …)
2. Numerical factorization $Z = LU$ or $Z = LDL^T$
3. Forward/backward solve $L\left(D\left(L^T q\right)\right) = F$



Sparse librairies : Umfpack (lu), MA57 (ldl), Pardiso, Mumps, BCS-Lib, Spooles, Taucs, …

Transfers : what subspace is needed ?





- Modes : high energy, load independent (no blister shape)
- Static response (influence of input=blister), important away from resonance

- Experimental modal analysis subspace :
  - Modes within bandwidth
  - Upper residual (residual flexibility, static correction, state-space D term)
  - Lower residual (rigid body inertia, …)

Nearby modes = poor representation of static

Ph.D. Corine Florens 2010

Normal modes of elastic structure

- Nominal model (elastic + viscous damping)

$$\left[Ms^2 + Cs + K\right]\{q(s)\} = [b]\{u(s)\}$$
$$\{y(s)\} = [c]\{q(s)\}$$

- Conservative eigenvalue problem

$$-[M]\{\phi_j\}\omega_j^2 + [K]_{N\times N}\{\phi_j\}_{N\times 1} = \{0\}_{N\times 1}$$

- M>0 & K≥0 ⇒ ϕ real
- Partial solvers exist

Normal modes of elastic structure

- Orthogonality
- Scaling conditions
  - Unit mass
  - Unit amplitude
- Principal coordinates

$$[\phi]^T[M][\phi] = \left[\diagdown \mu_j \diagdown\right]$$

$$[\phi]^T[K][\phi] = \left[\diagdown \mu_j \omega_j^2 \diagdown\right]$$

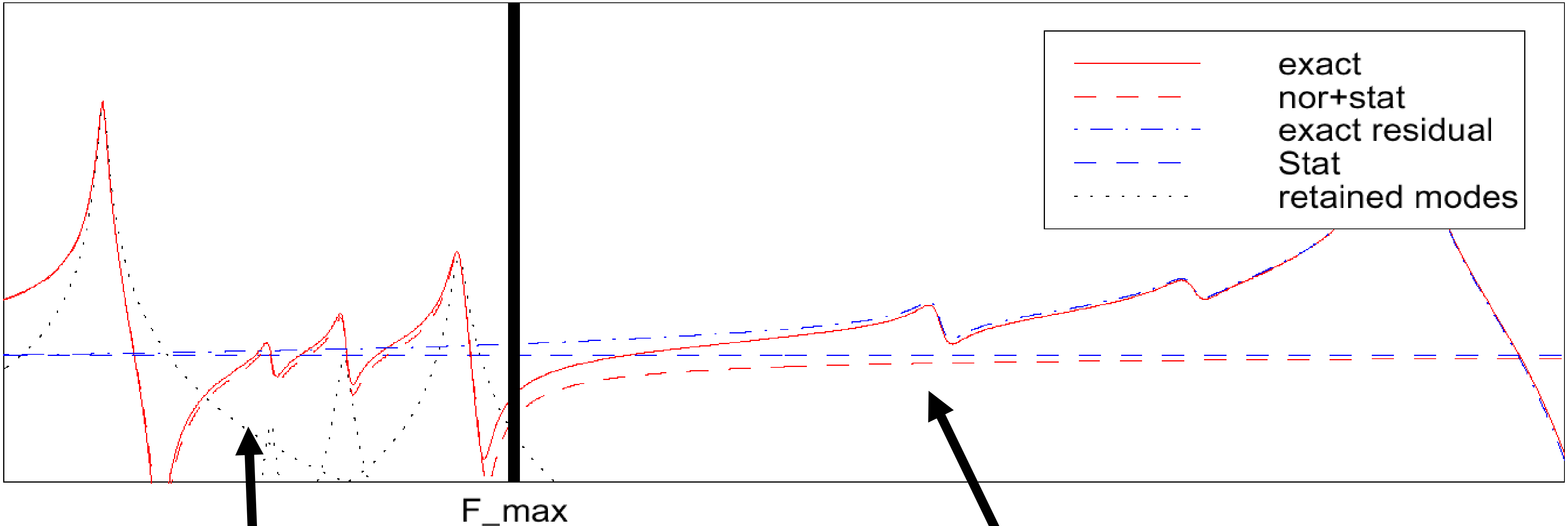
$$\{\phi_j\}^T[M]\{\phi_j\} = 1$$

$$[c_s]\{\tilde{\phi}_j\} = 1 \quad \mu_j(c_s) = \left([c_i]\{\phi_j\}\right)^{-2}$$

$$\left[[I]s^2 + [\Gamma]s + \left[\diagdown \omega_j^2 \diagdown\right]\right]\{p(s)\} = \left[\phi^T b\right]\{u(s)\}$$

$$\{y(s)\} = [c\phi]\{p(s)\}$$

Modal contributions & static correction



$$H(\omega) = [c]\left[-M\omega^2 + K\right]^{-1}[b] \approx$$

$$\sum_{j=1}^{N_M} \frac{[c]\{\phi_j\}\{\phi_j\}^T[b]}{-\omega^2 + \omega_j^2} + \sum_{j=N_M+1}^{N} \frac{[c]\{\phi_j\}\{\phi_j\}^T[b]}{\omega_j^2}$$

Reduction <-> Ritz analysis

Response is approximated

$$q(s) = \left[\phi_1 \ldots \phi_{NM} \quad [K_{Flex}]^{-1}[b]\right]_{N\times(NM+NA)} \left\{ \begin{array}{c} \vdots \\ \frac{\phi_j^T bu}{s^2+\omega_j^2} \\ \vdots \\ u \end{array} \right\}$$

- within subspace containing modes and flexibility

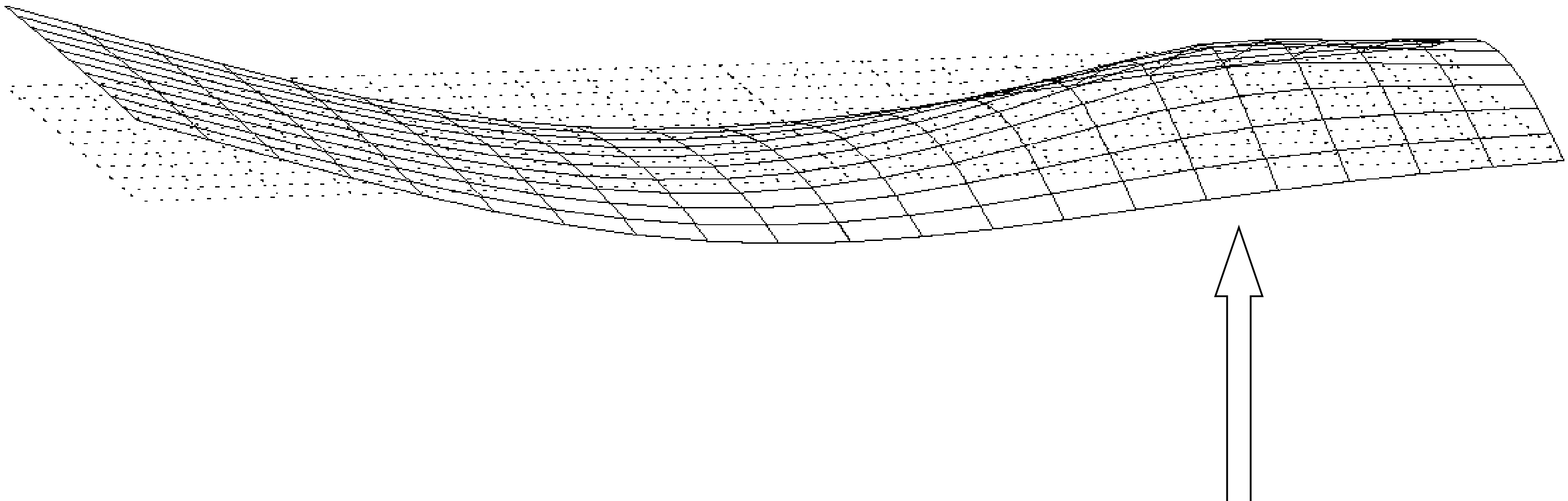
$$T = \left[\phi_1 \ldots \phi_{NR} \quad [K_{Flex}]^{-1}[b]\right]$$

- or modes and residual flexibility

$$T = \left[\phi_{1:NM} \quad [K_{Flex}]^{-1}\left[b - M\left[\phi_{1:NM}\right]\left[\phi_{1:NM}^T b\right]\right]\right]$$

Tuto : Krylov, orthog, …

Attachment modes



For free structure : static load implies deformation in a uniformly accelerating frame

$$\{q_F\} = [K]^{-1}_{Flex}\,[b] = \sum_{j=NB+1}^{N} \frac{\{\phi_j\}\left\{\phi_j^T b\right\}}{\omega_j^2}$$

Unit imposed displacement

Applied load : free modes + static correction = McNeal
Applied displacement : dynamic & Static/Guyan condensation

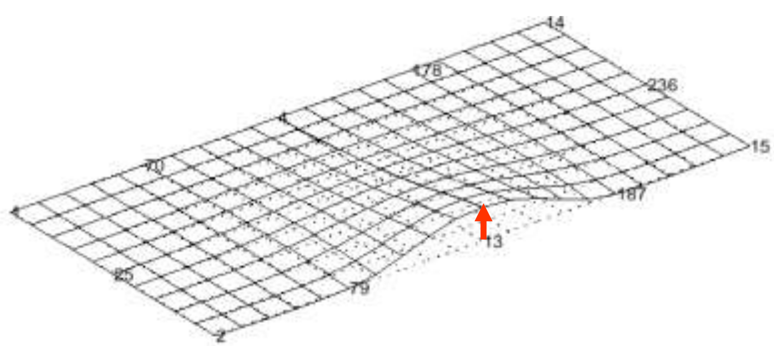
$$\begin{bmatrix} K_{II} & K_{IC} \\ K_{CI} & K_{CC} \end{bmatrix} \left\{ \begin{matrix} < q_I(s) > \\ q_C(s) \end{matrix} \right\} + \left[M s^2 \right] \{q\} = \left\{ \begin{matrix} R_I(s) \\ < 0 > \end{matrix} \right\}$$

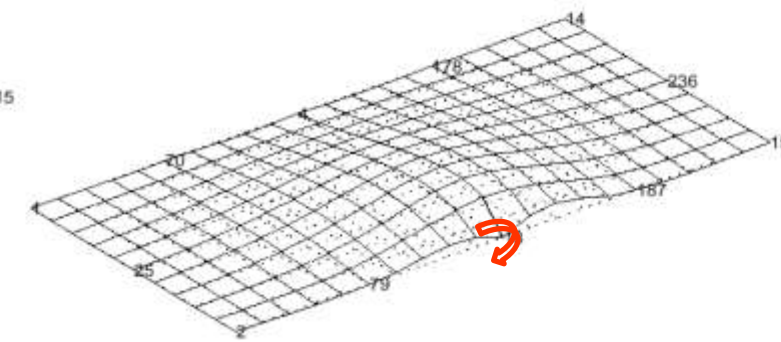
No interior load = dynamic condensation

$$[T(\omega)] = \begin{bmatrix} I \\ -Z_{CC}(\omega)^{-1} Z_{CI}(\omega) \end{bmatrix}$$

Inertia neglected = static/Guyan

$$[T] = \begin{bmatrix} I \\ -K_{CC}^{-1} K_{CI} \end{bmatrix}$$





Frequency limit -> Craig-Bampton

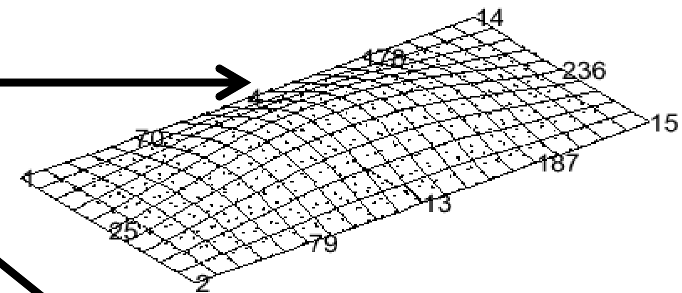
Inertia neglected : error associated with $M_{cc}q_c$

$$\begin{bmatrix} K_{II} & K_{IC} \\ K_{CI} & K_{CC} \end{bmatrix} \begin{Bmatrix} < q_I(s) > \\ q_C(s) \end{Bmatrix} + \left[Ms^2\right]\{q\} = \begin{Bmatrix} R_I(s) \\ < 0 > \end{Bmatrix}$$

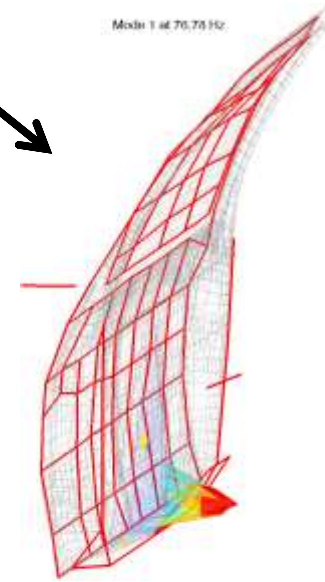
When $Z_{cc}(s)$ is singular
$\Rightarrow$
Approximation cannot be valid

$$\begin{bmatrix} 0 & 0 \\ 0 & Z_{CC}(\omega_j) \end{bmatrix} \begin{Bmatrix} 0 \\ \phi_{j,c} \end{Bmatrix} = \begin{Bmatrix} R_I \\ 0 \end{Bmatrix}$$

Fixed interface modes



Craig-Bampton = guyan/static + fixed interface

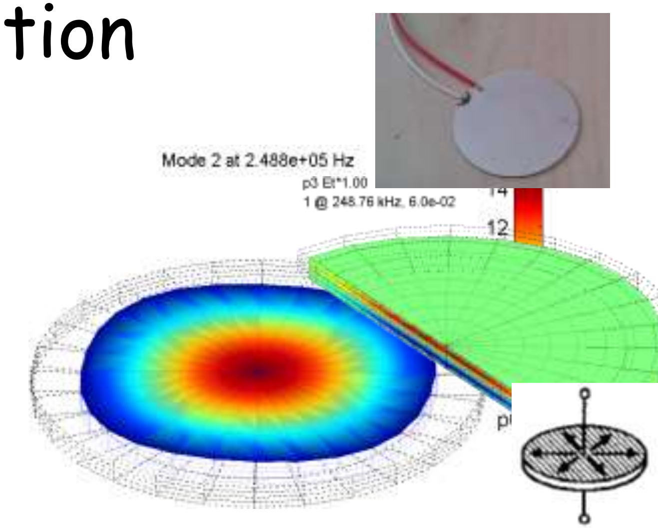


$$[T] = \left[\begin{bmatrix} I \\ K_{cc}^{-1}K_{ci} \end{bmatrix} \begin{bmatrix} 0 \\ \phi_{1:NM,c} \end{bmatrix} \right]$$

Learning strategy variants : POD

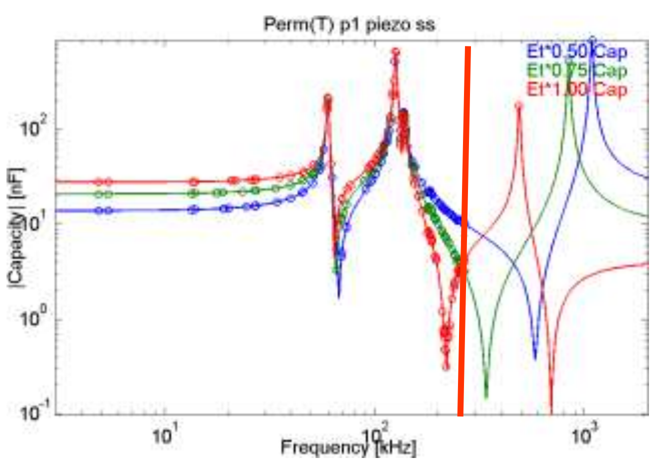
Traditional : modes + static correction

$$T = \begin{bmatrix} \phi(Zcc(\omega_j)) & K_{CC}(s)^{-1}K_{CV}(s)V_{In} \\ 0 & V_{In} \end{bmatrix}_{\perp M,K}$$

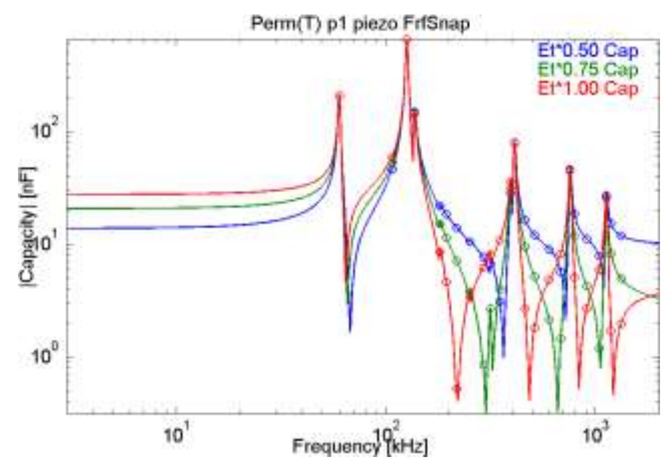


Snap-shot Ritz basis

$$T = \left[\left\{ \begin{matrix} Z_{CC}(s)^{-1}Z_{CV}(s)V_{In} \\ V_{In} \end{matrix} \right\}_{s \in i\omega_{target}} \right]_{\perp M,K}$$



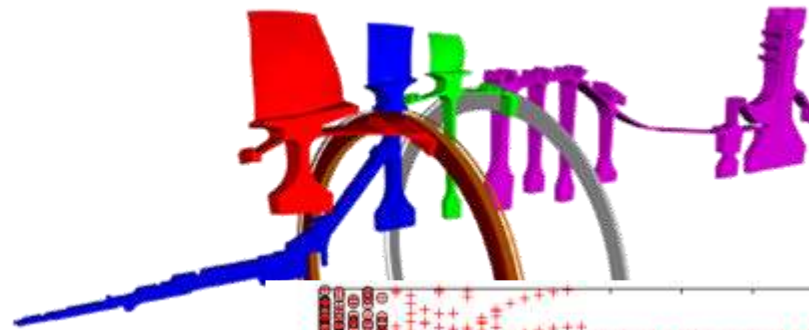
3 out of 100 useful modes
Relatively close static correction

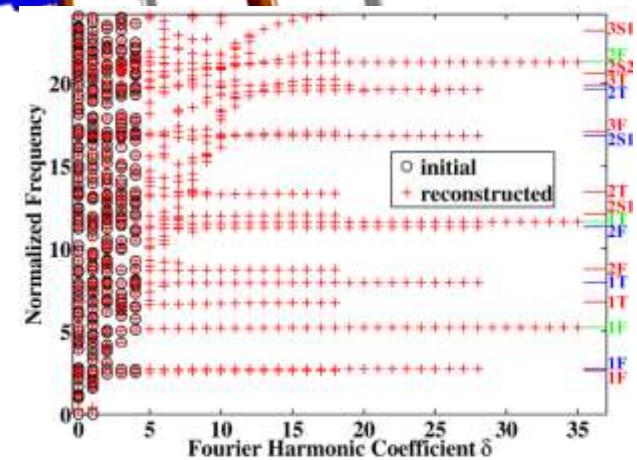


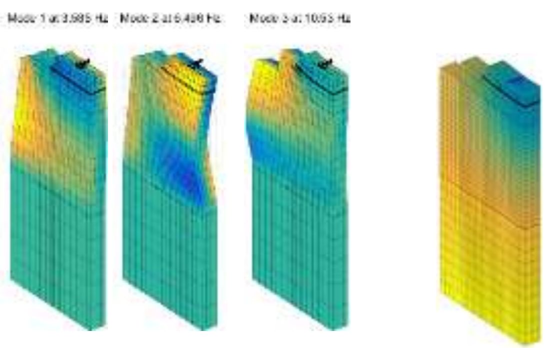
Easily captures wide range

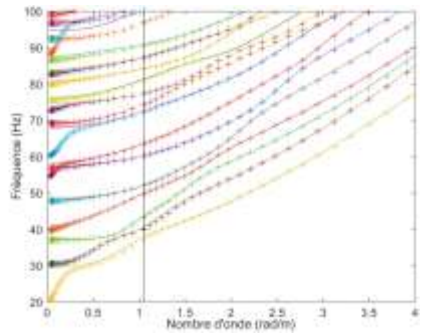
Learning strategy : wave/cyclic

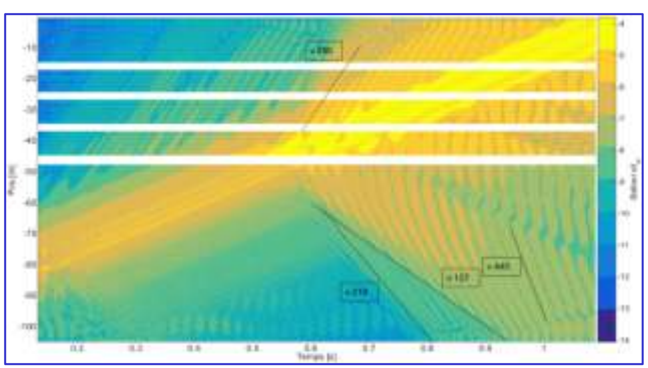
1. Learn using wave (Floquet)/cyclic solutions
2. Build basis with left/right compatibility
3. Assemble reduced model

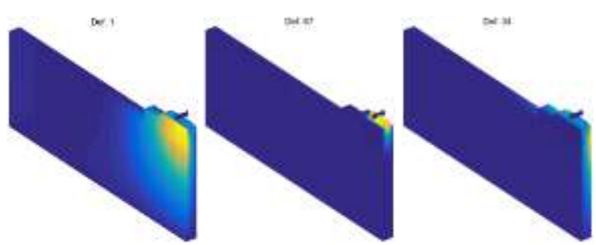




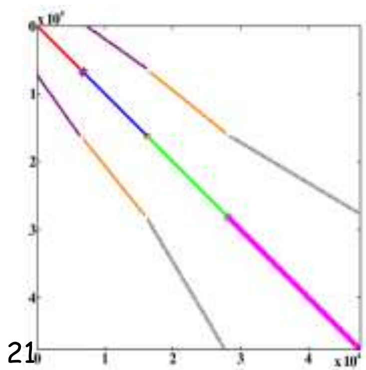








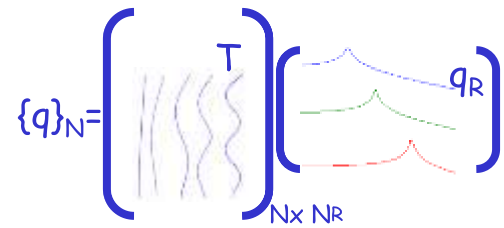
PhD Elodie Arlaud, 2016



PhD Sternshuss 2008

Outline

1. Learning phase
    1. modes & static responses (bandwidth, inputs) : McNeal, Guyan, Craig-Bampton
    2. POD
2. Basis generation DOF selection
    1. SVD (truncation)
    2. Gramm-Schmidt, conjugate-gradient (Lanczos)
    3. Piecewise learning (sparsity, superelements, Component mode synthesis)



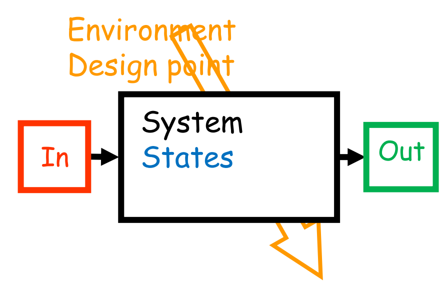
3. Model reduction/modal synthesis/Ritz-Galerkin/virtual work principle

$$\{q(x,t)\} = [T(x)]\{q_R(t)\} \Rightarrow Z_R(\omega,p) = T^T[Z(\omega,p)]T$$

Optimize reduced model usage

4. Beyond LTI (parametric, NL, time varying, …)

T independent of p

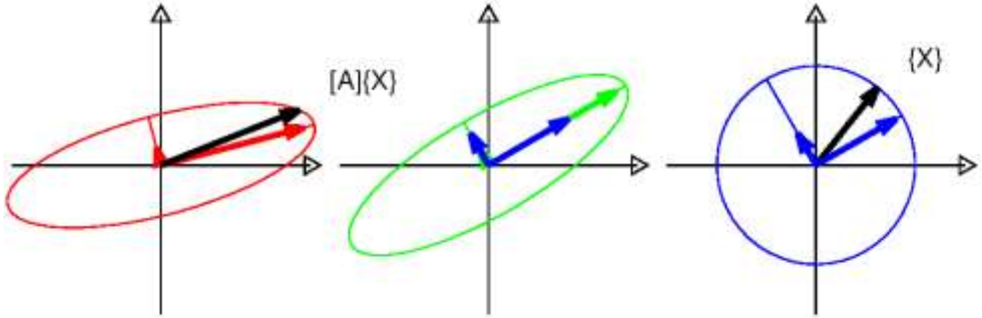


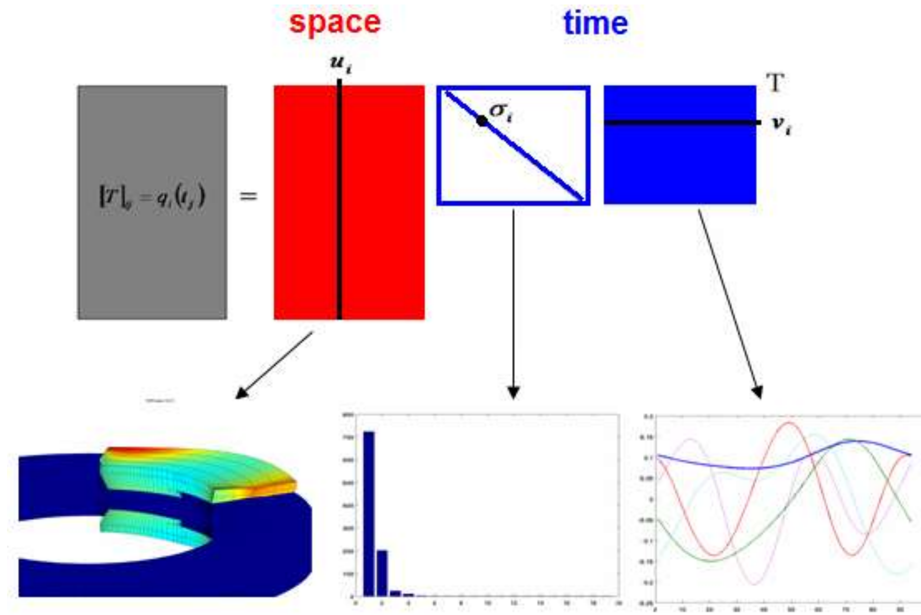
Tuto steps 3-4

- POD learning
- Rayleigh-Ritz / reduced solve

SVD & variants

$A = U\,S\,V^H$





shape DOF

SVD

- {X} on sphere in input space transformed in {Y}=[A]{X} ellipsoid
- Series of rank one contributions

Mode

- $\{\phi\}$ on unit strain energy sphere output is kinetic energy
- Singular value $\frac{1}{\omega_j^2} = \frac{\phi_j^T M \phi_j}{\phi_j^T K \phi_j}$ = 1/Rayleigh quotient

AIAA Journal, Balmes, 1996

Optimize reduced model computation

- Spectral decomposition

$$\{y\} = c_R[M_R\ s^2 + K_R]^{-1} b_R u = \sum_{j=1}^{NM} \frac{c\phi_j \phi_j^T bu}{s^2 + 2\zeta_j \omega_j s + \omega_j^2}$$

  - Cost is $O(N^3) \times N_F$ or $O(N^3) + O((NM \times N_a \times N_s) \times N_F)$
  - The second is obviously much lower for not very small N_F

- Transition matrix (or matrix exponential) : time domain (free response), space domain (Wave Finite Element)

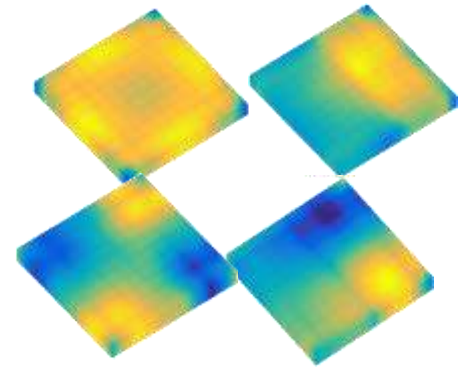
$$\{x_{n+1}\} = [A]\{x_n\} = \left[\Theta_R \left[\backslash \lambda_{j_\backslash}\right] \Theta_L^T\right]\{x_n\} = \left[\Theta_R \left[\backslash \lambda_j^{n+1}{}_\backslash\right] \Theta_L^T\right]\{x_0\}$$

  - The second is obviously much lower for not very small n

SVD, variants, related

Random fields Karhunen-Loeve :

- input-norm I for all DOFs
- output norm spatial correlation
  $C = \exp[-(|x_1 - x_2| + |y_1 - y_2|)]$



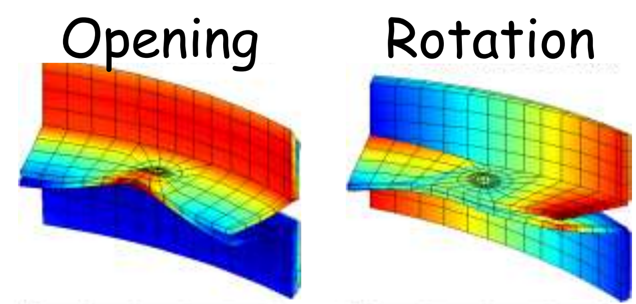
PCA Principal Component Analysis
POD based on snapshot-reduction :

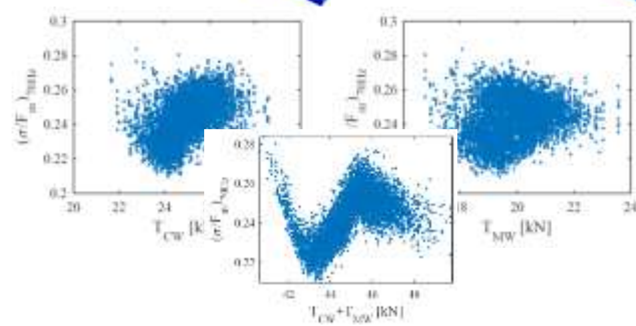
- input-norm I on snapshot vectors
- output norm I

[1] Chung, Gutiérrez, & all, “stochastic finite element models,” IJME, 2005.
[2] Kershen & al. “POD”, Nonlinear dynamics , 2005
[3] Balmes, Vermot, “Colloque assemblages 2015”,
[4] Bendhia 1-epsilon compatibility EJCM 2010
[5] Ph.D. Olivier Vo Van 2016

Junction modes

- input-norm I for modes
  or contact stiffness
- output norm local stiffness





Non-linear dimensionality reduction (manifold)

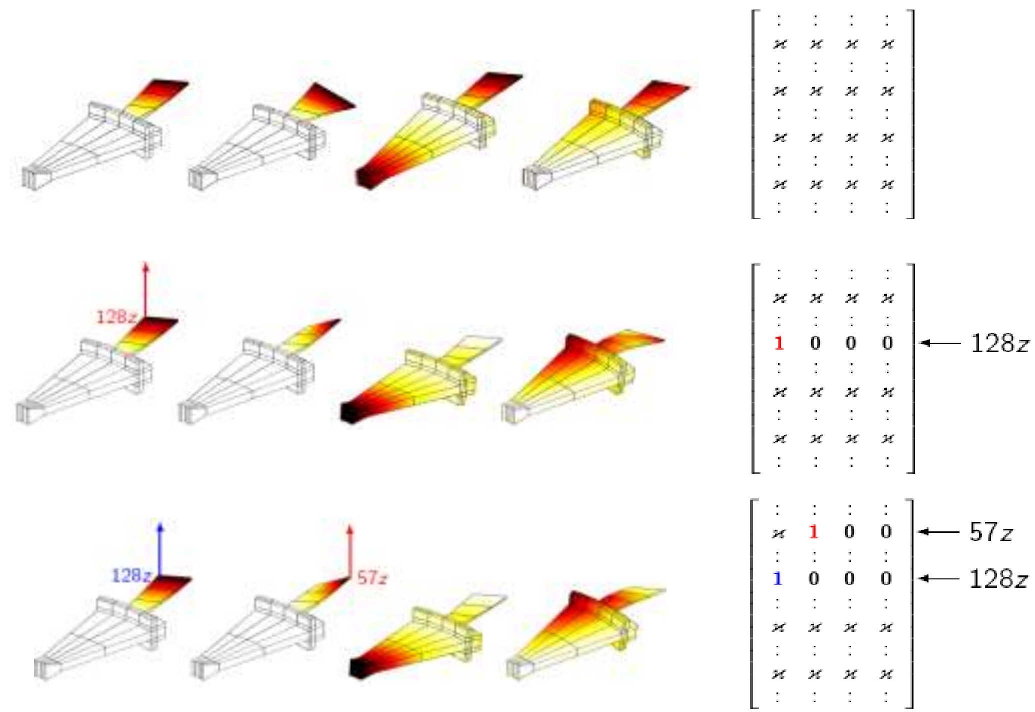
- More complex relation between parameters

From shapes to bases

Vector independence

- SVD
- Krylov/Lanczos (iterations & conditioning, step5)
- Gram Schmidt
- LU

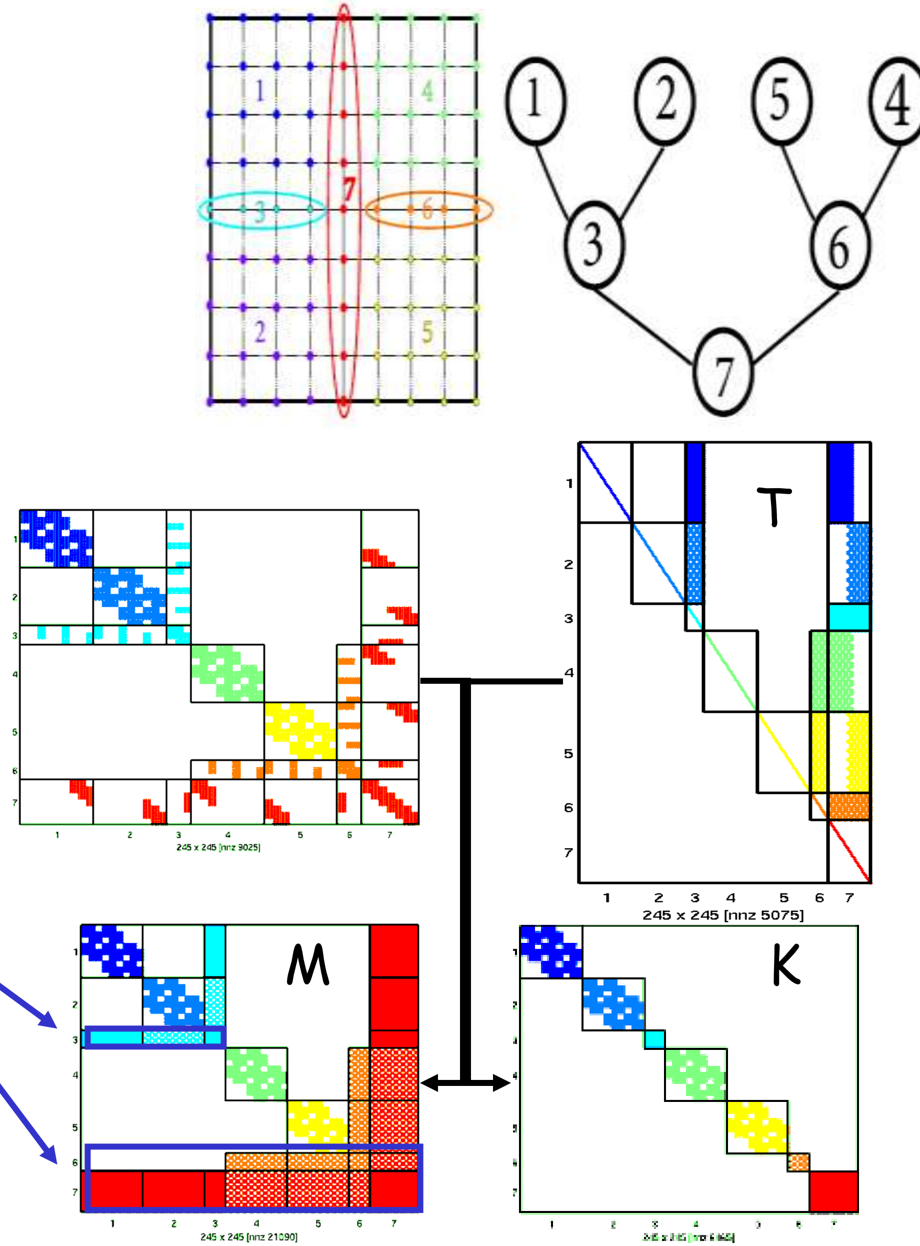
O. Boiteau, « Modal Solvers and resolution of the generalized problem (GEP) », *Code_Aster, Version 5.0, R5.01.01-C*, p. 1-78, 2001.



Multi-frontal solvers / AMLS

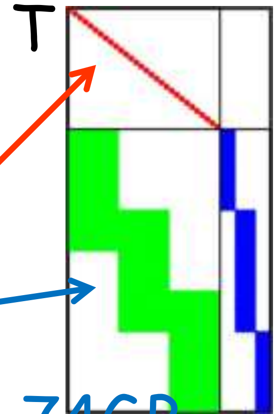
- Graph partionning methods ⇒ group DOFs in an elimination tree with separate branches
- Block structure of reduction basis
- Block diagonal stiffness
- Very populated mass coupling

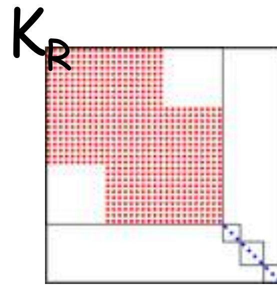
- Multi-frontal eigensolvers introduce some form of interface modes to limit size of mass coupling



Interface reduction / model size / sparsity

- Craig-Bampton often sub-performant because of interfaces

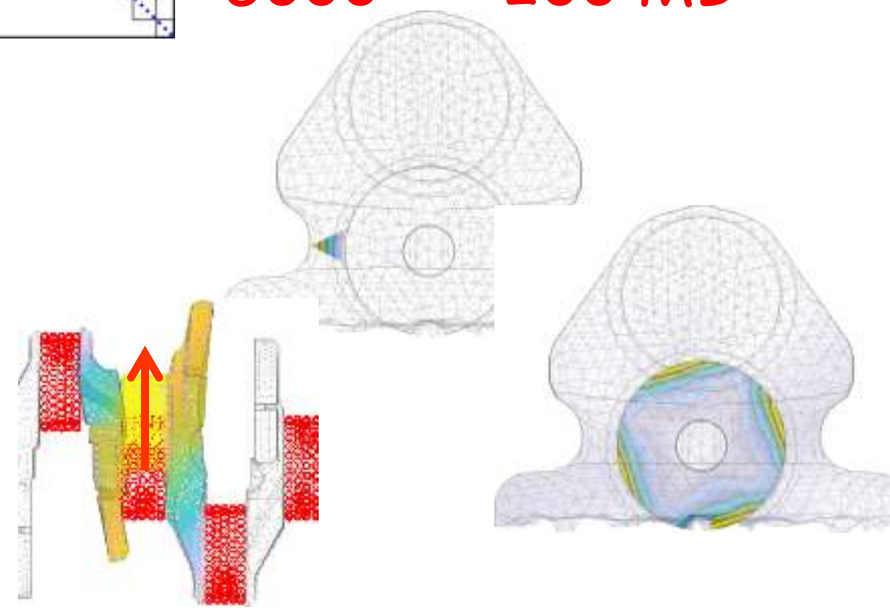




5000^2 = 200 MB

2^e6 rest x 5000 Int = 74GB

- Unit motion can be redefined : interface modes
  Fourier, analytic polynomials, local eigenvalue
  5000 -> 500 interface DOFs.
- Disjoint internal DOF subsets



Separate requirements for learning shapes & basis building :

bandwidth, inputs external & parameter
truncation, sparsity

MATLAB Tutorial : reduction, full operators

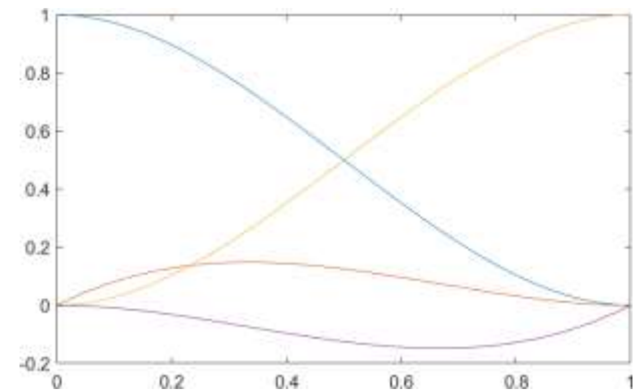
- Step 5 : Krylov
- Step 6 : sparse reduced model

- Step 7 : frequency limit CB
- Step 8 : an experimental case of SVD

DOF / sensor selection

Solutions depends on subspace NOT basis

Choose DOF you like or that make sense



Ex 1: beam shape functions

- Subspace $a + bx + cx^2 + dx^3$
- Observation $y = \{w_1, \theta_1, w_2, \theta_2\}^T$
- Condition of unit on observation gives shape functions N_i

Ex 2: multibody dynamics : use master nodes

needed $[c][T]$ full rank

$$\{y\} = [c][T]\{q_R\}$$
$$\Downarrow$$
$$\{y\} = [\hat{c}][\hat{T}]\begin{Bmatrix} y \\ q_c \end{Bmatrix} = [I\ 0]\begin{Bmatrix} y \\ q_c \end{Bmatrix}$$

Physical & Modal DOF

- Physical domain:

$$[M]\{\ddot{q}(t)\} + [C]\{\dot{q}(t)\} + [K]\{q(t)\} = \{f(q,\dot{q},t)\}$$

- Modal domain:

- mass orthogonality condition $\phi^T M \phi = I$
- stiffness orthogonality condition $\phi_j^T K \phi_j = \omega_j^2$
- Modal equation

$$[I]\{\ddot{\alpha}(t)\} + [\Gamma]\{\dot{\alpha}(t)\} + \left[{}^{\backslash}\omega_j^2{}_{\backslash}\right]\{\alpha(t)\} = \{f(\alpha,\dot{\alpha},t)\}$$

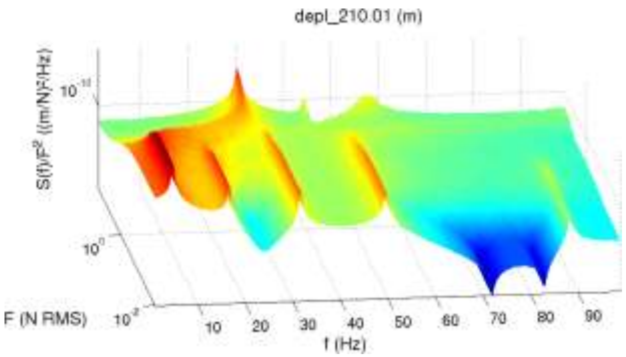
- Modal amplitudes $\{\alpha\} = [\boldsymbol{\phi}^{-1}]\{q\} = [\boldsymbol{\phi}^T \boldsymbol{M}]\{q\}$

  Associated concepts : force appropriation, modal filter

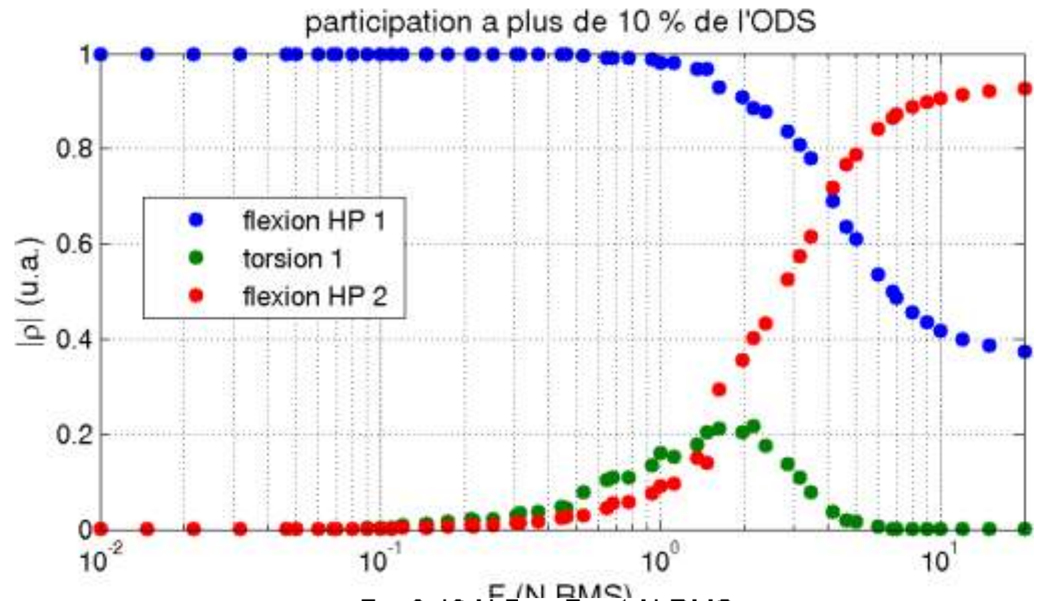
- Modal energies $e_j = \frac{1}{2}\left(\dot{\alpha}_j^2 + \omega_j^2 \alpha_j^2\right)$

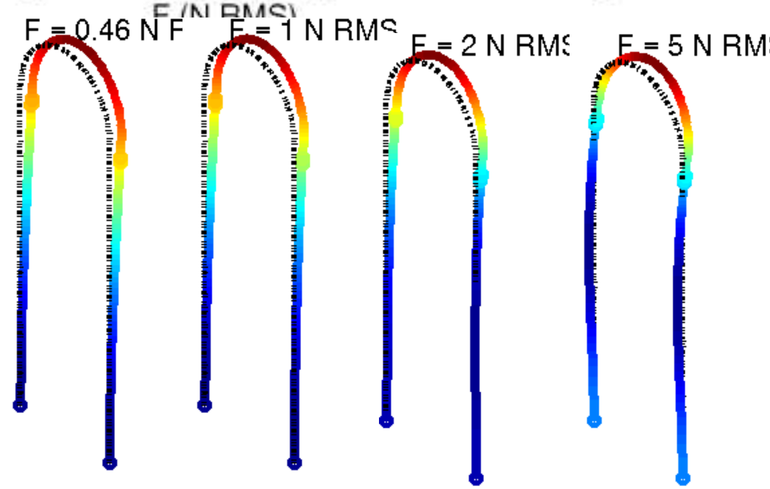
[1] J. P. Bianchi, E. Balmes, G. Vermot des Roches, et A. Bobillot, « Using modal damping for full model transient analysis. Application to pantograph/catenary vibration », in *ISMA*, Leuven, 2010.

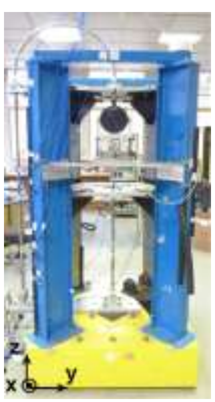
Modal participations in ODS

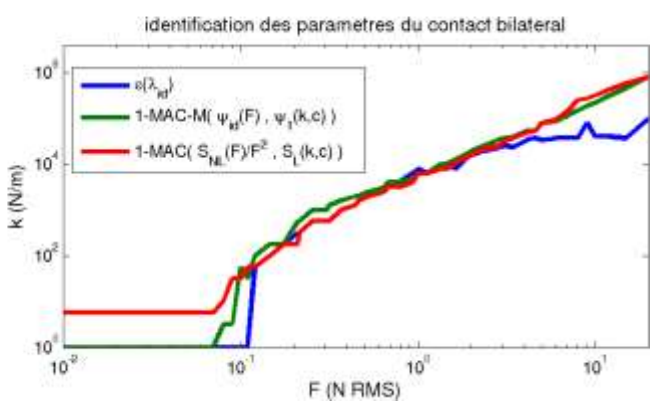


- Extract shape =
  SVD around « resonance »
- Obtain modal amplitudes of nominal modes
- Apparent stiffness/damping consistent for various methods





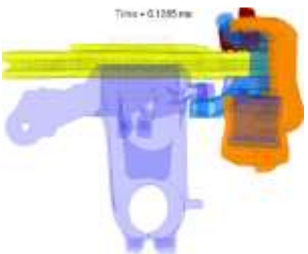


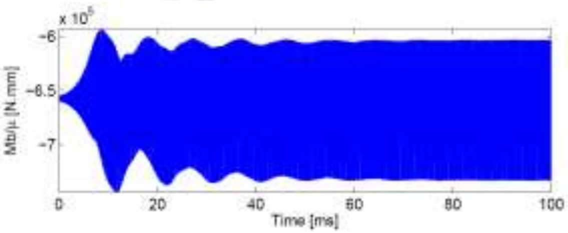


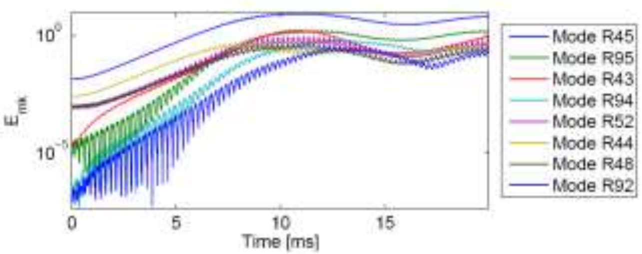
PhD. Thénint 2011

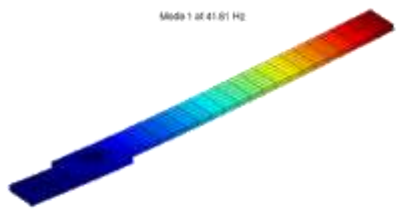
Modal energy computations

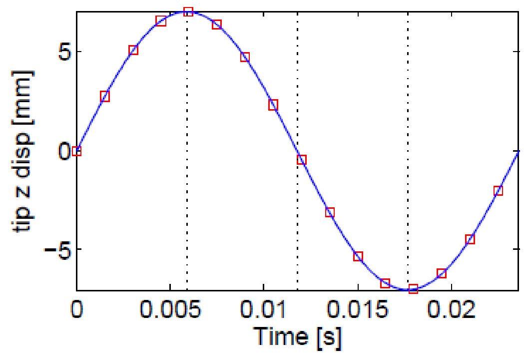
- Does the shape change in NL behavior

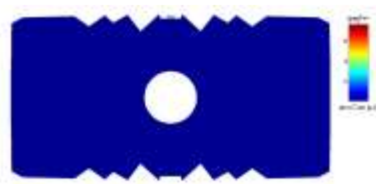


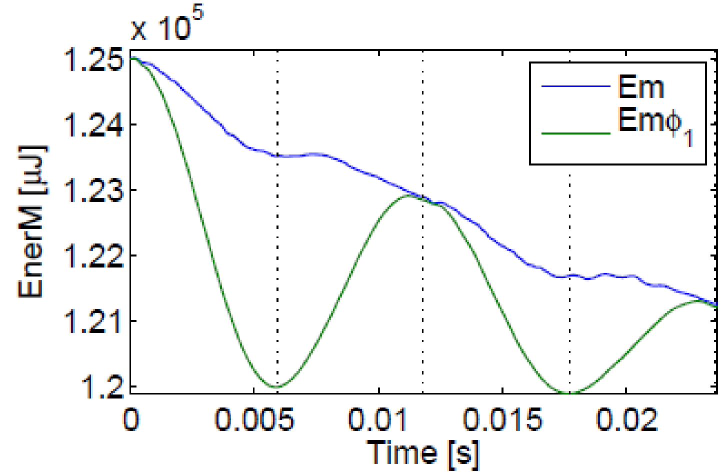






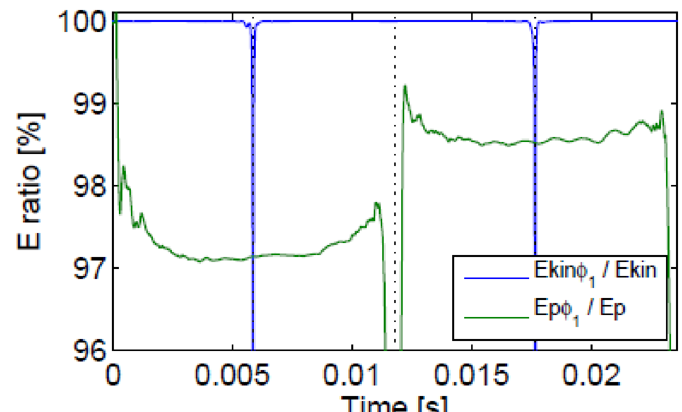






$$2E_{mj}(t) = \omega_j^2 \alpha_j^2(t) + \dot{\alpha}_j^2(t)$$

$E_p(t)$ $E_k(t)$

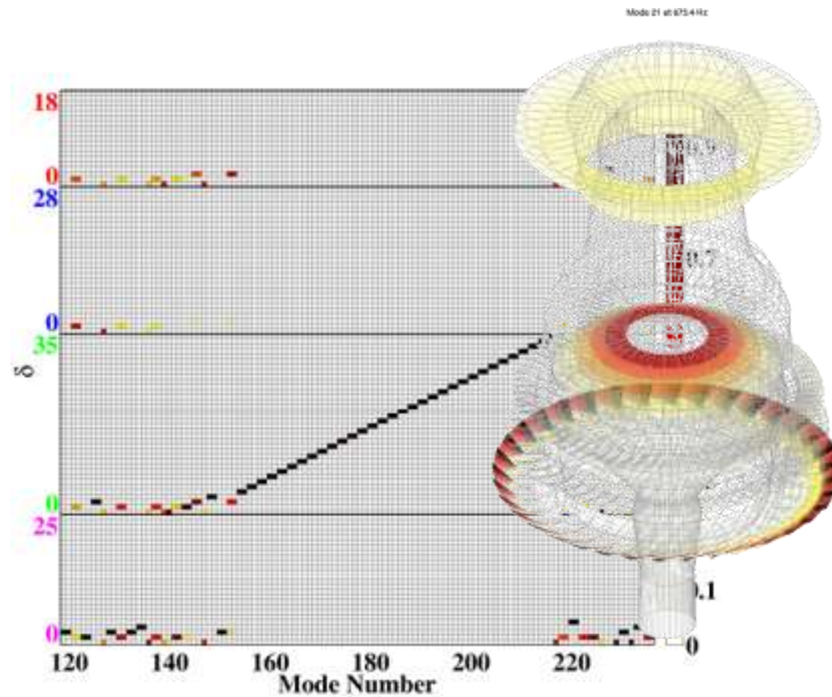


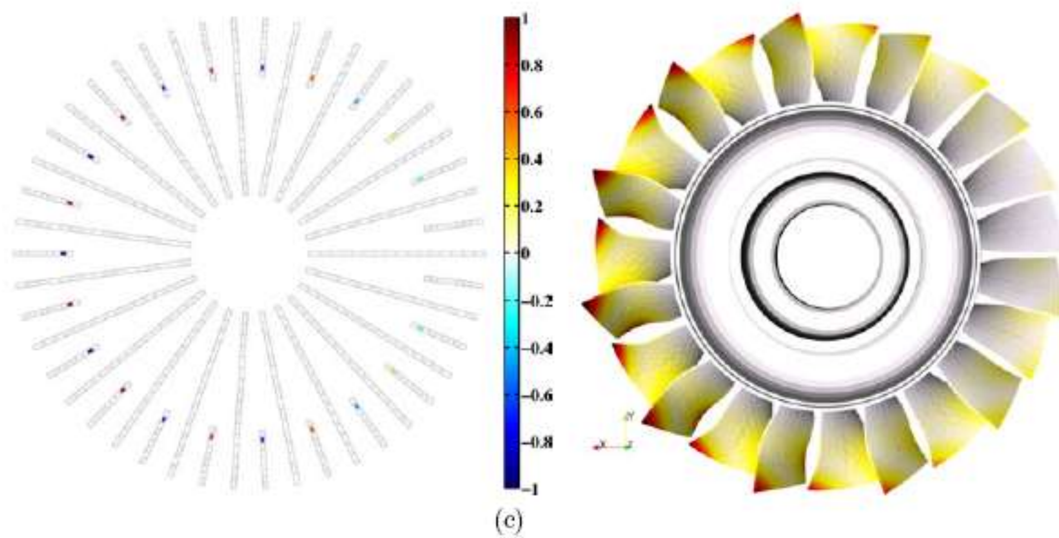
[1] G. Vermot Des Roches, « Frequency and time simulation of squeal instabilities. Application to the design of industrial automotive brakes », Ph.D. thesis, Ecole Centrale Paris, CIFRE SDTools, 2011.

[2] G. Vermot Des Roches et E. Balmes, « Understanding friction induced damping in bolted assemblies through explicit transient simulation », in *ISMA*, 2014, p. ID360.

Modal DOF

- Multi-stage cyclic symmetry (SNECMA).
  - Which stage, which diameter, …
  - Mistuning (which blade)



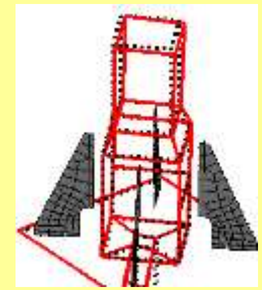


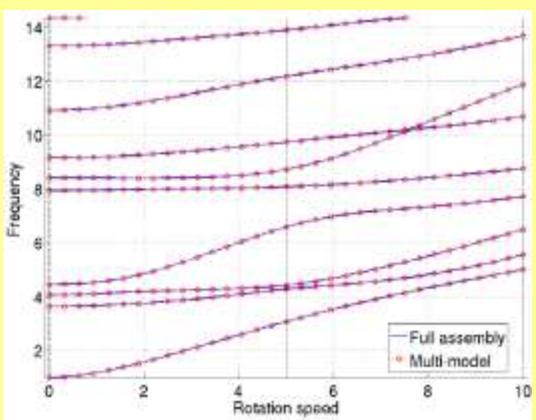
PhD Sternshuss 2008

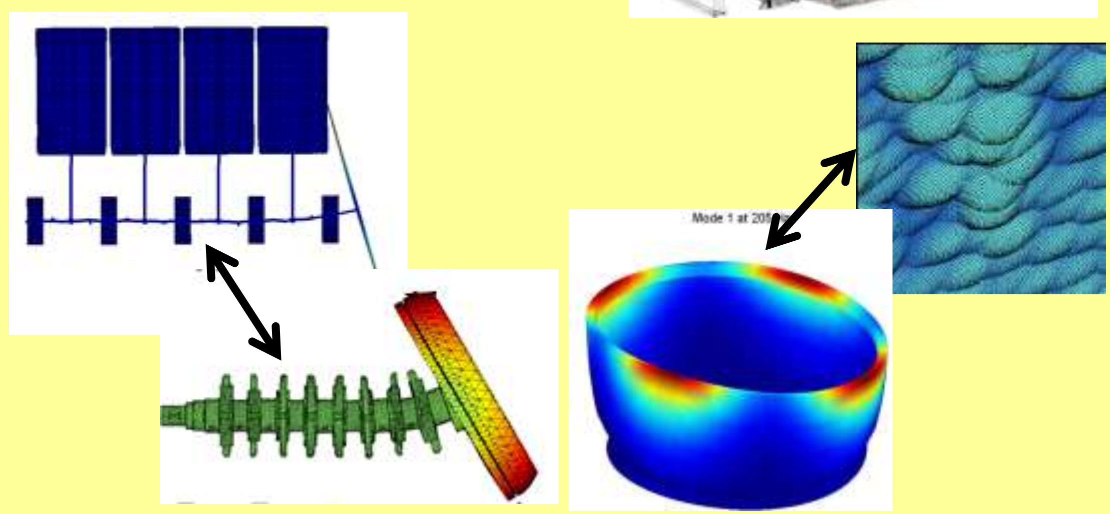
Dealing with NL/parameters/damping

In → Reduced model → Sensors

- Coupling : test/FEM, fluid/structure active control, …
- Local non-linearities : machining, bearings, contact/friction, …
- Optimization / uncertainty



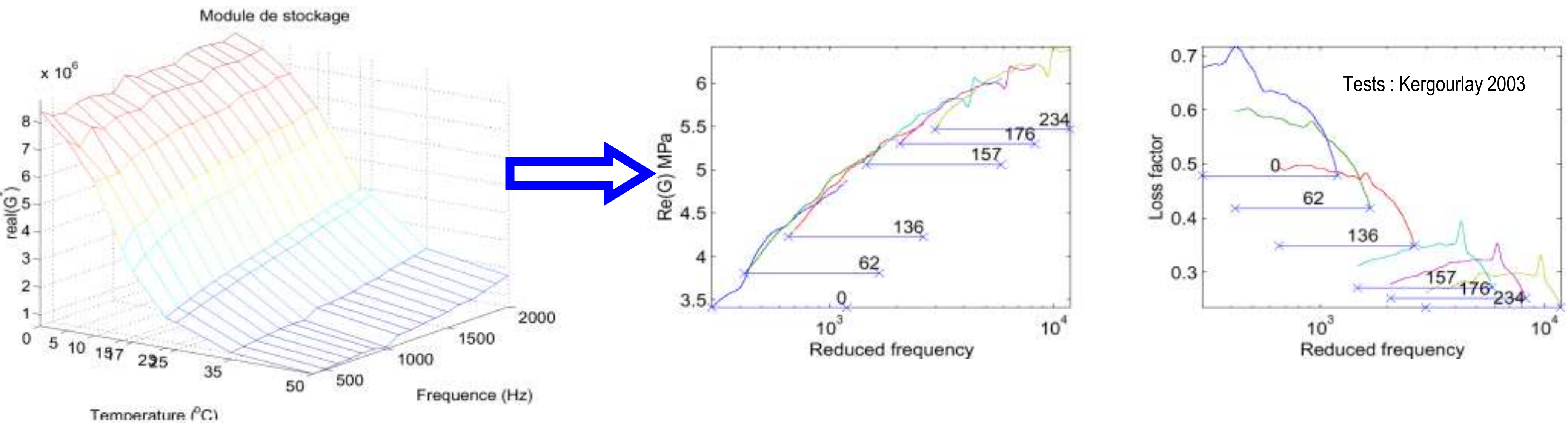




Viscoelastic constitutive relations

- Stress is a function of strain history
- Complex modulus in Laplace domain

$$\sigma(s) = E(s, T, \sigma_0)\varepsilon(s) = (E' + iE'')\varepsilon(s)$$



- Dynamic stiffness linear combination of fixed matrices

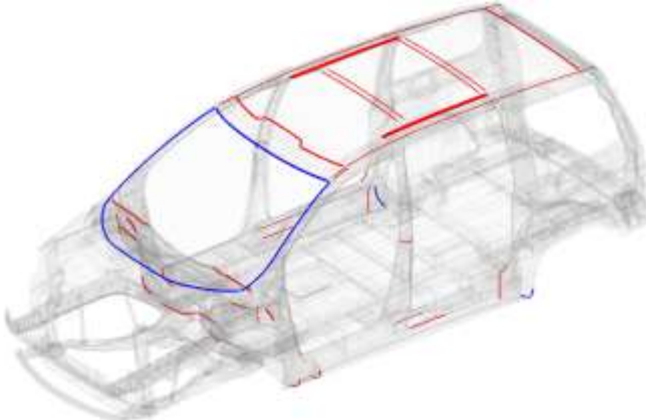
$$Z(E_i, s) = \left[Ms^2 + K_e + \sum_j E_i(s, T, \sigma_0)\left[\frac{K_{vi}(E_0)}{E_0}\right]\right]$$

Residue iteration : viscoelastic material

$[Z(E_i(s),s)]\{q\} = \{F\}$ Damped viscoelastic resp. rewritten as

$$[Z(E_0,s)]\{q\} = \{F\} - \sum_j (E_i(s) - E_0)\left[\frac{K_{vi}(E_0)}{E_0}\right]\{q\}$$

Tangent linear system, internal NL/parametric loads



Basis contains

- Modes to represent nominal resonances
- Flexibility to viscoelastic loads associated with nominal modes

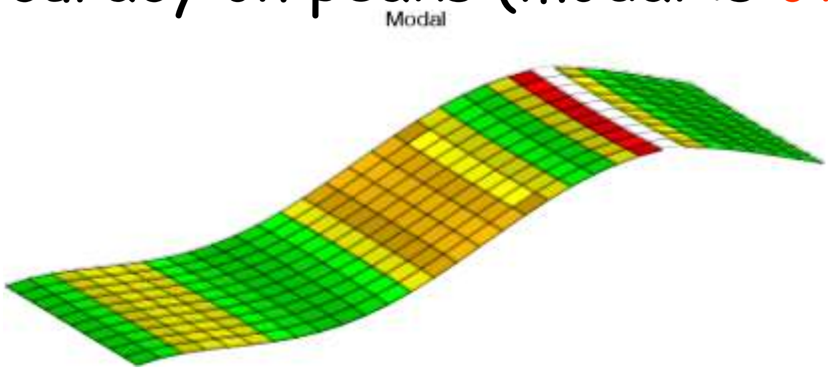
$$T = \left[\phi_{1:NM} \quad K_0^{-1}\left[Im(Z-Z_0)\right]\phi_{1:NM}\right]$$

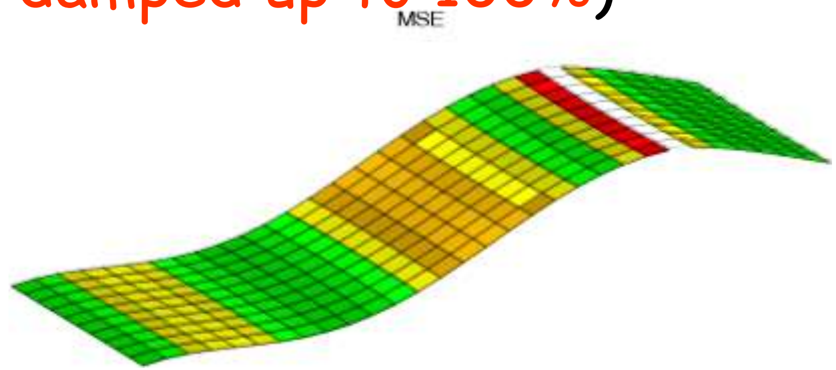
Modes static response to parametric load

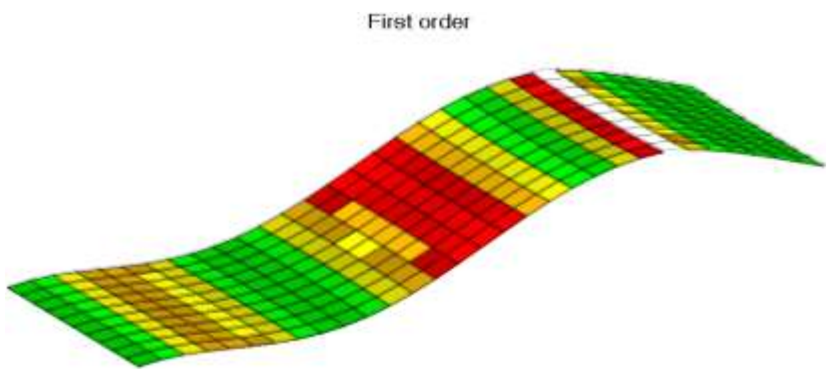
Principle of reduction (assumptions on excitation space & freq) unchanged

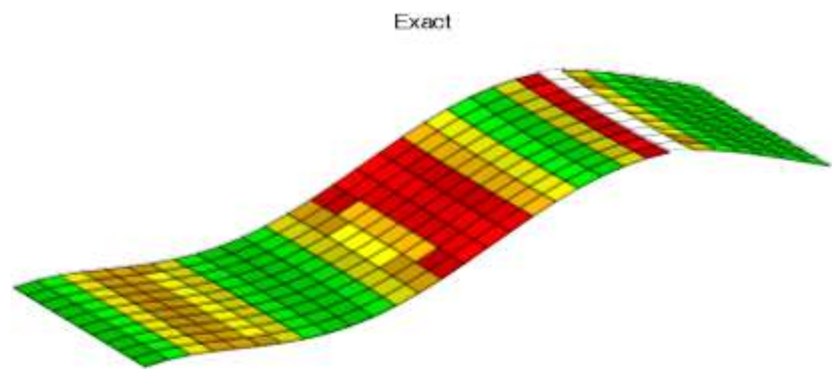
What does first order bring ?

- Correct energy distribution
- Accuracy on peaks (modal is over-damped up to 100%)

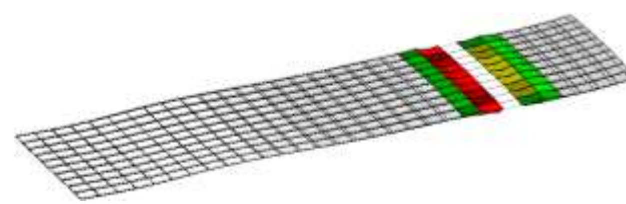




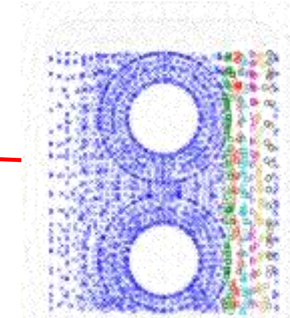




First order shape : $T = [K_o^{-1} \, [Im(Z-Zo)] \, \phi_4 \,]_{orth}$



Parametric loads & reduction



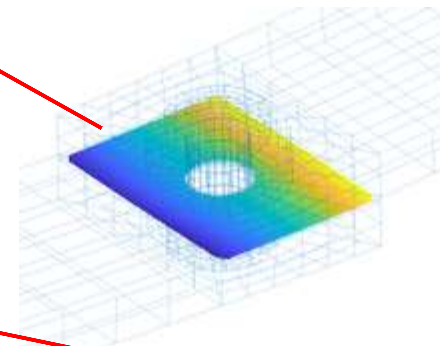
Space/time decomposition of load $[b_{Contact}]_{N\times \mathrm{Ng}}\{p(t)\}$

- Know nothing about $\{p(t)\}_{Ng}$ too large

- $\{p(t)\}$ associated with initial modes = $\left[[c_{NOR}][\phi_{1:NM}]\right]_{N\times \mathrm{NM}}\{q_r(t)\}$

  Static correction for pressure load of elastic normal modes

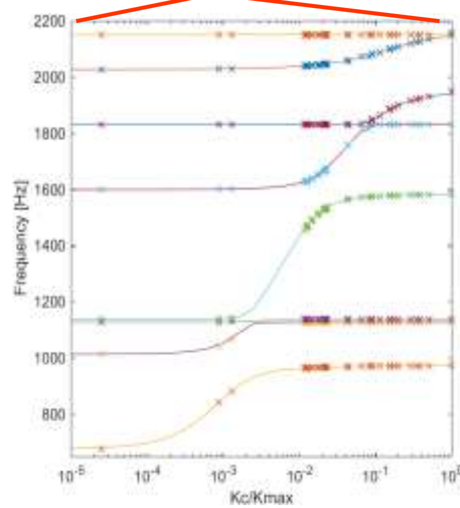
  $$T = [\phi(p_0)\ \ K^{-1}[b_c c_{NOR}\phi(p_0)]_{N\times\mathrm{NM}}]_{\perp}$$



- Multi-model learning $T = [\phi(p_1)\ \phi(p_2)_{N\times NM}]_{\perp}$

- Error control (residue iteration)

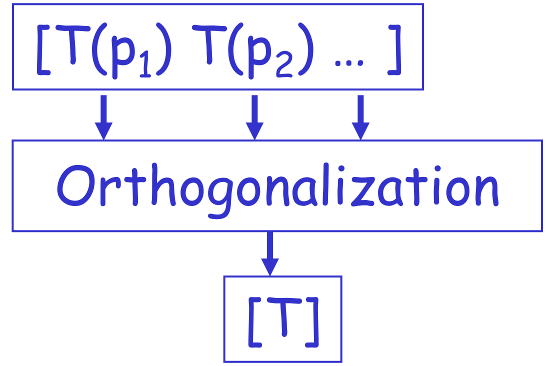
$$R_d = K_0^{-1}\left\{[M_0 s^2 + K_0]\{Tq_R\} - [b_{ext}]\{u_{ext}\} + \{f_p(Tq_R, p)\}\right\}$$



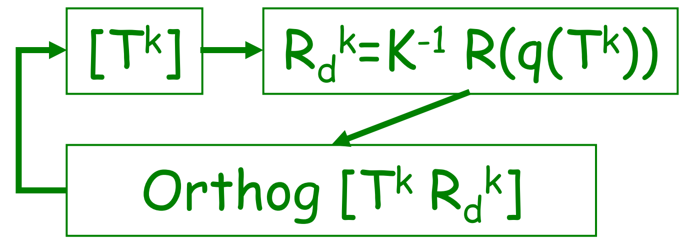
PhD A. Bobillot 2002

Bases for parametric studies

- Multi-model

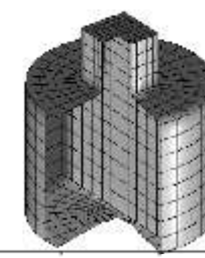


- Other + residue iteration



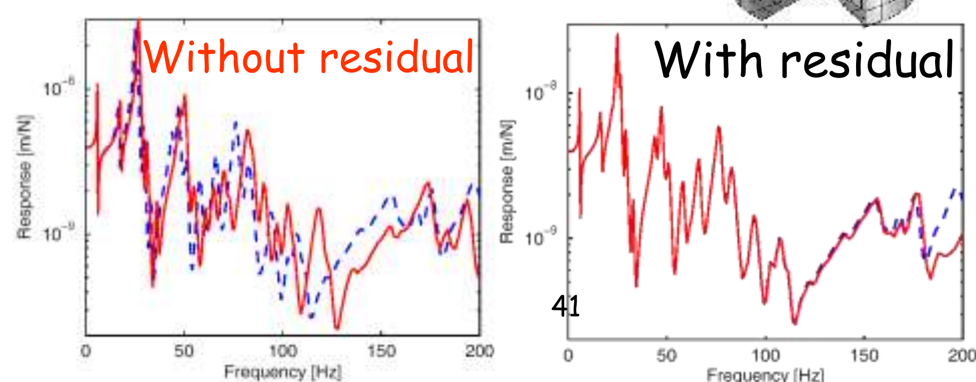
- Example : strong coupling
  With heavy fluids : modes of structure & fluid give poor coupled prediction

Example water filled tank



$$\left[\begin{bmatrix} M & 0 \\ C^T & K_p \end{bmatrix} s^2 + \begin{bmatrix} K(s) & -C \\ 0 & F \end{bmatrix}\right] \begin{Bmatrix} q \\ p \end{Bmatrix} = \begin{Bmatrix} F^{ex} \\ 0 \end{Bmatrix}$$

$$\left[\hat{T}^S\right] = \left[T^S \;\; [K_0]^{-1}[C][T^F]\right] \qquad \left[\hat{T}^F\right] = \left[T^F \;\; [F_0]^{-1}[C]^T[T^S]\right]$$



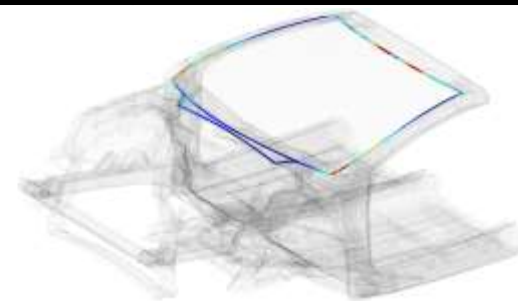
MATLAB/SDT TutoParametric

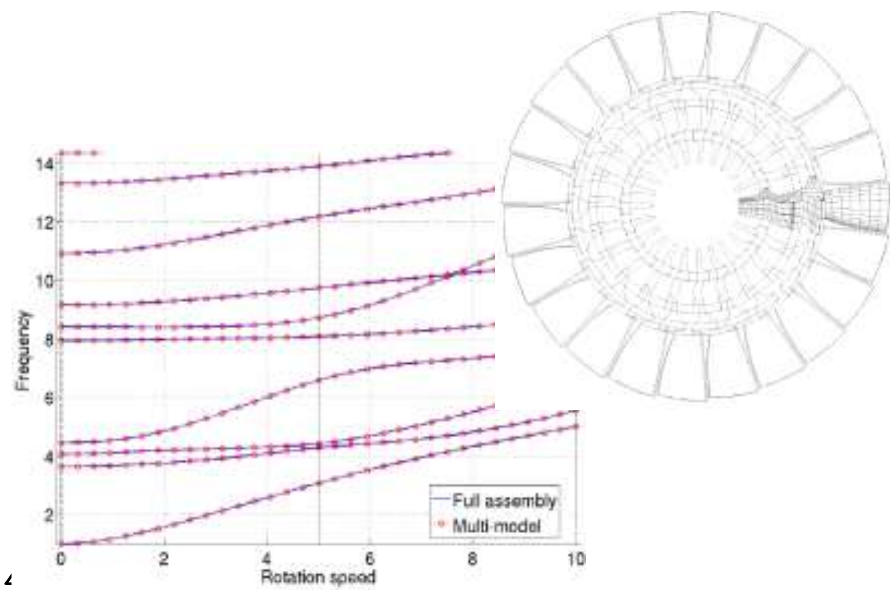
- Step 1 : Load model
- Step 2 : Multi-model reduction
- Step 3 : Analyze frequency/damping evolution
- Step 4 : Analyze MAC, use modal coordinates

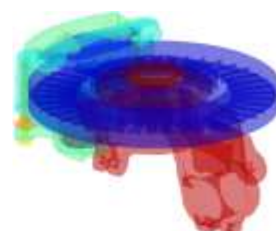
Fixed basis : enormous cost reduction

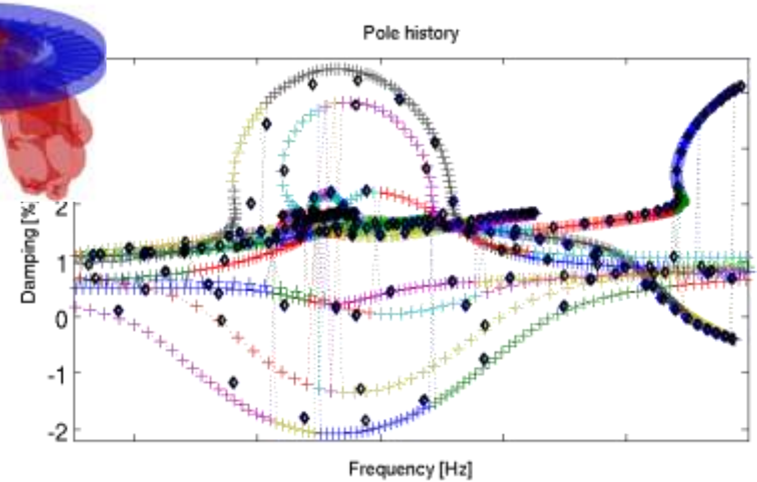
- Windshield joint complex modes at 500 design points for ½ cost of direct solver
- Campbell diagram : 200 rotations speeds for the cost of 4.
- Squeal instabilities as function of pressure : few pressures sufficient for interpolation

Ψ,λ	SOL107	2200s
Φ,ω	SOL103	300s
Ψ,λ Reduced	First order Error <4%	490s
Ψ,λ(500*T)	SOL107	~ 12 days
Ψ,λ(500*T) reduced	First order Error small	~1000s

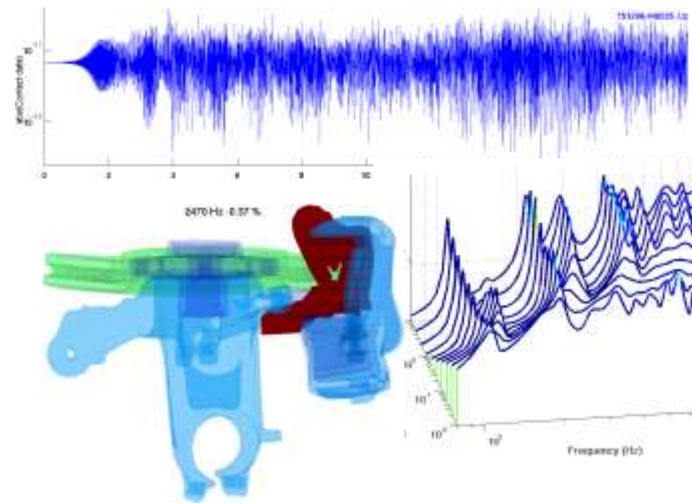








Reduction / response surface / HBM-PGD



Fixed basis reanalysis

- Response surface for system matrices

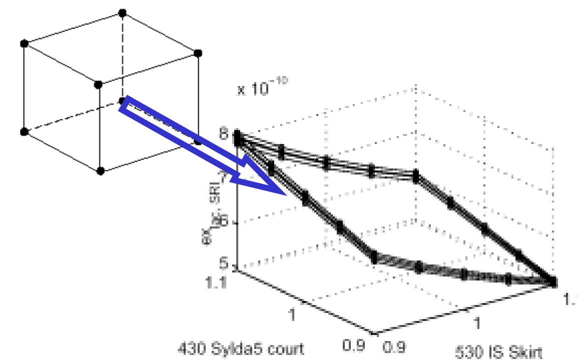
$$T^T Z(p) T \approx f(p, T^T M_i T)$$

But

- still dynamic model
- restitution $\{q\}=[T]\{q_R\}$
  provides estimates of all internal states

Response surface/meta-model methodologies

- also predict I/O relation
- but no knowledge of internal state



PGD & HBM methodologies : variable separation of higher dim

$$\{q(t,p)\} = \sum_i \{T_{i\,space}\}\{T_{i\,time}\} q_i(p)$$

Conclusions : solvers for dynamics

Continuous/discrete/reduced models (a brief reminder)

Full order model solvers

- Direct frequency resolution
- Direct time integration (implicit/explicit, first/second order, Newmark, … Gaël Chevallier)

Reduced order model + time/frequency resolution

- Basic reduction : modal superposition, static correction, Guyan, Craig-Bampton, …
- Modern vision of reduction: learning phase, basis building, DOF choice
- Substructuring
- Parametric model reduction, error control

https://savoir.ensam.eu/moodle/course/search.php?search=1874